Self-Ratings and Peer Review

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Agents have a **knowledge network**.

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A mechanism specifies a probability of getting the prize for every agent for every set of messages the principal might receive.

Two Specific Applications

Peer review processes in academia:

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- in 2013, 34% among 112 companies, including Boeing, Monsanto, PepsiCo, FedEx and Dell use 360 degree feedback (survey by 3D Group)

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How to interpret applications/self-evaluations and reports/peer-evaluations to always identify the most deserving candidate? Which “mechanism”?

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- the mechanism fully implements in expectation for every network, if communication is noisy

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Implementation with **partially honest** agents.

- Dutta and Sen (2012), no underlying knowledge network. Korpela (2014), Bayesian setting.
- This paper: knowledge network, no integer mechanisms, no incentive-compatibility.

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- Renou and Tomala (2012): implementation for different communication networks among agents and mechanism designer
- Dziubiński, Sankowski, Zhang (2016): optimal network protection when the network is unknown
- Bloch and Olckers (2018): elicit full ordinal ranking, one desirable equilibrium, incentive-compatibility
- This paper: communication network is a star with principal as center; network is public knowledge; cardinal problem, all equilibria, no incentive compatibility

Model

One Principal and Many Heterogeneous Agents

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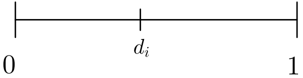

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- P's utility is maximized if and only if the prize is assigned to the *global minimum* g with $d_g = \min_{i \in N} d_i$

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- given θ_i, i's expectation that others' types are $\theta_{-i} \in \Theta'_{-i}$ is conditional probability $p(\Theta'_{-i} | \theta_i)$ derived from distribution of distances

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- the announcement of π induces a static game $\Gamma(\pi)$ among the agents

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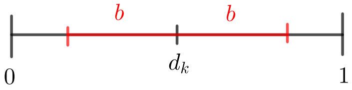

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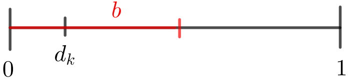

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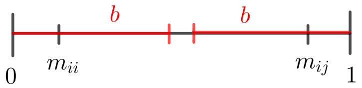

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  - agent i "proves" to be better than j if $m_{ij} - m_{ii} > 2b$
  - messages are sent privately to the principal

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Static Game $\Gamma(\pi)$ among the Agents

- agent i's action set at θ_i is message set $M_i(\theta_i) \subset M_i$
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- Goal: Design π such that π fully implements for as many L as possible.

Full Implementation

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Finally, let $\pi_i^{so} = \frac{1}{|B_2|}$ for all $i \in B_2$.

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Definition (Dominance)

A message $m_i \in M_i(\theta_i)$ is dominant at θ_i, if m_i is weakly better than any $m_i' \in M_i(\theta_i)$ for all m_{-i} which agent i believes the others can send.

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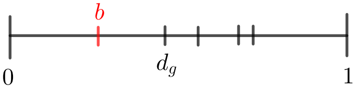

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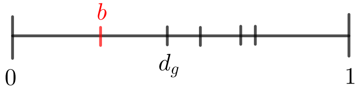

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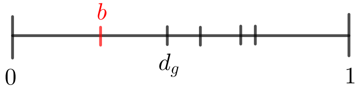

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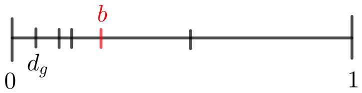

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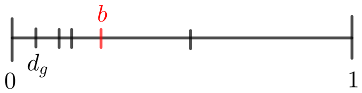

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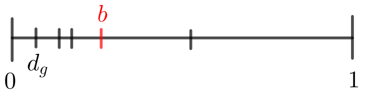

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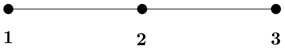

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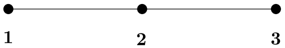

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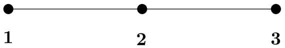

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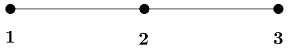

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Full Implementation with Partially Honest Agents

- Full implementation with π^{so} in L incomplete fails because agents who know they are losing are indifferent between references.
- Tie-breaking rule for this case such that references are "meaningful".
- Assume an agent only lies, if lying increases her exp. prob. of winning, otherwise she says the truth. (Dutta and Sen, 2012)
- Lexicographic preferences: Agents first care about maximizing exp. prob. of winning, and second about truth-telling.
- The game induced by π with partially honest agents and everything else as in base game is denoted by $\Gamma^h(\pi)$.

Strategy profile $\hat{m}$ is an equilibrium of $\Gamma^h(\pi)$ if and only if for all i, all θ_i and all $m_i \in M_i(\theta_i)$

1. $\Pi(\hat{m}_i(\theta_i), \hat{m}_{-i}|\theta_i) \geq \Pi(m_i, \hat{m}_{-i}|\theta_i)$, and
2. $\hat{m}_i(\theta_i) = \theta_i$, if $\Pi(\theta_i, \hat{m}_{-i}|\theta_i) \geq \Pi(m_i, \hat{m}_{-i}|\theta_i)$.

A tweak to π^{so} is needed for equilibrium existence.

If P uses π^{so} and agents are partially honest,
equilibrium existence of $\Gamma^h(\pi^{so})$ is not guaranteed for some L because of lexicographic preferences. (see paper)

$\Rightarrow$ need to adjust π^{so} to recover equilibrium existence for all L!

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Third, choose $B_3 \subseteq B_2$ such that $i \in B_3$ iff 1) or 2) is satisfied:

1. $m_{ij} - m_{ii} > 2b$ for all $j \in N_i$ (prove better than all neighbors)
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If $B_3 = \emptyset$, then "punishment allocations"... (see paper)

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Claim

For every L, strategy profile $\hat{m}^h$ is an equilibrium of $\Gamma^h(\pi^{soh})$ such that $\pi_g^{soh} = 1$ for all θ.

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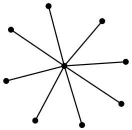
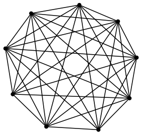
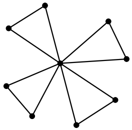
E.g. the complete graph, the star, the "Dutch windmill", ...







Full Implementation with π^{soh} is Non-Monotonic in Links

Claim

There is L which is a supergraph of the star and a subgraph of the complete graph such that π^{soh} does not fully implement in L.

There is an equilibrium such that
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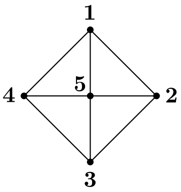

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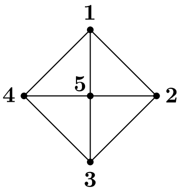

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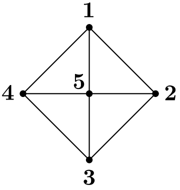

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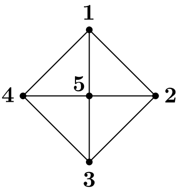

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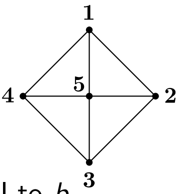

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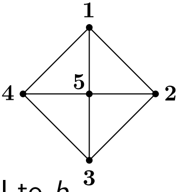

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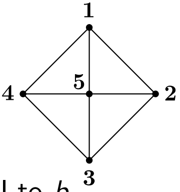

The property of full implementation not monotonic in links added.
More information and communication possibilities not always beneficial.

Conclusion

- develop mechanisms to allocate a prize to the best agent, when there are self-ratings, peer reviews and a limit to lying
- achieve full implementation for the complete network, and for a larger class of networks, if agents are partially honest
- full implementation via untruthful equilibria
  $\Rightarrow$ focus on truthful revelation is not necessary

Open questions:

- how can the principal ensure a specific limit to lying?
- other tie-breaking rules for indifference, e.g. favoritism? Imperfect knowledge about neighbors and network? ...

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    - failure of M-/ B- monotonicity implies that full implementation would be impossible in our setting, if there was no limit to lying (Maskin (1999), Jackson (1991))

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     It requires that agents' preference orderings over outcomes change across type realizations/states. In our setting, every agent always prefers a higher over a lower probability of getting the prize.
   - failure of M-/ B- monotonicity implies that full implementation would be impossible in our setting, if there was no limit to lying (Maskin (1999), Jackson (1991))
   - I use mechanisms for full implementation where agents lie in equilibrium (no truthful revelation).

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“Applications only” and “references only” fail for different states $\Rightarrow$ a mechanism which relies on both is successful!

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- thus for any m_{ik}, $\tilde{m}_{ik}$ can take any value in $[0, 1]$ with higher likelihood for values closer to m_{ik}
- If P applies the prize allocation function to $\tilde{m}$,
  then for every θ_i there is a pos. prob. that $\tilde{m}_i$ is a winning message

Full Implementation in Expectation for all L

- For any $\hat{m}_{-i}$ and any θ_i,
  m_i such that $m_{ii} = \max\{0, d_i - b\}$ and $m_{ij} = \min\{d_j + b, 1\}$ for all $j \in N_i$ uniquely maximizes i's exp. prob. of sending the best application, and receiving the min-max reference if the best application is zero.

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For any L, the unique equilibrium of $\Gamma^n(\pi^{so})$ is $\hat{m}_{ii} = \max\{0, d_i - b\}$ and $\hat{m}_{ij} = \min\{d_j + b, 1\}$ for all $j \in N_i$ and i. Thus π^{so} fully implements in expectation.

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π^{so} fully implements in expectation, because $E[\tilde{m}] = \hat{m}(\theta)$ and $\pi_g^{so}(\hat{m}(\theta)) = 1$ for all θ.

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Suppose $\hat{m}$ is an eq. and if $d_g \leq b$, then $\pi_i^s > \frac{1}{|D|}$ for some $i \in D$ with pos. prob. But then some $j \in D$ expects $\pi_i^s < \frac{1}{|D|}$ with pos. prob. ↯

Back

References Only: Formal Proof

Consider L complete.
If $d_g < 1 - b$ and $\pi_g^o < 1$,
g deviates to $m'_{gj} = \min\{d_j + b, 1\} = \bar{r}'_j > d_g + b \geq \bar{r}_g$ for all $j \neq g$ and $\pi_g^{o'} = 1$.
If $d_g \geq 1 - b$ and $\pi_i^o < \frac{1}{n}$,
i deviates to $m'_{ij} = 1 \geq \bar{r}_i$ for all $j \neq i$ and $\pi_i^{o'} \geq \frac{1}{n}$.

Back

Consensus relying on Maskin-/Bayesian-Monotonicity: Proof

- Rule 1:
  If there are no conflicting statements across all agents, allocate the prize according to the consensus.
- Rule 2:
  If $n-1$ agents claim state θ with outcome $\pi(\theta)$ and agent i claims state θ' with $\pi(\theta') \neq \pi(\theta)$,
  then P chooses $\pi(\theta)$, if i strictly prefers $\pi(\theta')$ in state θ, and $\pi(\theta')$, if i weakly prefers $\pi(\theta)$ in state θ.

Take our model with L complete. Consider a false consensus claiming θ with $\pi(\theta)$ when the true state is θ'.
For this not to be an equilibrium, there must be i who deviates to claim θ' with $\pi(\theta')$ and weakly prefers $\pi(\theta)$ in state θ. If this is the case, then i also weakly prefers $\pi(\theta)$ in state θ' and i does not deviate.

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Complete Graph: Formal Poof

Proposition

If L is complete, mechanism π^{so} *fully implements the principal's objective.*

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and if $d_g \leq b$, then $m'_{gg} = 0$ and $\bar{r}_j > d_g + b$ for all $j \neq g$.
In each case $\pi_g^{so'} = 1$ and $\hat{m}$ was not an eq.

Back

π^{so} partially implements for all other L: Formal Proof

Claim

For all L,
$\hat{m}_{ii} = \max\{d_i - b, 0\}$ *and* $\hat{m}_{ij} = \min\{d_j + b, 1\}$ *for all* $j \in N_i$*,* θ_i *and* i *is a dominant strategy equilibrium of* $\Gamma(\pi^{so})$ *such that* $\pi_g^{so} = 1$ *for all* θ*.*

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In each case $B_2 = \{g\}$.

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For some L, there is a dominant strategy equilibrium of $\Gamma(\pi^{so})$ such that $\pi_g^{so} = 0$ for some θ.

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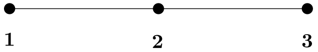

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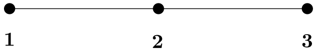

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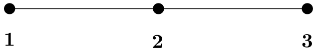

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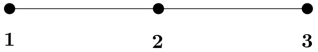

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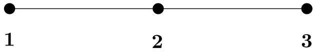

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If $d_2 > 0.8$ and $d_1, d_3 < 0.2$, then 2's best application will always be worse for any messages 1 and 3 can send, because $d_i + b < d_2 - b$ for $i = 1, 3$.

π^{so} does not fully implement for all L: Formal Proof

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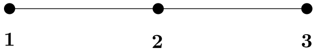

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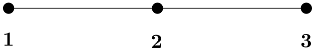

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If $d_2 > 0.8$ and $d_1, d_3 < 0.2$, then 2's best application will always be worse for any messages 1 and 3 can send, because $d_i + b < d_2 - b$ for $i = 1, 3$. Thus any message is dominant for 2 and suppose in this case $\hat{m}_{23} = d_1$ and $\hat{m}_{21} = d_3$.Then $\hat{m}$ is a dom. strat. eq. and if θ s.t. $d_1, d_3 < .2$ and $d_2 > .8$, then $\pi^{so}_g = 0$.

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