A Delegation Approach to Persuasion

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Introduction

- A *monotone persuasion problem* is the Bayesian persuasion problem of Kamenica-Gentzkow (2011) with
  - Interval state space
  - Monotone partitional experiments

Why Monotone Persuasion?

- Monotone experiments constitute an important subclass of persuasion mechanisms:
  - credit rating of financial institutions
  - consumer rating of services on AirBnB, Tripadvisor, Uber,...
  - hygiene certification of restaurants
  - grade conversion schemes from 100-point to ABC scale
- Two defining features of monotone experiments: Determinism and Monotonicity
- Conditions for optimality of monotone experiments: Ivanov (2016), Mensch (2016), Dworczak-Martini (2017), Inostroza-Pavan (2017)

Preview of Results

- We show equivalence of a monotone persuasion problem and a constrained delegation problem

- Why is it interesting?
  - Delegation problem is more intuitive and better understood
  - There are developed techniques how to address and solve delegation problems

Outline

- Description of monotone persuasion and constrained delegation problems

- Equivalence result

- Sketch of proof

- Illustration of how the existing techniques in delegation can be applied to address the persuasion problem

A Problem

- Principal (she) and Agent (he)

- Agent must make a decision $y \in [0, 1]$

- Payoffs depend on the state $\omega \in [0, 1]$

- No one observes ω; its distribution F is common knowledge

Payoffs

- Agent's and Principal's payoffs, $U(\omega, y)$ and $V(\omega, y)$, are twice continuously differentiable

- Agent's payoff function satisfies

$$\left.\frac{\partial}{\partial y} U(\omega, y)\right|_{y=\omega} = 0, \quad \frac{\partial^2}{\partial y^2} U(\omega, y) < 0, \quad \text{and } \frac{\partial^2}{\partial \omega \partial y} U(\omega, y) > 0.$$

- Distribution of states F admits a positive density f

- A triple (U, V, F) is called a *primitive*

- $\mathcal{P}$ is the set of primitives that satisfy the above assumptions

Monotone Persuasion Problem

- Principal chooses a monotone experiment $\pi : [0,1] \to \mathbb{R}$, where π is nondecreasing

- W.l.o.g., we focus on **diagonalized** experiments:
  $\pi(\omega) = \inf\{t : \pi(t) = \pi(\omega)\}$ and $\pi(1) = 1$

- Denote by Π^* the set of monotone diagonalized experiments

- Given a message m of an experiment π, Agent chooses

$$y^*_\pi(m) \in \arg\max_{y \in [0,1]} \mathbb{E}[U(\omega, y) \mid \pi(\omega) = m]$$

- Principal's problem:

$$\max_{\pi \in \Pi^*} \mathbb{E}[V(\omega, y^*_\pi(\pi(\omega)))]$$

Constrained Delegation Problem

- Principal chooses a compact subset $X \subset [0,1]$ of decisions such that X contains extreme decisions $\{0,1\}$

- Denote by $\mathcal{X}^*$ the set of all such delegation sets

- Agent observes $\tilde{\omega}$, and then chooses a decision from X

$$y^*_X(\tilde{\omega}) \in \arg\max_{y \in X} \tilde{U}(\tilde{\omega}, y)$$

- Principal chooses a delegation set $X \in \mathcal{X}^*$ to maximize her expected payoff

$$\max_{X \in \mathcal{X}^*} \mathbb{E}[\tilde{V}(\tilde{\omega}, y^*_X(\tilde{\omega}))]$$

Constrained Delegation Problem: An Interpretation

- A contractual relationship between Principal and Agent: Agent can always keep the contract unchanged or terminate the contract, but any other alterations must be permitted by Principal

Main Result

The monotone persuasion problem and the constrained delegation problem are “equivalent.”

Equivalence

- Consider a one-to-one mapping $\mu : \Pi^* \to \mathcal{X}^*$ that maps each experiment π into a unique delegation set $X = \mu(\pi)$.

- Primitives (U, V, F) and $(\tilde{U}, \tilde{V}, \tilde{F})$ are equivalent under μ,

$$(U, V, F) \sim_\mu (\tilde{U}, \tilde{V}, \tilde{F}),$$

  if, for all $\pi \in \Pi^*$,

$$\mathbb{E}_F\Big[V(\omega, y^*_\pi(\omega))\Big] = \mathbb{E}_{\tilde{F}}\Big[\tilde{V}(\tilde{\omega}, y^*_{\mu(\pi)}(\tilde{\omega}))\Big].$$

Equivalence

Theorem: Let

$$\mu(\pi) = \pi([0,1]).$$

Consider any primitives $(U, V, F) \in \mathcal{P}$ and $(\tilde{U}, \tilde{V}, \tilde{F}) \in \mathcal{P}$.

If, for all $(\omega, \tilde{\omega}) \in [0,1]^2$,

$$U_2'(\omega, \tilde{\omega}) f(\omega) = -\tilde{U}_2'(\tilde{\omega}, \omega) \tilde{f}(\tilde{\omega}),$$
$$V_2'(\omega, \tilde{\omega}) f(\omega) = -\tilde{V}_2'(\tilde{\omega}, \omega) \tilde{f}(\tilde{\omega}),$$
$$V(\omega, 0) = \tilde{V}(\omega, 1),$$

then $(U, V, F) \sim_\mu (\tilde{U}, \tilde{V}, \tilde{F})$.

Monotone Persuasion with a Privately Informed Agent

- Agent has private type $\tilde{\omega} \in [0, 1]$
- There is an unobserved state $\omega \in [0, 1]$
- Principal chooses a monotone experiment $\pi \in \Pi^*$
- State ω realizes; Agent receives message $m = \pi(\omega)$
- Agent decides between actions $a = 1$ and $a = 0$

Assumptions

- Principal and Agent's payoffs are $v(\omega, \tilde{\omega})$ and $u(\omega, \tilde{\omega})$ if $a = 1$ and zero if $a = 0$

- We assume that

$$\frac{\partial}{\partial \tilde{\omega}} u(\omega, \tilde{\omega}) < 0 \quad \text{and} \quad \frac{\partial}{\partial \tilde{\omega}} u(\omega, \tilde{\omega}) > 0$$

  and

$$u(\omega, \omega) = 0 \quad \text{for all } \omega \in [0, 1]$$

- ω and $\tilde{\omega}$ are independently distributed, with distributions F and $\tilde{F}$ that admit positive densities f and $\tilde{f}$

Equivalence to Monotone Persuasion

- Change the order:
  - Agent observes message $m = \pi(\omega)$
  - Agent makes decision
  - Agent learns type $\tilde{\omega}$

- Decision is a threshold type y, so $a = 1$ iff $\tilde{\omega} \leq y$

- Agent's payoff (before learning the type) is

$$\mathbb{E}_{\tilde{F}}[u(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\tilde{\omega} \leq y\}}] = \int_0^y u(\omega, \tilde{\omega}) \mathrm{d}\tilde{F}(\tilde{\omega}) =: U(\omega, y)$$

  Principal's payoff is

$$\mathbb{E}_{\tilde{F}}[v(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\tilde{\omega} \leq y\}}] = \int_0^y v(\omega, \tilde{\omega}) \mathrm{d}\tilde{F}(\tilde{\omega}) =: V(\omega, y)$$

  The problem (U, V, F) is a monotone persuasion problem.

Monotone Experiments as Menus of Cutoff Experiments

- A monotone experiment π can be described as a set

$$X = \pi([0, 1])$$

- X consists of the intervals where π continuously increases, the discontinuity points of π, and the endpoints 0 and 1.

- Principal offers a menu $X \in \mathcal{X}^*$ of cutoff experiments

- Agent chooses a cutoff $x \in X$ and is informed whether $\omega \geq x$ or $\omega < x$.

- *Key observation:* Agent of type $\tilde{\omega}$ is indifferent between observing a preferred cutoff $x^*_X(\tilde{\omega})$ or observing experiment π

Agent's Decision

- W.l.o.g., for a given $x \in X$, Agent chooses $a^*(x, \omega) = \mathbf{1}_{\{\omega \geq x\}}$

- The decision of Agent boils down to a choice of $x \in X$

$$x^*_X(\tilde{\omega}) \in \arg\max_{x \in X} \mathbb{E}_\omega\Big[u(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\omega \geq x\}}\Big]$$

- Principal chooses $X \in \mathcal{X}^*$ to maximize

$$\max_{X \in \mathcal{X}^*} \mathbb{E}_{\tilde{F}}\Big[\mathbb{E}_\omega\Big[v(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\omega \geq x^*_X(\tilde{\omega})\}}\Big]\Big]$$

Equivalence to Constrained Delegation

- For a given $X \in \mathcal{X}^*$, and a given cutoff $x \in X$, Agent obtains

$$\mathbb{E}_\omega\Big[u(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\omega \geq x\}}\Big] = \int_x^1 u(\omega, \tilde{\omega}) \mathrm{d}F(\omega) := \tilde{U}(\tilde{\omega}, x).$$

- Principal obtains

$$\mathbb{E}_\omega\Big[v(\omega, \tilde{\omega}) \cdot \mathbf{1}_{\{\omega \geq x\}}\Big] = \int_x^1 v(\omega, \tilde{\omega}) \mathrm{d}F(\omega) := \tilde{V}(\tilde{\omega}, x).$$

The problem $(\tilde{U}, \tilde{V}, \tilde{F})$ is a constrained delegation problem.

Equivalence: Summary

- The mapping μ between experiments and delegations sets

$$\mu(\pi) = \pi\Big([0,1]\Big)$$

- $(U, V, F) \sim_\mu (\tilde{U}, \tilde{V}, \tilde{F})$ if

$$\frac{\partial}{\partial \tilde{\omega}} U(\omega, \tilde{\omega}) \cdot f(\omega) = -\frac{\partial}{\partial \omega} \tilde{U}(\tilde{\omega}, \omega) \cdot \tilde{f}(\omega)$$

  and

$$\frac{\partial}{\partial \tilde{\omega}} V(\omega, \tilde{\omega}) \cdot f(\omega) = -\frac{\partial}{\partial \omega} \tilde{V}(\tilde{\omega}, \omega) \cdot \tilde{f}(\omega)$$

  for all $(\omega, \tilde{\omega}) \in [0,1]^2$, with the initial condition

$$V(\omega, 0) = \tilde{V}(\omega, 1) = 0.$$

Linear Persuasion Problem

- If Principal's payoff depends only on the expected state as in Gentzkow-Kamenica (2016), then wlog, we can set

$$U(\omega, y) = -(\omega - y)^2 \text{ and } V(\omega, y) = V(y).$$

- For such (U, V, F), we can construct equivalent $(\tilde{U}, \tilde{V}, \tilde{F})$ that satisfy Amador-Bagwell (2013) assumptions.

- We adapt their techniques to study constrained delegation.

Linear Persuasion Problem

Consider (U, V, F) where

$$U_2'(\omega, y) = \alpha(y)\omega + \alpha_0(y),$$

$$V_2'(\omega, y) = c\gamma(y)\omega + \gamma_0(y),$$

such that $U_{22}'' < 0$, $U_{12}'' > 0$, and $U_2'(\omega, \omega) = 0$.

Further, assume

$$\gamma(y) > 0 \quad \text{and} \quad c \geq 0.$$

Finally, w.l.o.g., we assume that F is uniform.

Separable constrained delegation.

Consider $(\tilde{U}, \tilde{V}, \tilde{F})$ where

$$\tilde{U}_2'(\tilde{\omega}, y) = D(\tilde{\omega}) - \beta(y)$$

$$\tilde{V}_2'(\tilde{\omega}, y) = C(\tilde{\omega}) - A\beta(y),$$

such that

$$\tilde{U}_{22}'' < 0,\ \tilde{U}_{12}'' > 0, \text{ and } \tilde{U}_2'(\tilde{\omega}, \tilde{\omega}) = 0.$$

(The problem of Amador and Bagwell, ECMA 2013)

Characterization of Interval Disclosure

Denote $\tilde{V}(\tilde{\omega}) := \tilde{V}(\tilde{\omega}, \tilde{\omega})$.

Denote $m_L = \mathbb{E}[\tilde{\omega}|\tilde{\omega} \leq \tilde{\omega}_L]$ and $m_H = \mathbb{E}[\tilde{\omega}|\tilde{\omega} > \tilde{\omega}_H]$

Proposition An optimal monotone experiment is interval disclosure with cutoffs $\tilde{\omega}_L < \tilde{\omega}_H$ iff

$$\tilde{V}(\tilde{\omega}) \text{ is convex for all } \tilde{\omega} \in (\tilde{\omega}_L, \tilde{\omega}_H)$$
$$\tilde{V}(\tilde{\omega}) \leq \tilde{V}(m_L) + \tilde{V}'(m_L)(\tilde{\omega} - m_L) \text{ for all } \tilde{\omega} \in [0, \tilde{\omega}_L] \text{ w/eqty at } \tilde{\omega}_L$$
$$\tilde{V}(\tilde{\omega}) \leq V(m_H) + V'(m_H)(\tilde{\omega} - m_H) \text{ for all } \tilde{\omega} \in [\tilde{\omega}_H, 1] \text{ w/eqty at } \tilde{\omega}_H$$

- Can Principal do better with non-monotone experiments under these conditions? — No

Conclusion

- The monotone persuasion problem is equivalent to the constrained delegation problem
- Both are equivalent to a monotone persuasion problem with an informed Agent who chooses between two actions
- Known techniques for the delegation problem can be adapted and applied to solve the monotone persuasion problem

THANK YOU!