Inequality, Portfolio Choice, and the Business Cycle

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December 1 2017

CIRANO Macro Brownbag Workshop

Questions

What shocks drive large declines in aggregate quantities during the GR?

- GDP $\downarrow$ 5.6%, consumption $\downarrow$ 4.1%, and investment $\downarrow$ 19%

How can we explain such large decline in aggregate consumption?

- Challenge for the existing business cycle model
(Guerrieri and Lorenzoni (2015), Krueger et al.(2016))
- Importance of consumers's expectations about their future income. (De Nardi et al. (2012))

Are these shocks consistent with households dynamics?

- Households increase their savings rates in the PSID.
- inconsistent with consumption smoothing in a standard model
- likely reflects a strong precautionary savings motive

Overview of the paper

Goal

- Build a Quantitative DSGE heterogeneous agent model in which the precautionary savings motive strengthens in a recession
- Trace out implications for aggregates and household dynamics

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Key ingredients of the model

- Idiosyncratic earnings and unemployment risk
- Asset heterogeneity: liquid and illiquid wealth $\Rightarrow$ affects ability to smooth consumption
- Aggregate disaster risk increases precautionary savings and drives portfolio adjustment
  - Disaster shock: normal, high risk of disaster, disaster
  - In a disaster state, there is an additional fall in TFP

Main Results

Main Findings

- Baseline model consistent with cross-sectional heterogeneity replicates observed aggregate dynamics over GR
  - consumption $\downarrow$ 5.3%, and investment $\downarrow$ 22%
- Disaster risk and asset heterogeneity are both important
  - without disaster risk: consumption $\downarrow$ 2.5%
  - without asset heterogeneity: consumption $\downarrow$ 2.5%

Mechanism

- Rise in aggregate risk $\rightarrow \downarrow$ consumption and $\uparrow$ savings (*wealth effect*)
- Illiquidity in wealth weakens substitution effect
- Asset heterogeneity $\rightarrow$ portfolio adjustment toward liquid assets

Related Literature

- Households heterogeneity and the business cycle

  Krueger et al. (2016), Guerrieri and Lorenzoni (2015)

- Heterogeneity in the composition of wealth

  Glover et al. (2014), Kaplan et al. (2015), Kaplan and Violante (2014), Kaplan et al. (2016), Huo and Ríos-Rull (2014), Ahn et al. (2017), Bayer et al. (2017)

- Time-varying risk and learning

  Barro (2006), Gourio (2012), Kozlowski et al. (2017), Bayer et al. (2017)

Financial asset markets

distribution of households $\mu(j, a, b, e, \varepsilon, \xi)$

- Illiquid asset (a)
  - dividend $d(z, \mu)$, ex-dividend price $p(z, \mu)$
  - fixed adjustment cost $\xi \sim H(\xi)$
- Liquid asset (b)
  - supplied by the government at price q
- Age-varying borrowing limits $\phi \underline{b}_j$
  - A natural debt limit for age j is

$$\underline{b}_j = q\left(\underline{b}_{j+1} - x_{j+1}\right)$$

where x_j is labor income for workers and social security income for retirees.

▸ Age-varying natural debt limits

Workers and Retirees

- Workers
  - unemployment risk $e(z) \in \{0, p_e, 1\}$ with probability $\pi_e(z)$
  - unemployment benefits with a replacement of θ_u
  - earnings risk ε where $Pr(\varepsilon' = \varepsilon_k | \varepsilon = \varepsilon_l) = \pi_{lk} \geq 0$
- Retirees
  - social security benefits $s(\epsilon^{J_r-1}) = \theta_s w(z, \mu) \frac{\sum_{j=1}^{J_r-1} l(j)}{J_r-1} \varepsilon^{J_r-1}$

Partial employment

Aggregate states, Production, Investment firm and Government

- Exogenous aggregate states $z = (\eta, d)$ follows Markov chain
  - TFP shock η (AR(1))
  - Disaster shock (d): (i) normal, (ii) high risk of disaster, (iii) disaster
  - In a disaster, TFP falls by an additional λ percent only **if** $\eta = \underline{\eta}$.

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- Investment firm
  - sells shares of capital to households at $p(z, \mu)$ and pays dividend $d(z, \mu)$
  - rents capital to a production firm at a rental rate $r^k(z, \mu)$
  - faces a convex capital adjustment cost $\Phi(k', k)$

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- Government: social security benefits, unemployment benefits, interest on B, and government spending

Competitive investment firm

$$J(k, z_f, \mu) = \max_{k'} \Big((r^k(z_f, \mu) + 1 - \delta)k - (p(z_f, \mu) + d(z_f, \mu))k$$
$$+ p(z_f, \mu)k' - k' - \Phi(k', k) + \sum_{g=1}^{n_z} \pi_{fg} r(z_g, z_f, \mu) J(k', z_g, \mu') \Big)$$

- discounts future earnings by marginal rate of substitution of households $r(z_g, z_f, \mu)$.
- F.O.C is

$$p(z_f, \mu) - 1 - \Phi_1(k', k) + \sum_{g=1}^{n_z} \pi_{fg} r(z_g, z_f, \mu) D_2 J(k', z_g, \mu') = 0$$

- Benvensite and Scheinkman condition is

$$D_2 J(z_f, k, \mu) = r^k(z_f, \mu) + 1 - \delta - (p(z_f, \mu) + d(z_f, \mu)) - \Phi_2(k', k)$$

Competitive investment firm

- In equilibrium, competitive investment firm satisfies its optimality condition when

$$p(z_f, \mu) = 1 + \Phi_1(k', k)$$

and $D_2 J(k', z_g, \mu') = 0$.

- This implies

$$d(z_f, \mu) = r^k(z_f, \mu) - \delta - \Phi_1(k', k) - \Phi_2(k', k)$$

where $k' = G_k(z, \mu)$.

- Note that these prices imply $p(z_f, \mu) = 1$ and $d(z_f, \mu) = r^k(z_f, \mu) - \delta$ in a steady state

Households' problem

Discrete portfolio adjustment choice

$$v_j(a, b, e, \varepsilon_I, \xi; z, \mu) = \max\left\{ v_j^a(a, b, e, \varepsilon_I, \xi; z, \mu), v_j^n(a, b, e, \varepsilon_I; z, \mu) \right\}$$

- v_j^a : adjusting household, v_j^n : a non-adjusting household.
- Define $v_j^0(a, b, e, \varepsilon_I; z, \mu) = \int_0^{\overline{\xi}} v_j(a, b, e, \varepsilon_I, \xi; z, \mu) H(d\xi)$

Value of a household adjusting its portfolio

Epstein-Zin preferences

$$v_j^a(a, b, e, \varepsilon_l, \xi; z_f, \mu) =$$

$$\max_{c,\, a',\, b'} \left[(1-\beta)c^{1-\sigma} + \beta \left\{ \sum_{k=1}^{n_\varepsilon} \pi_{lk} \sum_{g=1}^{n_z} \pi_{fg}^z \sum_{e=0}^{1} \pi_e(z_g) v_{j+1}^0(a', b', e', \varepsilon_k; z_g, \mu')^{1-\gamma} \right\}^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}$$

subject to

$$c + q(z_f, \mu)b' + p(z_f, \mu)a' \leq b + (p(z_f, \mu) + (1-\tau_a)d(z_f, \mu))a + x - \xi$$

$$x = \begin{cases} (1-\tau_n)w(z_f, \mu)l(j)\varepsilon(e + (1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n)s(\epsilon^{J_r - 1}) & \text{otherwise} \end{cases}$$

$$b' \geq \phi \underline{b}_j, \;\; a' \geq 0, \;\; c \geq 0$$

$$\mu' = \Gamma(z_f, \mu)$$

Value of a non-adjusting household

$$v_j^n(a, b, e, \varepsilon_l; z_f, \mu) =$$

$$\max_{c,\, b'} \left[(1-\beta)c^{1-\sigma} + \beta \left\{ \sum_{k=1}^{n_\varepsilon} \pi_{lk} \sum_{g=1}^{n_z} \pi_{fg} \sum_{e=0}^{1} \pi_e(z_g) v_{j+1}^0(a', b', e', \varepsilon_k; z_g, \mu')^{1-\gamma} \right\}^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}}$$

subject to

$$c + q(z_f, \mu)b' \leq b + x$$

$$a' = (1 + (1-\tau_a)d(z_f, \mu))a$$

$$x = \begin{cases} (1-\tau_n)w(z_f, \mu)l(j)\varepsilon_l(e + (1-e)\theta_u) & \text{if } j < J_r \\ (1-\tau_n)s(\epsilon^{J_r - 1}) & \text{otherwise} \end{cases}$$

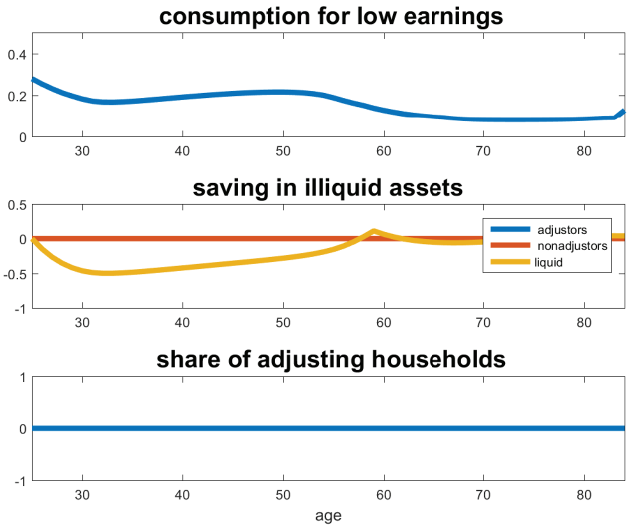
$$b' \geq \phi \underline{b}_j, \;\; c \geq 0$$

$$\mu' = \Gamma(z_f, \mu)$$

▸ Recursive Competitive Equilibrium

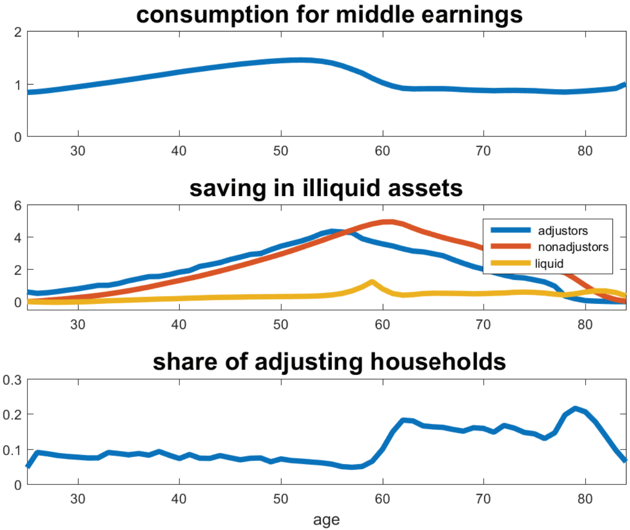
Life-cycle simulation

low earnings through working life



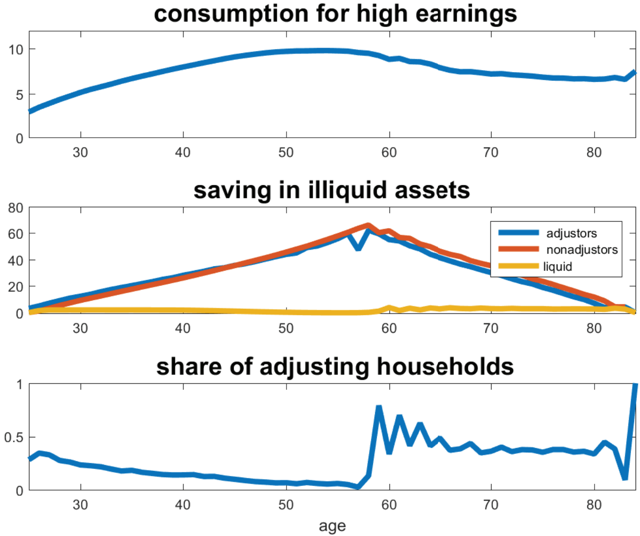
Life-cycle simulation

middle earnings through working life



Life-cycle simulation

high earnings through working life



Numerical Method

"*Given the high-frequency OLG structure, solving a ... two-asset model with aggregate shocks and asset returns determined endogenously is* **not numerically feasible**."

-Kaplan and Violante (2014)-

Two-stage approach : allows me to use EGM to solve for liquid assets.

▸ Two-stage approach

Backward Induction : applied to OLG economy with a six-dimensional distribution $\mu(j, a, b, e, \varepsilon, \xi)$ ▸ Backward Induction

- Reiter (2002, 2005)
- State-space reduction (Kim, 2017)
- vs. Continuous linear method (Ahn et al., 2017)

Calibration

household data

- Enter the labor market at age 25 and retire at age 60 (yearly model)
- fixed cost $\xi \in U(0, \overline{\xi})$ and borrowing limit $\phi \underline{b}_j$ ▸ Borrowing limits
  - total liquid asset to output ratio $= 0.3531$
  - share of households holding zero or negative net worth $= 0.10$
- β: capital (illiquid wealth) to output ratio $= 2.66$
  - productive illiquid asset = business equity + stocks + net equity in non-residential real estate $+0.4$ net housing $+ 0.4$ net consumer durables (Kaplan et al, 2016)
- unemployment shock : (i) mean (ii) median duration of unemployment, and (iii) overall unemployment rate. ▸ unemployment risk

Calibration

Earnings process and disaster shock

- idiosyncratic productivity shock (persistent + transitory) and $l(j)$ are estimated from PSID
- Epstein-Zin preferences: (CRRA, IES) $= (2, 1.5)$
- disaster shock

$$\pi^d = \begin{pmatrix} 0.909 & 0.091 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

  - The probability of disaster to happen is 3 percent. (Barro, 2006)
  - λ is chosen to match the change in expenditure rates between normal times and the GR

income shock estimation | labor market experience | Table

Distribution of wealth

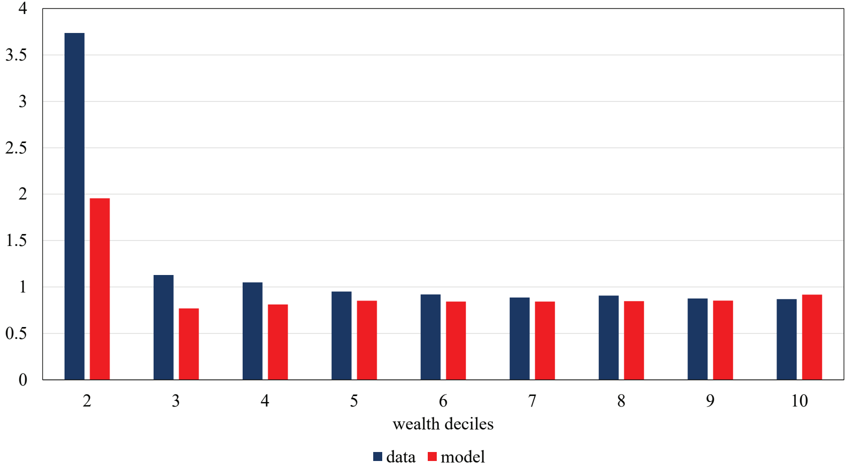
Steady state

Net worth	Q1	Q2	Q3	Q4	Q5	≤ 0	Gini
2007 SCF	-0.3	1.4	5.7	14.1	79.1	10.3	0.78
Benchmark	-0.0	3.4	9.3	18.0	69.5	13.0	0.67
Illiquid wealth	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**		**Gini**
2007 SCF	0.14	1.6	5.9	14.4	78.0		0.76
Benchmark	0.0	2.1	8.5	17.5	71.9		0.69
Liquid wealth	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**		**Gini**
2007 SCF	-11.7	-0.53	0.92	7.8	103		0.92
Benchmark	-6.8	-1.2	5.46	22.7	79.8		0.85

▸ Distribution

Share of illiquid wealth as a fraction of net worth

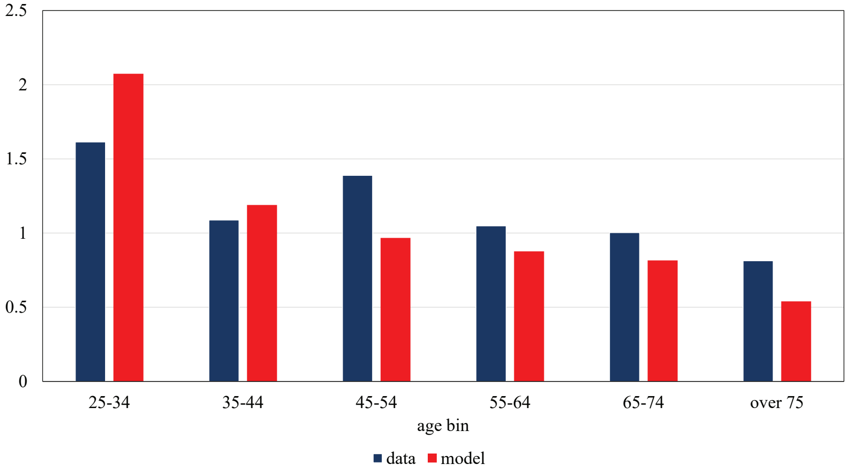
Steady state



▸ Share as a fraction of asset

Share of illiquid wealth as a fraction of net worth

Steady state



Share as a fraction of asset

Peak-to-trough declines in the Great Recession

With disaster risk

- Measured TFP fell by 2.18 percent.

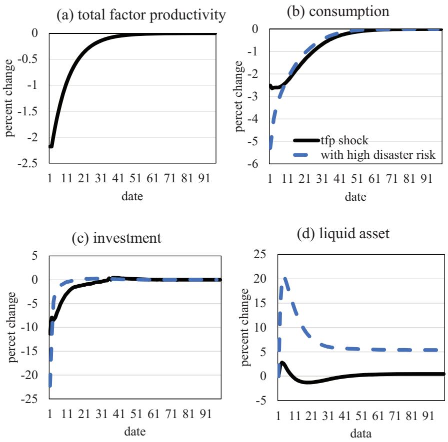
	GDP	I	N	C
data	5.59	18.98	6.03	4.08
single asset	5.71	8.99	5.58	2.38
elastic supply of liquidity	5.71	22.24	5.58	5.29

  - Unemployment risk raises unemployment rate to 10 percent.
  - Disaster risk rises

Impulse responses to TFP and disaster risk

With elastic supply of liquidity

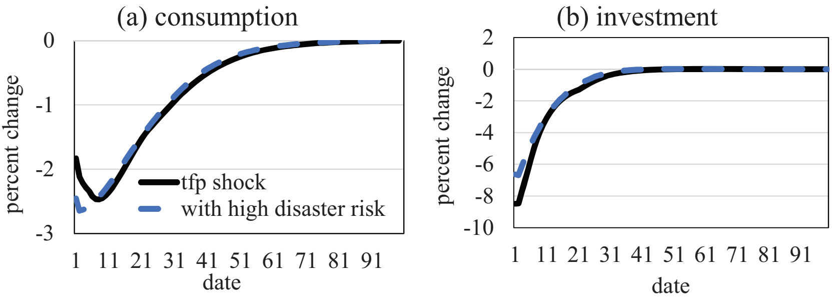
▸ Business cycle moments ▸ with fixed supply of liquid assets



Notes: y-axis measures percentage deviations from simulation mean.

Impulse responses to TFP and disaster risk

Single asset economy



- With single-asset, disaster risk changes aggregate dynamics little.

▸ Without disaster risk

- A rise in aggregate risk
  - *wealth effect*: households expect lower future income
  - consumption falls and savings increase
  - savings rate rise
- Illiquidity in wealth weakens substitution effect
  - With a single asset, households decrease consumption less in a response to a fall in the return on savings. (*substitution effect*)
  - More than 85 percent of households do not respond to the fall in the return on capital

Liquid assets to total net worth

	steady state	impact dates	
NW Quintiles	t=0	t=1	t=2
all	0.12	0.12	0.13
Q1(poor)	-3.09	0.23	0.42
Q2	0.33	0.30	0.29
Q3	0.19	0.18	0.19
Q4	0.16	0.15	0.16
Q5(wealthy)	0.09	0.09	0.10

- Higher risk of economic disaster increases precautionary savings by the poorest 20 percent of households

Data

Adjusting households

- Share of households who actively adjust their portfolio

	steady state	impact dates	
NW Quintiles	t=0	t=1	t=2
all	0.13	0.13	0.13
Q1(poor)	0.05	0.06	0.07
Q2	0.10	0.09	0.10
Q3	0.13	0.13	0.12
Q4	0.15	0.15	0.14
Q5(wealthy)	0.23	0.24	0.23

- Share of adjustors who monetize illiquid assets

	steady state	impact dates	
NW Quintiles	t=0	t=1	t=2
Q1(poor)	0.04	0.05	0.05
Q2	0.41	0.39	0.34
Q3	0.64	0.68	0.68
Q4	0.77	0.82	0.83
Q5(wealthy)	0.80	0.86	0.86

Changes in the distribution of households

Normal and recession times

Normal	Net worth		Disp. Income		Expenditure		Exp. Rate (pp)	
NW Quintiles	PSID	model	PSID	model	PSID	model	PSID	model
Q1(*poor*)	n/a	n/a	3.3	7.7	11	5.8	3.6	-1.4
Q2	25	12.9	1.9	5.2	7.1	1.5	2.3	-3.1
Q3	22	3.2	2.0	1.6	3.0	0.2	0.5	-1.3
Q4	11	1.2	2.9	-1.2	2.9	-2.7	0.0	-1.4
Q5(*wealthy*)	2.1	-0.1	1.2	-1.6	4.3	0.4	1.5	2.0
all	6.8		2.2		5.1		1.4	-0.1

Recession	Net worth		Disp. Income		Expenditure		Exp. Rate (pp)	
NW Quintiles	PSID	model	PSID	model	PSID	model	PSID	model
Q1(*poor*)	n/a	n/a	2.5	5.7	0.2	2.5	-2.3	-2.4
Q2	11	12.7	0.8	3.7	1.9	-1.0	1.0	-4.1
Q3	-5.1	2.4	-0.1	0.5	-0.2	-2.0	-0.1	-2.3
Q4	-0.5	0.1	-0.1	-2.0	-1.6	-4.5	-1.7	-2.5
Q5(*wealthy*)	-3.5	-1.3	0.3	-2.0	-2.5	-1.3	-2.8	0.7
all	-1.7		0.1		-0.1		-0.1	-1.2

Go back

Changes in the distribution of households

First difference

	Net Worth		Disp. Income		Expenditure		Exp. Rate (pp)	
NW Quintiles	PSID	model	PSID	model	PSID	model	PSID	model
Q1(*poor*)	n/a	n/a	-0.8	-2.0	-11	-3.3	-5.9	-1.0
Q2	-14	-0.2	-1.1	-1.5	-5.2	-2.5	-1.3	-1.0
Q3	-27	-0.8	-2.1	-1.1	-3.2	-2.2	-0.6	-1.0
Q4	-12	-1.1	-3.0	-0.8	-4.5	-2.5	-1.7	-0.9
Q5(*wealthy*)	-6	-1.2	-0.9	-0.4	-6.8	-1.7	-4.3	-1.3
all	-5.1		-2.1		-5.2		-1.5	-1.1

- Slowdown in the growth of net worth, disposable income, and consumption during the GR
- A marked fall in consumption rates during the GR
  $\Rightarrow$ hard to generate in a standard model!
  $\Rightarrow$ explained by a rise in disaster risk with an elastic supply of liquidity

▸ Data ▸ without rising disaster risk

Concluding Remarks

- A rise in risk and illiquidity in wealth are important to explain
  - the large falls in consumption and investment in the GR
  - consistent with a rise in savings rates in the GR
- Illiquidity in wealth weakens substitution effect that accompanies a fall in the expected return to high-yield assets
  - Negative wealth effect is a driving force to explain a sharp drop in aggregate consumption
- Consumption response is driven by wealth poor households who raise their precautionary savings in liquid assets.

The end

Natural debt limit in OLG economy

Borrowing limits are a common percentage of age-varying natural debt limits $\phi \underline{b}_j$

At last age, borrowing is not allowed, $\underline{b}_J = 0$.

Given natural debt limit $\underline{b}_{j+1}$ and lowest possible earnings x_{j+1} conditional on labor endowment, a natural debt limit for age j is

$$\underline{b}_j = q\left(\underline{b}_{j+1} - x_{j+1}\right)$$

where x_j is labor income for workers and social security income for retirees.

Age-specific natural debt limits allow households to borrow against their future income.

Go back

Idiosyncratic unemployment shocks

Khan (2016)

- Working hours $e(z) \in \{0, p_e, 1\}$

$$\text{a full-time worker} : 1 - \pi_u(z)$$
$$\text{a part-time worker} : \pi_u(z)\pi_p(z)$$
$$\text{unemployed} : \pi_u(z)(1 - \pi_p(z))$$

- Households can be partially employed for a fraction $p_e(z)$ of a model period (partially unemployed)
- $\pi_e(z)$ changes with an aggregate state to increase unemployment rate and duration of unemployment in a recession
- Partially and fully unemployed workers receive unemployment benefits proportional to their possible earnings with a replacement rate of θ_u

Go back

Recursive Competitive Equilibrium

1. Households maximize their utility
2. The government budget is balanced
3. Markets clear
4. Prices are competitively determined

$$w(z,\mu) = (1-\alpha)(1-\lambda(d))\eta k^{\alpha} n^{-\alpha}$$
$$r^k(z,\mu) = \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha}$$
$$p(z,\mu) = 1 + \Phi_1(G_k(z,\mu), k)$$
$$d(z,\mu) = \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha} - \delta - \Phi_1(G_k(z,\mu), k) - \Phi_2(G_k(z,\mu), k)$$

   where $G_k(z,\mu)$ is the aggregate law of motion for aggregate capital.

5. Distribution of households $\mu' = \Gamma(z,\mu)$

▸ Resource constraint ▸ Go back

Recursive Competitive Equilibrium

1. Households maximize their utility
2. The government budget is balanced

$$G(z,\mu) + B_s + \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}}(1-\tau_n)(s(\varepsilon_l \mathbf{1}_{j\geq J_r}) + (1-e)\theta_u wl(j)\varepsilon \mathbf{1}_{j<J_r})\mu(j, da, db, e, \varepsilon_l)$$

$$= \tau_a d(z,\mu)k + \tau_n w(z,\mu)n + q(z,\mu)B_s^{'}$$

3. Markets clear

$$n(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} l(j)\varepsilon_l e\mu(j, da, db, e, \varepsilon_l)$$

$$k(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} a\mu(j, da, db, e, \varepsilon_l)$$

$$B(z,\mu) = \sum_{e=0}^{1}\sum_{j=1}^{J}\sum_{l=1}^{n_\varepsilon}\int_{\mathbf{A}}\int_{\mathbf{B}} b\mu(j, da, db, e, \varepsilon_l)$$

Recursive Competitive Equilibrium

4 Prices are competitively determined

$$
\begin{aligned}
w(z,\mu) &= (1-\alpha)(1-\lambda(d))\eta k^{\alpha} n^{-\alpha} \\
r^k(z,\mu) &= \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha} \\
p(z,\mu) &= 1 + \Phi_1(G_k(z,\mu),k) \\
d(z,\mu) &= \alpha(1-\lambda(d))\eta k^{\alpha-1} n^{1-\alpha} - \delta - \Phi_1(G_k(z,\mu),k) - \Phi_2(G_k(z,\mu),k)
\end{aligned}
$$

where $G_k(z,\mu)$ is the aggregate law of motion for aggregate capital. Φ_1 and Φ_2 are the derivatives of Φ with respect to G_k and k, respectively.

5 Distribution of households

$$
\mu'(j+1, A_0, B_0, e', \varepsilon_k) = \pi_{e'}(z) \sum_{l=1}^{n_\varepsilon} \pi_{lk} \left(\int_{\Delta_1} \mu(j, da, db, e, \varepsilon_l) H(d\xi) + \int_{\Delta_2} \mu(j, da, db, e, \varepsilon_l) H(d\xi) \right) \quad \forall\, j
$$

where $\Delta_1 =$

$\{(a,b,e,\varepsilon_l,\xi) | h^a(j,a,b,e,\varepsilon_l,\xi;z,\mu) \in A_0,\ \ b^a(j,a,b,e,\varepsilon_l,\xi;z,\mu) \in B_0$ and $\chi(j,a,b,e,\varepsilon_l,\xi;z,\mu) = 1\}$ and

$\Delta_2 = \{(a,b,e,\varepsilon_l,\xi) | h^n(j,a,b,e,\varepsilon_l;z,\mu) \in A_0,\ \ b^n(j,a,b,e,\varepsilon_l;z,\mu) \in B_0$ and $\chi(j,a,b,e,\varepsilon_l,\xi;z,\mu) = 0\}$,

$(j,a,b,e,\varepsilon_l) \in S$ and $\xi \in \mathbf{\Xi}$.

Recursive Competitive Equilibrium

Resource constraint

- Abstracting from government and liquid assets, aggregate budget constraint for all adjusting households

$$c_a + p(z,\mu)k_a' \leq w_a(z,\mu) + (p(z,\mu) + d(z,\mu))k_a - \xi_a$$

$$c_a + (1 + \Phi_1(k',k))k_a' \leq w_a(z,\mu) + (\alpha\eta k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_2(k',k))k_a - \xi_a$$

- Aggregate budget constraint for all non-adjusting households

$$c_n \leq w_n(z,\mu)$$

$$k_n' = (1 + \alpha\eta k^{\alpha-1}n^{1-\alpha} - \delta - \Phi_1(k',k) - \Phi_2(k',k))k_n$$

- Imposing $x_a + x_n = x$ and $\Phi_1(k',k)k' + \Phi_2(k',k)k = \Phi(k',k)$,

$$c + k' + \Phi(k',k) \leq y + (1-\delta)k - \xi_a$$

Go back

Decision rules: Golden section search + EGM

Define $v_j^m(m, a', e, \varepsilon_i)$ as the intermediate value defined over cash-on-hand, m

The illiquid asset problem (Golden Section Search)

$$v_j(a, b, e, \varepsilon_i, \xi) = \max\left\{\max_{0 \le a' \le m} v_j^m(m - pa', a', e, \varepsilon_i), v_j^m(x_i + b, (1 + (1-\tau_a)d) a, e, \varepsilon_i)\right\}$$

subject to

$$m = x_i(j, \varepsilon, e) + (p + (1-\tau_a)d) a + b - \xi$$

The consumption and liquid wealth problem (EGM)

$$v_j^m(m, a', e, \varepsilon_i) = \max_{b'}\left(u(c) + \beta v_{j+1}^0(a', b', e, \varepsilon_i)\right)$$

subject to

$$c + qb' \le m$$

Go back

Aggregate solution

Let $z = \{z_1, ..., z_{n_z}\}$ be the grid for aggregate state and $m = \{m_1, ..., m_{n_m}\}$ be the grid for an approximate aggregate state

1. Choose the full *reference distribution* $r^{\mu}(j, a, b, \varepsilon; z, m)$
2. State-space reduction (Kim, 2017): aggregate full *reference distribution* into a small subset of age and idiosyncratic type groups and save weights for this mapping.

$$\omega_0(j, a, b, \varepsilon, \tilde{\varepsilon}; z, m) : r_0^{\mu}(a, b, \tilde{\varepsilon}; z, m) \rightarrow r^{\mu}(j, a, b, \varepsilon; z, m)$$

3. Choose a DSF which gives the *proxy distribution*, $p_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$
   - minimizes the distance to the reduced distribution $r_0^{\mu}(a, b, \tilde{\varepsilon}; z, m)$
   - satisfies moment consistency constraints
4. Using weights in (2), restore the full proxy distribution over age and idiosyncratic shocks, $p^{\mu}(j, a, b, \varepsilon; z, m)$.

Aggregate solution

5. Simultaneously solve for households' decision rules and an intrameporally consistent future approximate aggregate m'.
   - Guess the aggregate law of motion $G_k(z, m)$ for approximate aggregate states
   - Solve for decision rules and value functions backwards by age over aggregate states
   - Compute the end-of-period aggregate state m' and update $G_k(z, m)$
   - Iterate until $G_k(z, m)$ converges.
6. Simulate the model to update the reference distribution in (1).
7. Iterate (1)-(6) until no additional accuracy is achieved.

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Calibration: household data

Unemployment risk (CPS data)

- unemployment shock $e(z) \in \{0, p_e, 1\}$ with $\pi_e \in [\overline{\pi}_e - \varepsilon_e, \overline{\pi}_e + \varepsilon_e]$
  - p_e: median unemployment duration of 12 weeks (1981-2016)
  - $(\overline{\pi}_p, \varepsilon_p)$: mean unemployment duration of 25 weeks (1981-2016) and the rise in the mean duration to 36 weeks after 2008
  - $(\overline{\pi}_u, \varepsilon_u)$: 5 percent unemployment rate and an additional 5 percentage point rise in unemployment rate over the Great Recession
  - θ_u : 43.5 percent replacement rate of unemployment benefits (Nakajima, 2012)

Go back

Estimation of earning shock process

Source: 1968 -2011 PSID data

- OLS regression

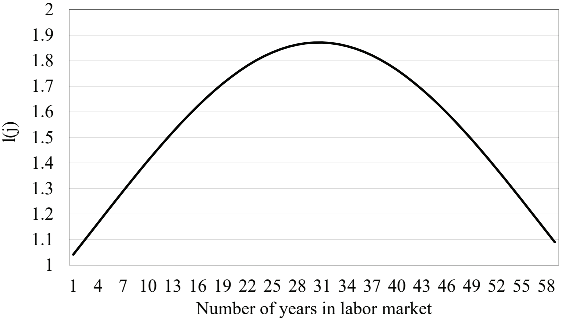
$$\log w_{i,j,t} = \beta_{t,0} + \beta_{t,1} D_i^e + \beta_3 \theta + \beta_4 \theta^2 + \widehat{\varepsilon}_{i,j,t}$$

  - $w_{i,j,t}$: labor earnings of sample i with age of head j in year t

- Minimum distance estimation
  - persistent and transitory shock variances $\{\sigma^2_{s_t}, \sigma^2_{v_t}\}$
  - persistence $\{\rho\}$
  - variance of persistent shocks for initial age $\{\sigma^2_{\pi}\}$

Go back

Labor market experience premium



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Summary of parametrization

Parameters set externally				Value
α		capital share of output(NIPA)		0.36
δ		depreciation rate(NIPA)		0.069
τ_n, τ_a		labor and capital income taxes		0.27, 0.25
(ρ_η, σ_η)		TFP shock process (Khan and Thomas, 2013)		(0.909, 0.014)
γ		coefficient of relative risk aversion		2.0
σ		inverse of EIS		$\frac{1}{1.5}$
$\lambda(d=3)$		additional drop in tfp a in disaster state		0.6
θ_s		replacement rate of avg pre-tax earnings for social security		0.45
θ_u		replacement rate of avg pre-tax earnings for unemployment benefit		0.435
$(\sigma_s^2, \sigma_v^2, \rho_\varepsilon)$		earnings shock process		(0.0512, 0.1143, 0.9776)
$l(j)$		male hourly wage life-cycle profile		see text
Parameters calibrated	Value	moments to match	data	model
β	0.942	capital to output ratio	2.66	2.66
p_e	0.2287	median unemployment duration as a fraction of a model period	0.2287	0.2287
π^d	see text	transition probability matrix for disaster state		
$\pi_e(z)$	see text	unemployment rate		
ξ	1.7	share of liquid asset to output	0.3531	0.35
ϕ	0.2	share of hhs holding zero or negative net worth	0.103	0.13

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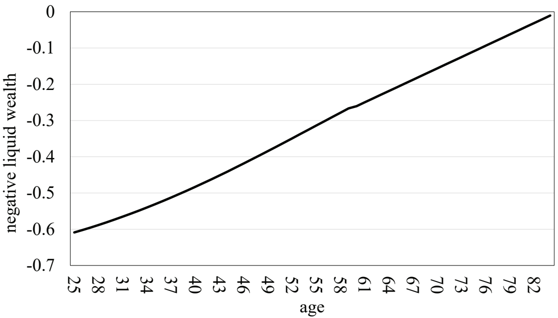
Borrowing limits



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Recession without rising disaster risk

Changes in growth rates

	Net Worth			Earning			Income			Expend.			Exp. Rate (pp)		
Quintile	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)	PSID	(1)	(2)
Q1(*poor*)	n/a	n/a	n/a	-1.2	-1.9	-1.9	-0.8	-1.9	-1.9	-11	-1.6	-1.7	-5.9	0.2	0.1
Q2	-14	-1.0	-2.0	-0.6	-1.6	-1.5	-1.1	-1.6	-1.6	-5.2	-1.2	-1.3	-1.3	0.2	0.2
Q3	-27	-0.8	-0.9	-2.1	-1.4	-1.3	-2.1	-1.3	-1.4	-3.2	-1.1	-1.2	-0.6	0.2	0.1
Q4	-12	-0.9	-1.0	-2.0	-1.2	-1.2	-3.0	-1.2	-1.3	-4.5	-0.9	-0.9	-1.7	0.3	0.4
Q5(*wealthy*)	-6	-0.9	-1.0	-2.6	-1.0	-1.0	-0.9	-1.1	-1.2	-6.8	-0.8	-0.9	-4.3	0.4	0.4

(1) elastic supply of liquidity(2) fixed supply of liquid assets

Go back

Business cycle statistics with disaster

- elastic supply of liquidity

$x =$	Y	C	I	K	B_s	N	$E(r)$	r_f	w
mean(x)	2.01	1.50	0.40	5.31	0.66	1.17	0.07	0.0	1.09
σ_x/σ_y	(9.36)	0.44	1.77	0.17	1.27	0.25	0.1	n/a	0.94
$corr(x,y)$	1.0	0.91	0.89	-0.19	-0.16	0.43	-0.76	n/a	0.97

- fixed supply of liquid assets

$x =$	Y	C	I	K	B_s	N	$E(r)$	r_f	w
mean(x)	2.01	1.50	0.40	5.29	0.73	1.17	0.08	0.01	1.09
σ_x/σ_y	(9.36)	0.57	2.03	0.17	n/a	0.25	0.21	0.23	0.94
$corr(x,y)$	1.0	0.99	0.98	-0.17	n/a	0.43	-0.82	-0.83	0.97

Business cycle statistics with disaster

- elastic supply of liquidity

$x =$	$E(r) - r_f$	adjusting pop	$\frac{B}{K+B}$
mean(x)	0.075	0.130	0.110
σ_x/σ_y	0.1	0.219	1.191
$corr(x, y)$	-0.76	0.662	-0.133

- fixed supply of liquid assets

$x =$	$E(r) - r_f$	adjusting pop	$\frac{B}{K+B}$
mean(x)	0.069	0.128	0.123
σ_x/σ_y	0.023	0.180	0.155
$corr(x, y)$	0.528	0.321	0.206

Go back

Business cycle statistics without disaster

- Two asset economy (r_f fixed)

$x =$	Y	C	I	K	B	N	$E(r)$	r_f	w
mean(x)	2.04	1.53	0.40	5.37	0.65	1.17	0.08	0.0	1.11
σ_x/σ_y	(2.76)	0.43	1.7	0.18	0.23	0.86	0.04	n/a	0.29
$corr(x, y)$	1.0	0.98	0.98	-0.00	-0.51	0.96	0.88	n/a	0.60

- Two asset economy (B fixed)

$x =$	Y	C	I	K	B	N	$E(r)$	r_f	w
mean(x)	2.04	1.53	0.40	5.37	0.73	1.17	0.07	0.0	1.11
σ_x/σ_y	(2.75)	0.40	1.78	0.20	n/a	0.86	0.05	0.02	0.29
$corr(x, y)$	1.0	0.96	0.99	-0.03	n/a	0.96	0.91	0.78	0.60

Go back

Business cycle statistics without disaster

- Two asset economy (r_f fixed)

$x =$	adjusting pop	$\frac{B}{K+B}$
mean(x)	0.130	0.109
σ_x/σ_y	0.214	0.235
$corr(x,y)$	-0.209	-0.433

- Two asset economy (B fixed)

$x =$	adjusting pop	$\frac{B}{K+B}$
mean(x)	0.128	0.12
σ_x/σ_y	0.205	0.168
$corr(x,y)$	-0.283	0.036

Go back

PSID data

- Construct a panel of taxable earnings, income, wealth and expenditure data using PSID.
  - Net worth = assets - debt
  - Expenditure is total spending on nondurable goods and services.
- Using this panel, I document joint distribution of these variables before and during the GR (Krueger et al, 2016)
- I follow the same households in each wealth quintile and calculate growth rates as the change in the average of each variable.

PSID data

sample selection

- SRC sample (KMP used both SRC and SEO sample) - id number is less than 3000 in the year of 1968
- drop the samples with head and wife who have positive income but zero hours worked
- expressed in 2013 dollars
- income is not from self-employment
- hourly wage is more than half of the minimum wage
- drop if any item in wealth is missing.
- drop sample with wealth less than equal to -99 million dollars

Households data

2007-2009 SCF panel

- Net worth = total assets - total debt
- Illiquid wealth: stocks, business equity, net residential property, net equity in non-residential real estate and net consumer durables.

2007-2009 PSID data

- total expenditure is total spending on nondurable goods and services.

Year	Gini	top 1%	5%	10%	50%	90%	≤ 0
2007 SCF	0.78	29.1	52.3	64.3	96.8	100	10.3
2007 PSID	0.76	25.8	47.9	62.1	96.3	101	10.2
2009 SCF	0.79	29.8	53.2	65.5	98.2	101	14.8
2009 PSID	0.79	28.5	50.1	64.6	98.1	101	14.3

Go back

Impulse responses with disaster risk

With fixed stock of liquid asset

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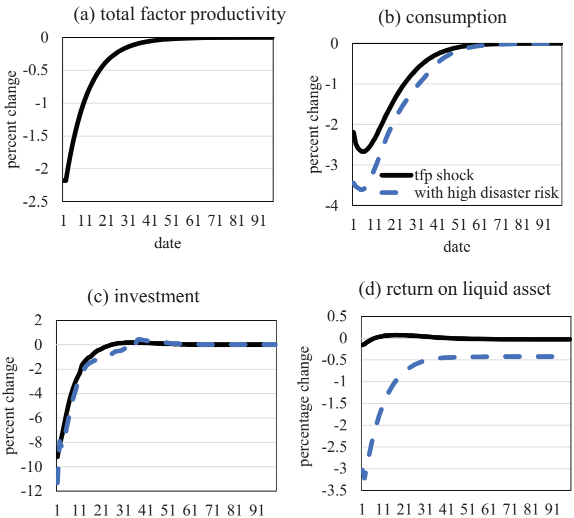
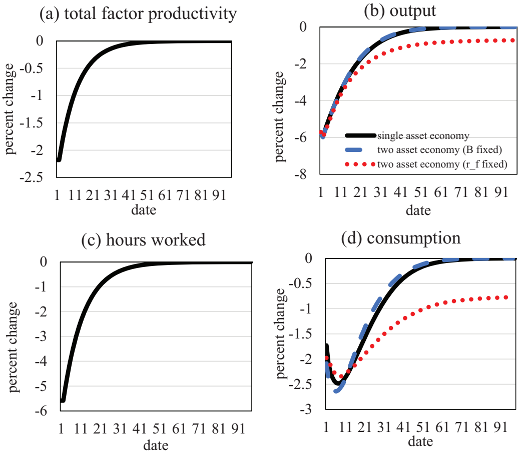
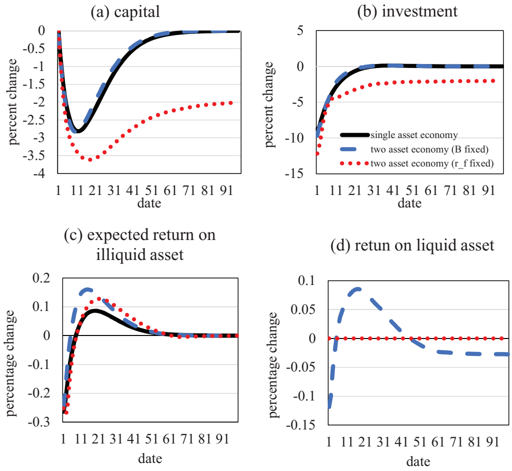


Impulse responses without disaster risk



Notes: Y-axes measure percent deviations from simulation mean.

Impulse responses without disaster risk



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Distribution of wealth

Steady state

Net worth	Q1	Q2	Q3	Q4	Q5	top 1%	5%	10%	≤ 0	Gini
2007 SCF	-0.3	1.4	5.7	14.1	79.1	29.1	52.3	64.3	10.3	0.78
Benchmark	-0.0	3.4	9.3	18.0	69.5	9.4	32.8	51.3	13.0	0.69
Illiquid wealth	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**top 1%**	**5%**	**10%**		**Gini**
2007 SCF	0.14	1.6	5.9	14.4	78.0	28.2	51.2	63.2		0.76
Benchmark	0.0	2.1	8.5	17.5	71.9	9.8	34.9	53.9		0.69
Liquid wealth	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**top 1%**	**5%**	**10%**		**Gini**
2007 SCF	-11.7	-0.53	0.92	7.8	103	47.1	76.2	90.5		0.92
Benchmark	-6.8	-1.2	5.46	22.7	79.8	11.5	36.7	55.1		0.85

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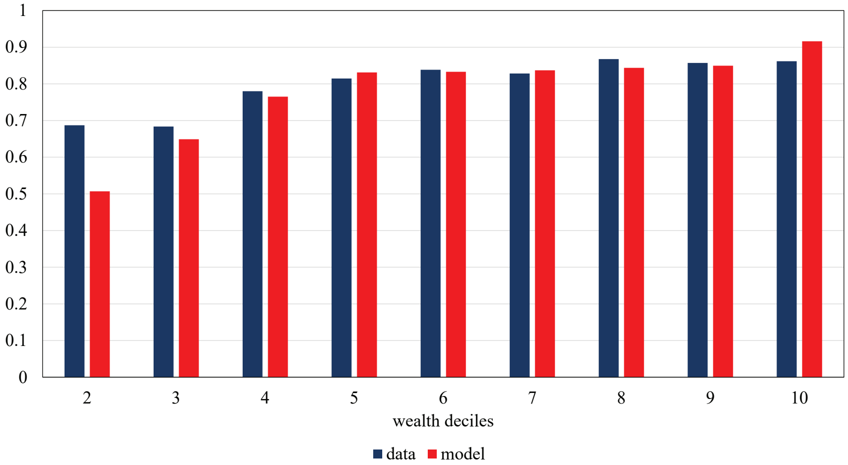
Share of illiquid wealth as a fraction of total asset

Steady state



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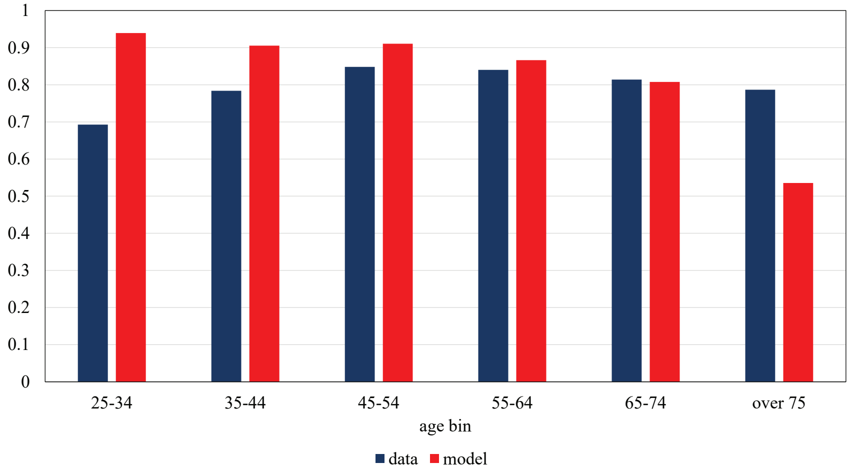
Share of illiquid wealth as a fraction of total asset

Steady state



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Sensitivity Analysis

$$(1)\ \pi^d = \begin{pmatrix} 0.909 & 0.091 & 0 \\ 0.25 & 0.5 & 0.25 \\ 0 & 0.5 & 0.5 \end{pmatrix}$$

$$(2)\ \pi^d = \begin{pmatrix} 0.909 & 0.091 & 0 \\ 0.25 & 0.65 & 0.1 \\ 0 & 0.091 & 0.909 \end{pmatrix}$$

	I	C
baseline	19.89	3.82
(1) low persistence in disaster	18.64	3.02
(2) low probability of disaster	12.37	2.75

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Liquid assets to total net worth

	steady state	impact dates		SCF	
NW Quintiles	t=0	t=1	t=2	07	09
all	0.12	0.12	0.13	0.12	0.15
Q1(poor)	-3.09	0.23	0.42	-1.94	-3.61
Q2	0.33	0.30	0.29	-0.05	-0.08
Q3	0.19	0.18	0.19	0.07	0.02
Q4	0.16	0.15	0.16	0.10	0.10
Q5(wealthy)	0.09	0.09	0.10	0.14	0.18

- Higher risk of economic disaster increases precautionary savings by the poorest 20 percent of households

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