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Sequential Auctions with Multi-Unit Demand: Theory, Experiments and Simulations*

Jacques Robert†, *Claude Montmarquette*‡

Résumé / Abstract

Ce travail concerne la théorie des enchères séquentielles où les participants peuvent acheter plusieurs unités d'un bien homogène. Les prédictions théoriques des modèles sont comparées à des données expérimentales. Nous estimons des modèles structuraux qui supposent que les mises des participants sont tirées d'un processus stochastique. Le comportement observé dépend largement de la mise théorique, bien que la variance des mises demeure inexpliquée. L'analyse du modèle et des données expérimentales est complétée par des simulations. Les mises gagnantes issues des modèles structuraux montrent des prix décroissants, ce qui est contraire à la prédiction du modèle théorique.

This paper presents the theory of sequential auctions when participants desire more than one unit. The theoretical predictions are compared to experimental data. We estimate structural models which presume that participants' bids are drawn from some stochastic processes. The observed behavior depends highly on the theoretical bid, however some variance in participant bids remains unexplained. The analysis of the model and of the experimental data is completed by some simulations. While the theoretical model predicts increasing prices, on average both the data and the winning bids generated by our structural models exhibit declining prices.

Mots Clés : Enchères séquentielles multi-unitaires, données expérimentales, simulations, modèles économétriques

Keywords: Sequential auctions, multi-unit demands, econometric analysis of experimental data, simulations

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I Introduction.

In the vast literature on auctions, until recently scant attention was devoted to cases where bidders have multi-unit demands. [1] While little theoretical work modelling has been done with multiple-unit demand in auctions, it is important in practice. Most empirical data on auctions come from cases where more than one unit are on sale, and where most participants seek to buy many units. Livestock, wool , wine, and art (prints) timber-export permits, airwaves U.S. treasury auctions are examples discussed in the literature. Recently however, authors squirted to tackles the issue of multi-unit auctions. Chakravorti et al.(1995) and Klemperer (1999) provide interesting surveys of this literature. Ganal (1997) has studied of a sequential auction of multiple independent objects in the context of area cable television licences in Israel. Chanel and Vincent (1997) examined the cause and measurements of decreasing prices in sequential auctions of multiple objects. Donald, Paarsch and Robert (1997) constructed and estimated a theoretical model of participation and bidding at a sequential, oral, ascending-price, open-exit auction with multi-unit demand. Katzman (1999) assuming that bidders are symmetrically informed has shown that when only two objects are sold, the sequence of second price auctions achieve an efficient allocation. Experimental studies (Keser and Olson (1996), Kagel and Levin, (1998); Alsemgeest, Noussair and Oslon, (1998); Manelli, Sefton and Wilner, in progress) have explored the various designs for multi-unit demand auctions.

The objective of this paper is twofold. First, this paper examines the strategic behavior in sequential auctions when participants may desire multiple units (up to 15 units). We compute the strategic equilibrium for sequential auctions with multi-unit demands and we confront the theory with experimental data. The theoretical section provide the basis to understand strategic behavior in these games and offer some insights on the dynamic pattern of winning prices in the sequence of auctions.

The second objective of the paper is more methodological. Game theory has its limits. Much like the laws of physics in a frictionless world, it makes precise behavior predictions under extreme (unrealistic) assumptions. Experimental economics has most of the time invalidated the game theoretical predictions. A strategic equilibrium describes a stable social rule of behavior under the assumption that rationality , the rules of the game and the equilibrium rule of behavior are common knowledge. In practice none is guaranteed. Participants is a complex game are unlikely to be able to compute the strategic equilibrium of the game (most participants have at best only a vague idea of what it represents). Even if some are able to compute the strategic equilibrium, they need not believe that others are playing according to it .

Participants in a complex game such as a sequence of auctions are likely to experiment, test one strategy and another. Sometimes they make smart moves, sometimes they make strategic mistakes, and often they cannot figure out whether they did one or the other. Part of participants behavior is simply random and cannot be explained by any rational decision theory. One can take the point of view that a theory ought to be rejected if in one instance it is rejected by experience. If we seek this very high standard, game theory will never pass the test. Nevertheless, despite its limitations, the predictions of game theory are still taken seriously. Among all the good reasons to do so, one is simply that there is no good alternative to predict behavior in games.

In this paper, we exploit experimental data to test to what extent the behavior of participants complies with the predictions of the theory and to what extent their behavior remains random and unexplained. We believe that such approach is meaningful to assess the degree of bounded rationality of players, the complexity of the trading game and eventually address market design issues.

The experimental procedure is presented in Section II. We explained there the nature of the game played by the participants. We solve for the strategic equilibrium in Section III, for the sequential descending-price, ascending-price and mixed auctions. We show how to compute the equilibrium bid function for the experiments. In Section IV, we report some descriptive experimental results. In particular, we note that the allocation of units on sale is very efficient while contrary to the theory declining winning prices are observed. The rest of the paper seeks to understand why. Structural estimations are presented in Section V. We assume that the bids selecting by bidders are randomly drawn from some distributions. We provide maximum likelihood estimates of idiosyncratic parameters of these distributions which include as special cases the theoretical predictions and purely random behavior. Our estimations are naturally between these two extremes, but the theoretical predictions seems nevertheless to determine a large part of the observed behavior. Finally, we have performed simulations in order

[1] A large proportion of the literature consider the case where only one unit of a indivisible good was assumed on sale (as in the classical references on auctions and bidding: Milgrom and Weber, 1982; McAfee and McMillan, 1987; Vickrey, 1961) or when multiple units were on sell, buyers were assumed to desire at most one unit (see Weber, 1982; McAfee and Vincent, 1993; Laffont and Robert, 1997). In papers where multi-unit demands are allowed, demands are parametrized by a one-dimensional type (see Maskin and Riley, 1989).

to better evaluate the implications of the estimated structural models and other alternatives on the winning prices and welfare losses. The simulation results are presented in Section V. We conclude in Section VI.

II Experimental procedure

The experiments concern descending-price auctions, ascending-price and mixed (both descending and ascending) auctions . The experiments were conducted with a group of eight undergraduate students from the department of economics at the Université de Montréal. Each session began with a presentation of the experimental program indicating the type of auctions, and the number of rounds to be completed during the session. A period was devoted to allow the participants to get familiar with the computerized routines and to understand the different auction mechanisms. All participants had hands-on training with each auction mechanism prior to the actual experimentations. Each participant earned $5Cdn for showing up plus their experimental gains. The expected earnings for each participant was around 25$Cdn per meeting which lasts 2 to 3 hours.

In a round of auctions, 15 units of a fictitious good are sold. At the beginning of each round, participants receive some private values for the units on sale. We provide electronically to each participant i a vector of valuations of the form: $W^i = \{w_1^i, w_2^i, ..., w_m^i, 0, 0, ...\}$, where w_j^i represents i's valuation of her j^{th} unit of good, and m the number of positive valuations. Participant have decreasing marginal utility, so that $w_1^i \geq w_2^i \geq ... \geq w_m^i$. The vector of private valuations for each participant or bidder are generated as follows: (i) the number of positive valuations, m, is drawn from a Poisson process[2] with an average frequency of $\lambda = 7$, (ii) then m valuations are drawn identically and independently from an uniform distribution between 0 and 100, (iii) these m valuations are finally ranked in descending order. The highest valuation, w_1^i, corresponds to the gain accruing to buyer i from her first unit, while her j^{th} valuation corresponds to the marginal utility of her j^{th} unit. With each purchase a participant earns a number of the experimental currencies which is calculated as the difference between the participant‘s current highest valuation and the price she pays. For example, if in a round participant i obtains three units and pays respectively p_1, p_2 and p_3, then her gain is given by $(w_1^i - p_1) + (w_2^i - p_2) + (w_3^i - p_3)$. The computer calculates as the auction go on the incremental gain the participant would make if she were to win the current unit at the current price. The computer also calculates the total gain made by the participant so far in the session. Experimental gains were converted into monetary gains with a factor of 0.05$ for each experimental currency unit.

The vector W^i and the number of positive valuations, m, are private information to bidder i. Note that in the above specification, it is possible that a bidder draws no positive valuation. Whether a specific buyer has a positive valuation or not is not observable by other participants.

A round of descending-price (or Dutch) auctions is conducted as follows. The first unit is offered at an initial price of 100. The tendered price is then lowered by one unit of experimental currency after every delay of two seconds, until a participant signals her willingness to purchase the unit. The second unit is next offered at an initial selling price of 5 units above the winning price of the first auction. The process continues until all 15 units are sold. This ends the round. Participants within a round observe the number of units sold and left to be sold and the history of winning price for the current round. Participants do not know who won the unit not won by themselves. We collected data for 8 rounds of 15 sequential descending-price auctions.

In a round of ascending-price (or English) auctions, the first unit is initially offered at the initial price of 40. If one or more participants bid for the good with a delay of three seconds, the price of increased by one currency unit. So long as one participant is willing to purchase the unit at the current price, the price keeps moving up. The auction ends if after a delay of three seconds no bid is received, the unit is allocated to the last bidder at the previous price. The system is set up such that participants cannot bid above their own best price. The second unit is offered at an initial price of 10 currency units below the previous selling price. The process continue until all 15 units are sold. We collected data for 8 rounds of 15 sequential ascending-price auctions.

In a round of mixed (Dutch-English) auctions, the first unit is initially offered at a price of 100. The price decreases by steps of two units until one participant signals her willingness to purchase the unit, then the price moves up much like in the ascending-price mechanism. The next unit is initially offered at a price equal to the winning price of the previous auction. Data for 8 rounds of 15 sequential ascending-price auctions were collected.

Eight rounds of 15 sequential auctions yield 120 observations by type of auctions. These data will be used to obtain maximum likelihood estimates of the structural model.derived from our theory We are fully aware,

[2]Although this assumption may seem quite natural, it turns out to be crucial. As we shall argue latter it is necessary to maintain symmetry between bidder through out the game and to solve for the equilibrium of the auction game.

however, of the limits if our experimental procedure. Keeping the same 8 participants, in all the experiments has fitted our goal of confronting our theory with experienced people playing a difficult game but, this approach is unequivocally restrictive for the number of independent observations.available Nonparametric tests are excluded, but within the context of parametric estimates that specifies interactions among players, 120 observations appears an acceptable.sample. Due to time and cost constraints, another limit of our experimental design is the absence of different treatments in the number of units sold and the number of positive valuations. Thus, our experimental procedure is to be considered a pilot experiment serving an illustrative purpose.

III The Game Theoretical Equilibria

In this section, we present the theory for the auction games described above. For more generality, we shall find the strategic equilibria for an arbitrary number of participants, $n = |N|$, and number of units on sale, T. Further we allow the parameter λ of the Poisson process which determines the number of positive valuations for each participants to be arbitrary. Finally, the distribution from which these positive valuations are drawn is given by some distribution function $F(.)$. The parameters and function n, T, λ and $F(.)$ are assumed common knowledge.

We assume that bidders are risk-neutral and that they seek to maximize the total value of the units purchased minus the payment necessary to purchase them. If bidder i has a private vector $W^i = \{w_1^i, w_2^i, ..., w_m^i, 0, 0, ...\}$ and if she obtains k units for the total cost of p, her gain is given by $\left(\sum_{j=1}^{k} w_j^i\right) - p$.

The value w_j^i represents i's j^{th} highest valuation or the monetary value $j^{th}$of her unit. Expanding this notation, we can defined W^S as the vector of descending valuations which ranks valuations of all bidders in coalition S, where w_k^S denotes the k^{th} highest valuation among all bidders in S. In particular w_k^{-i} denotes the k^{th} highest valuation among all bidders but i.

The process by which these values are generated guarantees the following properties:

A: For all $j < k$, we have $Prob\left[w_k^i = x | w_j^i \le y, w_{j-1}^i, ..., w_1^i\right] = Prob\left[w_k^i = x | w_j^i \le \min\left[y, w_{j-1}^i\right]\right]$.

B: (symmetry) For all k, and i and $l \in N$ we have $Prob\left[w_k^i = x | w_1^i \le y\right] = Prob\left[w_k^l = x | w_1^l \le y\right]$.

C: For all bidder $i \in N$, all $j \le k$, we have:

$$Prob\left[w_k^i = x | w_j^i \le y\right] = Prob\left[w_{k-j+1}^i = x | w_1^i \le y\right]. \tag{1}$$

Property A is direct from the assumption of independent draws and the properties of order-statistics. Property B follows directly from the assumption of symmetric between players. Property C is more involved; it states that the conditional distribution is invariant to reindexing of order-statistics. This is not a general property of order-statistics, it follows here from the assumption that the number m of draws follows a Poisson process. A formal proof is provided in Appendix. Property C proves to be very useful.

The vectors W^i's represent the valuations at the beginning of the auctions. Suppose that bidder i, wins a unit, then her valuation for the next extra unit becomes w_2^i rather than w_1^i as initially. Also through her winning bid, i has revealed her value w_1^i. Property C states that the distribution of i's second valuation is no different then the distribution of the highest valuation of those who have not already won a unit (conditional on been less than w_1^i); i.e. $Prob\left[w_2^i = x | w_2^i \le w_1^i\right] = Prob\left[w_1^l = x | w_1^l \le w_1^i\right]$ for all i and l.

In what follows we shall reindex valuations to take into account the units already purchased by each bidder. Let $h(t)$ be the history prior to auction t, we denote $V^i(h(t))$ as i's reindexed vector of types such that if i won l_i units prior to auction t then $v_j^i(h(t)) = w_{j+l_i}^i$. Similarly, we let $V^S(h(t))$ be the reindexed version of W^s.When t and $h(t)$ are unambiguous, we shall drop the argument $h(t)$ and simply use the notations V^i or V^S for the reindexed vectors. Given this notation, properties B and C imply:

D: (Robust symmetry) For all bidder i and $l \in N$, all k and after all history $h(t)$, we have: $Prob\left[v_k^i(h(t)) = x | v_1^i(h(t)) \le y\right] = Prob\left[v_k^l(h(t)) = x | v_1^l(h(t)) \le y\right]$.

The result D states that the distribution of valuations remains symmetric between players through out the game irrespective of the history of the game. This assumption is instrumental in solving for the game.

III.1 Equilibrium for the sequential descending-price auctions.

We are first looking for the equilibrium of the game induced by the sequence of T descending-price auctions. Prior to auction t, each participant observe the list of all previous winning bids $h(t) = \left\{b_1^w, ..., b_{t-1}^w\right\}$ and how many units she has already won. A strategy specifies for each auction t, possible history at t, $h(t)$, and for each agent's vector of (reindexed) valuations a bid, i.e. at what price should a participant accept to stop the descending-price process and purchase the unit. For some well-defined bidding strategy, we can specify after each history equilibrium beliefs for the bidders. If bidding strategy are monotonically increasing, beliefs can be parametrized by some number z_t where z_t corresponds to the highest (reindexed) valuation of auction t's winner.

We characterize in the theorem below, a strategic equilibrium of the T-repeated auction game.[3] Given all the information available to the bidders in the process of the game the strategy specifies the equilibrium bidding rule. We now state the main result of this section.

Theorem 1 (Descending-price auctions): *For all auction t, where $s = (T - t + 1)$ corresponds to the number of units remaining to be sold (including the unit currently on sale), let $v^* \equiv \min\left[z_1, ..., z_{t-1}, v_1^i\right]$ given some vector $V^i(h(t))$ and history summarized by the vector $z^t = \{z_1, ..., z_{t-1}\}$, in equilibrium each bidder i bids*:

$$\begin{aligned} b_s^D(v^*) &= E\left[v_s^{-i} | v_1^{-i} \leq v^*\right] \\ &= \int_0^{v^*} x(\lambda(n-1))^s \frac{[F(v^*) - F(x)]^{s-1}}{(s-1)!} f(x) e^{-\lambda(n-1)[F(v^*)-F(x)]}\, dx \end{aligned} \tag{2}$$

The equilibrium characterized above has important properties:

(i) The equilibrium allocation is efficient, i.e. the T units are allocated to the bidders with the T highest valuations overall. Bids are increasing function of bidders' first (reindexed) valuation. The bidding function are symmetric, so in each round the unit is allocated to the bidder with the highest (reindexed) allocation.

(ii) The expected price paid by a buyer i for her $(l+1)^{th}$ unit is equal to the expected value of w_{T-l}^{-i},i.e. the $(T-l)^{th}$ highest value among all values of the other participants. Indeed, if before auction t bidder i has already won l units, then $v_s^{-i} = v_{T-t+1}^{-i} = w_{T-l}^{-i}$.

(iii) The bidding strategy has a recursive structure built into it. We have for all s:

$$b_s^D(v^*) = E\left[b_{s-1}^D(v_1^{-i}) | v_1^{-i} \leq v^*\right] = E\left[b_{s-2}^D(v_2^{-i}) | v_1^{-i} \leq v^*\right] = ... = E\left[b_0^D(v_s^{-i}) | v_1^{-i} \leq v^*\right]$$

Where we let $b_0^D(v^*) = v^*$. To see this, notice that by definition, this is true for $s = 1$. We need to show that if it is true for some $s-1$, then it is true for s.We have:

$$b_{s-1}^D(v^*) = E\left[b_0^D(v_{s-1}^{-i}) | v_1^{-i} \leq v^*\right] = E\left[b_0^D(v_s^{-i}) | v_2^{-i} \leq v_1^{-i} = v^*\right]$$

the second property follows from Property A and C above. Hence, we obtain:

$$\begin{aligned} E\left[b_{s-1}^D(v_1^{-i}) | v_1^{-i} \leq v^*\right] &= E\left[E\left[b_0^D(v_s^{-i}) | v_2^{-i} \leq v_1^{-i}\right] | v_1^{-i} \leq v^*\right] \\ &= E\left[b_0^D(v_s^{-i}) | v_1^{-i} \leq v^*\right] = b_s^D(v^*) \end{aligned} \tag{3}$$

(iv) Along the equilibrium path, we have: $\min\left[z_T, ..., z_{t+l'}, v_1^i\right] = v_1^i$ for all i. Hence, a bidder will use information from past auctions only if she has deviated from her equilibrium strategy in some previous auction, that is for one auction the winning bid was lower than i's equilibrium bid: $b_t^w < b_{T-t+1}\left(v_1^i\right)$. Hence, under the presumption that others follow the equilibrium, no one can influence future bidding by deviating, lowering or increasing her bid, in ways that do not change the identity of the winner. Implicit in this is that bid manipulations design only to signal information to others are ignored.

We are ready to prove Theorem 1.

Proof of Theorem 1: The proof proceeds by backward induction. In the last auction, auction 1, the game corresponds to the usual static first-price auction. Following Property D, we know that the distributions

[3] We make no statement on unicity of the equilibrium, nor try to characterize all equilibria of the game if there are others.

of bidders' willingness to pay $\left(v_1^i\left(h\left(t\right)\right)\right)$ are identical. The game corresponding to the last game is therefore symmetric. It is well known that for symmetric game, the equilibrium bidding strategy is to bid:

$$b_1^D\left(v\right) = E\left[v_1^{-i}|v_1^{-i} < v\right] = \int_0^v x\ d\ \left[e^{-\lambda(n-1)[F(v)-F(x)]}\right]$$

This is exactly what Theorem 1 recommends. The strategy prescribed by Theorem 1 forms the equilibrium when $s = 1$.

We show now that if it is a best-response for each player to follow the proposed equilibrium strategy in auction $t+1$, then it is her best-response to follow it in auction t. Let $s = T - t + 1$ and let denote by y the highest (reindexed) valuation of all bidders but i. Consider two alternative strategies for some bidder i with higher (reindexed) valuation v in some auction t. The alternative A consists of bidding $b_s^D\left(m\right)$ in auction t and following the equilibrium strategy in the subsequent auctions, which is in particular to bid $b_{s-1}^D\left(\min\left(m,v_2^i\right)\right)$ in auction $t+1$ if she win auction t and $b_{s-1}\left(v\right)$ is she loses. The Alternative B consists of following the proposed equilibrium bidding rule in auction t, i.e. to offer $b_s^D\left(v\right)$, and in auction $t-1$ to do the following:

(i) offer $b_{s-1}^D\left(y\right)$ if $m > y > v$, i.e. $b_s^D\left(m\right) >$ the winning bid at auction $t > b_s^D\left(v\right)$;

(ii) offer $b_{s-1}^D\left(\min\left(v_2^i, m\right)\right)$ if i wins auction t,

(iii) otherwise offer $b_{s-1}^D\left(v\right)$. Finally in all subsequent auctions, follows the equilibrium strategy.

We can verify that Alternative B yields the same expected payoff as Alternative A for all values of $m \neq v$ and y. Suppose that $m > v$. If $y > m$, they lead to the same outcome: i loses the first auction and then follows his equilibrium strategy. If $m > y > v$, he will win auction t in Alternative A and will lose auction $t+1$; in B, he will win auction $t+1$ but not auction t. If $m > v > y$, she will win auction t in both alternatives and will bid $b_{s-1}^D\left(v_2^i\right)$ in auction $t+1$. Because bidding strategies of others depend only on how many units each bidder has previously won, the choice between these two alternatives do not affect the outcome of auctions $t+2$ to T. The only thing that matters is the expected price that i will need to pay in order to win in auction t or auction $t+1$. In Alternative B, i will pay $b_s^D\left(m\right)$, and in Alternative A she would pay $b_{s-1}^D\left(y\right)$ if $y > v$ and $b_s^D\left(v\right)$ if $v > y$. In expected terms, these payments are the same. This result follows from the result in comment (iii) above. Recall that $v_1^{-i} = y$, we have:

$$\begin{aligned} b_s^D\left(m\right) &= E\left[b_{s-1}^D\left(y\right)|y \leq m\right] \\ &= E\left[b_{s-1}^D\left(y\right)|v \leq y \leq m\right] Prob\left[v \leq y \leq m\right] + E\left[b_{s-1}^D\left(y\right)|y \leq v\right] Prob\left[y \leq v\right] \\ &= E\left[b_{s-1}^D\left(y\right)|v \leq y \leq m\right] Prob\left[v \leq y \leq m\right] + b_s^D\left(v\right) Prob\left[y \leq v\right] \end{aligned} \tag{4}$$

Using a similar argument we can show that alternative A and B are equivalent in expected terms when $v > m$. If $y > v > m$, i loses auction t in both alternatives with no further consequence. If $v > y > m$, she will win auction t in Alternative B and will lose auction $t+1$; in A, she will win auction $t+1$ but not auction t. If $v > m > y$, she will win auction t in both alternatives and will bid $b_{s-1}^D\left(\min\left(m, v_2^i\right)\right)$ in auction $t+1$. In Alternative B, i will pay $b_s^D\left(v\right)$; in Alternative A she would pay $b_{s-1}^D\left(y\right)$ if $y > m$ and $b_s^D\left(m\right)$ if $m > y$. Following the argument in (2) these payments are the same in expected terms.

Hence deviating in auction t and then following the equilibrium strategy in auction $t+1$ is not better than following the equilibrium strategy in auction t and deviating in auction $t+1$. This in turn, under the assumption of equilibrium, cannot be better than following the equilibrium strategy in both auctions. Hence, i has no incentive to deviate from the strategy prescribed by Theorem 1. Q.E.D.

III.2 Equilibrium for the sequential ascending-price and mixed auctions.

We are now looking for the equilibrium of the game induced by the sequence of T ascending-price auctions. From a strategic point of view, the ascending-price auctions and the mixed auctions are not necessarily identical. In a mixed auction, the prices initially moves down until one participant accepts the ongoing price and allow the clock to move up again. The particular moment when a participant decides to stop the first phase of the auction may act as a signal (see Avery, 1998) for a discussion on the strategic impact of jump bidding). Formally, a strategy in a mixed auction must specifies when participants end the first phase of the auction; we do not do so here. We restrict our attention to an equilibrium for the mixed auction where strategies are invariant to the prices at which the first phase stops. So a strategy specifies for each auction t, for each history of previous winning bids, and for each agent's vector of (reindexed) valuations, a bid, i.e. at what price should a participant withdraw from the ascending-price process.

We characterize in the theorem below, a strategic equilibrium of the T-repeated auction game.[4] It is very similar to the case with descending prices.

Theorem 2 (Ascending-price and mixed auctions): *For all auction t, where $s = (T-t+1)$ corresponds to the number of units remaining to be sold (including the unit currently on sale), let $v \equiv v_1^i$ given some vector $V^i(h(t))$, in equilibrium each bidder i bids:*

$$b_s^A(v) = E\left[v_s^{-i} | v_1^{-i} = v\right] = b_{s-1}^D(v) \tag{5}$$

The equilibrium characterized above has properties similar to the one described in Theorem 1: (i) The equilibrium allocation is efficient since bids are increasing function of bidders' first (reindexed) valuation. (ii) The expected price paid by a buyer i for her $(l_i+1)^{th}$ unit remains equal to the expected value of $w_{T-l_i}^{-i}$. We have revenue equivalence between the two auction mechanism.. (iii) The bidding strategy still has the same recursive structure so that for all s: $b_s^A(v) = E\left[b_{s-1}^A(v_1^{-i}) | v_1^{-i} \leq v\right]$. (iv) Finally, bidders will not use information from past auctions even if they have deviated in some previous auction.

The structure of the proof of Theorem 2 is hence very similar to the proof of Theorem 1. [5] In the last auction, auction T where $s = 1$, the game corresponds to the usual single-unit ascending-price auction. In this case, it is indeed the equilibrium bidding strategy is to bid: $b_1^A(v) = E\left[v_1^{-i} | v_1^{-i} = v\right] = v$. The strategy prescribed by Theorem 2 forms indeed the equilibrium when $s = 1$.

Now for the earlier auctions, consider any other alternative strategy. A player will be indifferent between two distinct strategies which give to her the same number of units. It follows from the result (iii) above which states that the expected price one needs to pay to buy her l^{th} is constant in expected terms whenever she purchase it. So for any deviation in some auction t, there exists an alternative strategy which consists of following the equilibrium strategy in auction t and deviating in latter auctions and which yields the same expected payoffs. This in turn, under the assumption of equilibrium, cannot be better than following the equilibrium strategy in all auctions.

III.3 Main theoretical conclusions

From this theoretical section, we can draw a number of predictions about the strategic behavior of agents. First, there exists an equilibrium which generates the efficient allocation. There is no guarantee that this is the unique equilibrium and that there is no equilibrium which lead to an inefficient allocation. However, this first result means that the sequential auctions provides sufficiently rich strategies to induce the efficient allocation.

Second, the sequential descending-price, ascending-price and mixed auctions generate the same expected revenue. So there is no a priori reason to believe that one should lead to more revenues than the other.

Finally, the theoretical model offers sharp predictions about the winning bids. In each auction, the winner is the individual with the highest (reindexed) valuation. So given vectors of valuations W^i's, we can predict both who should win and what would be her winning price. In particular, we can calculate the equilibrium bidding functions when $\lambda = 7$, $n = 8$, and the distribution $F(\cdot)$ is a uniformed between 0 and 100: $F(v) = v/100$. If we integrate equation (1), we obtain:

$$\begin{aligned} b_s^D(v) &= v\left[1 - \sum_{m=0}^{s-2} \frac{[\lambda(n-1)]^m}{m!} \left(\frac{v}{100}\right)^m e^{-\lambda(n-1)\left(\frac{v}{100}\right)}\right] - \frac{100s}{\lambda(n-1)}\left[1 - \sum_{m=0}^{s-1} \left(\frac{v}{100}\right)^m e^{-\lambda(n-1)\left(\frac{v}{100}\right)}\right] \\ &\approx \max\left[0, v - \frac{100s}{\lambda(n-1)}\right] \end{aligned} \tag{6}$$

Similarly, we have:

$$b_s^A(v) \approx \max\left[0, v - \frac{100(s-1)}{\lambda(n-1)}\right] \tag{7}$$

[4] Again, we make no statement on unicity of the equilibrium, nor try to characterize all equilibria of the game if there are others.

[5] In order to prevent repetitions, the extensive proof is not presented here.

Note that we propose a simple approximation for the bid functions. The approximation error decreases in v and is negligeable, when v is sufficiently large ($v > 50$). This simple structure allows us to invert the bidding function: so we can write $z_t = b_t^w + \frac{100(T-t+1)}{\lambda(n-1)}$ if $b_t^w$is the winning price of the t^{th}descending-price auction. So for the descending-price auctions, we can calculate for each auction $t+1$ and bidder i:

$v^* = \min\left[b_1^w + \frac{100(T)}{\lambda(n-1)}, b_2^w + \frac{100(T-1)}{\lambda(n-1)}, ..., b_t^w + \frac{100(T-t+1)}{\lambda(n-1)}, v_1^i\right]$ and i's bid $b^D_{(T-t)}(v^*) = \max\left[0, v^* - \frac{100(T-t)}{\lambda(n-1)}\right]$.

IV Descriptive Experimental Results

In this section, we examine the experimental data from 8 rounds of 15 sequential data for the descending, ascending-price and mixed auctions. We first asked how these auctions experiences performed in terms of efficiency and revenue generation. Are these sequence of auctions efficient ? How the revenues generated by these auctions compare with the theoretical predictions ?

When a single unit is to be sold, an auction is inefficient if the unit is not won by the participant with the highest evaluation; in this case, the welfare losses can be measured as the difference between the highest valuation and the winner's valuation. When there are multiple unit on sale, inefficiency occurs when the units are not allocated to those with the highest valuations for these units. For our experiments, the welfare losses are given by the difference between the 15 highest valuations and the valuations of the winners. For the descending-price auctions, the welfare losses amount on average to 10.5 currency units, this corresponds to 14.4% of the average valuation of the last unit purchased with average value of 73 currency units. These numbers increase to 12 currency units (16.4% of the value of the last unit) for the ascending-price auctions while it is 5.75 currency units (7.8% of the average value of the last unit) for the mixed auctions. These levels of welfare losses are relatively small with respect to the total optimal surplus. Indeed, we can calculate an inefficiency rate given by the ratio of the welfare losses over the total surplus (the sum of the 15 highest evaluations). For the descending-price auctions, the mean inefficiency ratio is 0.00838. The ratio is 0.00933 for the ascending-price auctions and 0.0045 for the mixed auctions.

There is an alternative way to look at the inefficiency issue for sequential multi-unit auctions. For the last unit to be auctioned, we are in the single unit case situation. Given that 14 units has been already sold, if the person who wins this last unit has not the highest evaluation, then the selling of the 15th unit of the good is inefficient. Similarly, an inefficiency exists if the 14th unit has been not been sold to one of the two persons that enjoyed the two highest evaluations. And so on. Here, for the descending-price auctions, a situation of inefficiency has occurred 13 times in 120 events (8 rounds of 15 units sold) compared to 10 times for the ascending-price auctions and 9 for the mixed auctions. Figures 2 to 4 show on average that inefficient results appeared in the last few units sold with the differences in currency units between the winning evaluation and the lowest evaluation among the efficient groups being relatively small.

The average revenues generated are comparable between the three types of auctions with 1016.125 currency units per round relatively to 1113 for the ascending-price auctions and 1046 for the mixed auctions. Given the vectors of valuations for these experiments, the theoretical revenues (provide that the participants had followed the equilibrium strategy) are 1008.48 currency units for the descending-price auctions, 1044.21 for the ascending-price auctions and 1035.71 for the mixed auctions. The actual revenue numbers are relatively close to the predicted revenues for the descending-price auctions. The main difference between the theory and the actual data are in the price dynamics. The theory predicts that the average winning prices increase with the number of units sold. The data clearly exhibits the reserve: the average winning prices are declining dynamically (see Figures 5 to 7).

These descriptive statistics demonstrate the relative efficiency of the descending and ascending-price auctions. As such the results are interesting, but an institution or a trading mechanism can be declared efficient only with respect to alternatives. One of the alternative is to allocate the goods randomly. The questions are then to what extend the efficiency results are the reflection of rational behavior on the part of the agents, and more precisely to what extent do they follow the theoretical model of the preceding section. Finally, do the trading rules matter ? To investigate these questions, as well as to give an explanation of the decreasing prices anomaly, we need to estimate a parametric model of the participants decisions.

V Structural Estimations

We wish to investigate the experimental data more precisely. In order to do so, we performed structural estimations.

Our estimation technique relies on the assumption that bids are randomly drawn from some distribution with support between 0 and the bidder's willingness to pay. For practical purposes, we let bids be drawn from a Beta distribution. We have selected the Beta distribution because its support can be delimited to a finite interval, the main statistical softwares calculate directly its cumulative distribution, and finally it admits two parameters so the mean and variance are not necessarily linked.

A Beta distribution with support between 0 and some v and parameterized by $p > 0$ and $q > 0$, has mean $\mu = v * p/(p+q)$ and variance $\sigma^2 = (v-\mu)\mu/(p+q+1)$. Its cumulative density is given by:

$$(8) \qquad I(\frac{x}{v}, p, q) = \frac{\int_0^{x/v} t^{p-1}(1-t)^{q-1}\,dt}{\int_0^1 t^{p-1}(1-t)^{q-1}\,dt}$$

For the purpose of our estimations, we assume the bid of a participant i in a given auction are generated by a Beta Distribution with parameters:

$$(9) \qquad p_i = \frac{(\alpha_1)v_i + (\alpha_2)BT_i}{\alpha 3 v_i}$$

$$(10) \qquad q_i = \frac{(1-\alpha_1)v_i - \alpha_2 BT_i}{\alpha_3 v_i}$$

where v_i is i's highest virtual valuation and BT_i is her theoretical bid in this auction. For each bidder i, the expected value of her bid and the variance of a bid are given respectively by:

$$(11) \qquad \mu_i = \alpha_1 v_i + \alpha_2 BT_i$$

$$(12) \qquad \sigma_i^2 = \frac{\alpha_3}{1+\alpha_3}[\alpha_1 v_i + \alpha_2 BT_i][(1-\alpha_1)v_i - \alpha_2 BT_i)]$$

This formulation allows us to measure whether the theory is meaningful to explain the experimental data. Note that the theoretical model is a limit case of this formulation. Indeed, if $\alpha_2 = 1$ $\alpha_1 = 0$ and if α_3 converges to zero, then participants will almost always bid according to the theoretical prediction. Conversely, if $\alpha_2 = 0$, bids are drawn in a way which is independent of the theory. One extreme case is when $\alpha_1 = \alpha_3 = 1/2$ and $\alpha_2 = 0$. In this latter case, $p = q = 1$ and bids are drawn uniformly between 0 and v_i.

V.1 Maximum likelihood

We estimate the parameters α_1 to α_3 using the maximum likelihood estimator. For each auction, we have the identity of the winner and the winning bid. We also have for each bidder, her v_i and her theoretical bid. So we can calculate for each participant, the probability that she bids less then some value b^*.

For the descending-price auction, the likelihood that i wins at a price of b^* is equal to the probability that i accept the price b^* and was not ready to accept the price b^*+1 (recall that price are decreasing discretely) times the probability that all others were not ready to accept price b^*. It is given by:

$$(13) \qquad [I(\frac{(b^*+1)}{v_i}, p_i, q_i) - I(\frac{b^*}{v_i}, p_i, q_i)] \prod_{j \neq i} I(\frac{b^*}{v_j}, p_j, q_j)$$

We are looking for the values of α's which best explain why the winner of every particular auctions have bid the winning price but also why all the other were bidding a lower price.

For the ascending-price and mixed auctions, the likelihood that i wins at a price of b^* is equal to the probability that i was ready to bid up to b^* or more but that no other participants were ready to bid b^*+1 or above, but one of i's opponent was ready to bid b^*-1. The likelihood is given by:

$$(14) \qquad \left[1 - I(\frac{b^*}{v_i}, p_i, q_i)\right] \cdot \left(\sum_{j \neq i} \left[I(\frac{b^*+1}{v_j}, p_j, q_j) - I(\frac{b^*-1}{v_j}, p_j, q_j) \right] \prod_{\substack{k \neq i \\ k \neq j}} I(\frac{b^*+1}{v_k}, p_k, q_k) \right)$$

The structural estimations can be used to measure how close the process generating the bid is close to the theoretical prediction. Moreover, the variance of the Beta distribution generating the participants' bid measures the degree of randomness in the participants' behavior. It can be interpreted has evidence of some form of bounded rationality and the impossibility for players to compute the strategic equilibrium of the game. Hence, using experimental data we can measure this bounded rationality using the variance of the bids' distribution. When there is only one unit to sale, the subgame is objectively simpler than when there are many forthcoming auctions. Hence, we conjecture that the variance of bids' distributions decreases as the number of units left to be sold decreases. Similarly, we conjecture that as players become more experienced, the variance also decreases.

V.2 The structural maximum likelihood estimates

The purpose of our econometrics estimations is to derive from experimental data, the idiosyncratic parameters p_i and q_i of a Beta Distribution as defined in equations (9) and (10) . We have constrained $\alpha_3 > 0$, to guarantee identification and that p_i and q_i are > 0.

Table 1 reports the results of maximum likelihood estimates of the structural parameters for the descending-price auctions. The first column of Table 1 is a simple model with no unit effects. In column 2, the structural parameters are allowed to vary as different units are sold sequentially. This is captured by introducing for the three structural parameters to be estimated, a dummy variable taking the value one for the first ten units auctioned and zero otherwise.

Table1

Econometric Estimation of Structural Parameters for the Descending-Price Auctions*

	Without unit effects	With unit effects	
		1-10	11-15
α_1	-0.1108^a	0.1553	-0.1935^c
	(0.0520)	(0.115)	(0.113)
α_2	1.0404^a	0.6918^a	1.119^a
	(0.0620)	(0.129)	(0.129)
α_3	0.0475^a	0.0396^a	0.0734^a
	(0.0118)	(0.0188)	(0.0292)
Loglikelihood**	-1085.1300	-1081.9956	
N observation	120	120	

Standard errors are in parentheses.

a: significant at 0.01 level. b: significant at 0.025 level. c: significant at 0.05 level

* With constrained $\alpha_3 > 0$.

** Loglikelihood with the parameters set for the uniform distribution : -1152.5664.

- Loglikelihood for constant parameters p and q ($\alpha_2 = 0$): -1133.1084 .

- The p-value for the likelihood ratio test between the with and without unit effect models is 0.0987.

The results support the theory. Parameter α_2 is closed to unity and α_1 to zero. Furthermore, the parameter α_3 is relatively small. It is worth noting how the likelihood value for the function evaluated with the parameters corresponding to the uniform distribution (such that $p_i = q_i = 1$) is much lower at -1152.5664 than the maximum value obtained with our maximum likelihood procedure. When we restrict α_2 to be equal to 0, the loglikelihood increases to -1133.1084. Our model is significantly better than these constrained models. The theoretical bidding rule is useful to predict the participants' behavior.

In column 2, we note the effect of adjusting for the auction index. For the first 10 units auctioned, the participant does not closely follow the theoretical model as for the last 5 units. We can interpret this as a consequence of the complexity of the game the participant faces at the beginning of the auction.

The results for the ascending-price auctions are reported in Table 2. Again our model is significantly better than the models where α_2 is equal to 0 and where the parameters correspond to the uniform distribution (such that $\alpha_1 = \alpha_3 = 0.5, \alpha_2 = 0$). Here the difference between the two specifications with and without unit effects are more striking than before. For the first ten units, the participants are fixed at their evaluation value and with $\alpha_2 \approx 0$, bids are drawn independently from the theory. However, for the last five units, the participants use their theoretical bid as predicted by the model.

Table2

Econometric Estimation of Structural Parameters for the Ascending-Price Auctions*

	Without unit effects	With unit effects	
		1-10	11-15
α_1	0.4608^a	0.7089^c	-0.0929
	(0.0673)	(0.408)	(0.392)
α_2	0.4463^a	0.1456	1.0229^a
	(0.0769)	(0.407)	(0.0402)
α_3	0.1369^a	0.09176^a	0.2263^a
	(0.0245)	(0.0477)	(0.0733)
Loglikelihood**	-492.0516	-485.7216	
N observation	120	120	

Standard errors are in parentheses.

a: significant at 0.01 level. b: significant at 0.025 level. c: significant at 0.05 level

* With constrained $\alpha_3 > 0$.

** Loglikelihood with the parameters set for the uniform distribution : -671.8704.

- Loglikelihood for constant parameters p and q ($\alpha_2 = 0$): -513.600.

- The p-value for the likelihood ratio test between the with and without unit effect models is 0.00543.

The results for the mixed auctions are reported in Table 3. Without the unit effects, the estimations follow the predicted model, as α_2 is very closed to 1. However, with the unit effects the estimation are quite different. Again the behavior for the first 10 units is independent of the theoretical bidding function, while for the last units, the coefficient α_2 is larger than one. As in the other auctions, the likelihood values for the function evaluated with the parameters corresponding to the uniform distribution and the constant p and q are significantly lower than the likelihood value obtained with our model. Tests not presented in this paper conclude that the structural estimations are significantly different for the ascending and mixed auctions. In particular, the variance of the bidding distribution, measured by α_3 is greater in the latter.

Table 3
Econometric Estimation of Structural Parameters for the Mixed Auctions*

	Without unit effects	With unit effects	
		1-10	11-15
α_1	-0.1053	0.7267	-0.8403^c
	(0.090)	(0.434)	(0.423)
α_2	1.0121^a	-0.0216	1.785^a
	(0.101)	(0.432)	(0.432)
α_3	0.2389^a	0.1246	0.3586^a
	(0.0396)	(0.0811)	(0.118)
Loglikelihood**	-481.3872	-459.5100	
N observation	120	120	

Standard errors are in parentheses.
a: significant at 0.01 level. b: significant at 0.025 level. c: significant at 0.05 level

* With constrained $\alpha_3 > 0$.

** Loglikelihood with the parameters set for the uniform distribution : -598.3368.

- Loglikelihood for constant parameters p and q ($\alpha_2 = 0$): -522.2904.
- The p-value for the likelihood ratio test between the with and without unit effect models is 0.0987.

V.3 Simulations

We cannot directly compare the estimations of the structural model with the data. The bids of each participant are not directly observable. In the descending-price auctions, only the bid of the winner is observed; in the ascending-price auctions, the winning price reflects the bid of the second highest bidder whose identity is unknown. Using simulations, we can generate data which are compatible with the structural estimations.

The estimated structural models specify stochastic processes generating individual bids. We have assumed that bids are drawn randomly according to some Beta distributions. We use these estimated Beta distribution to generate bids for rounds of 15 sequential auctions. Given some random realizations of bids, we can (i) calculate the winning bids, (ii) identify winners, (iii) calculate levels of inefficiencies, and (iv) estimate standard deviations of bid distributions. The simulation numbers are generated as follows. We first generate valuation vectors through the same stochastic process as presented in Section 2. We generate 100 times 8 vectors of valuations for 8 virtual participants. For each set of valuation vectors, we simulate 100 rounds of 15 auctions. Within a sequence of auctions, valuations and the theoretical bids are adjusted according to the outcome of previous auctions. Overall, 10 000 rounds of 15 auctions were simulated. We have made simulations for 6 models different models.

First, we have considered the case where the allocation is purely random. In each auction, each participant has an equal probability of obtaining the unit on sale irrespective of the participants' valuations or the allocation of previous units. This method of allocation is the dummest way to allocate resources among competing agents. The simulation allows us to compute the degree of inefficiency for this extreme allocative procedure. It is plotted in Figure 1. Naturally, the level of inefficiencies is high relative to the other simulations we have made. The welfare losses measure the difference between the total consumer surplus at the optimal allocation and the realized consumer surplus. The total welfare losses correspond to 184 currency units; since the valuation of the last purchased unit is on average 73 currency units, the welfare lost corresponds to the destruction of the last 2 to 3 units. This is 25 times more than the observed values in the experiments. Figure 1 illustrates how the inefficiencies arise dynamically. A welfare losses arise in the first auction, if the unit is allocated to an individual whose valuation is not among the 15 highest valuations. Clearly, if all participants ought to obtain at least one unit, no welfare losses can arise. When there are s units left to be sold, inefficiencies arise when the unit is allocated to an individual whose valuation is not among the s highest valuations. Naturally, inefficiencies are more likely in the latter stage of the game, when the number of participants who should receive one of the remaining units decreases. Note that the average inefficiency ratio reaches now 14.3% compared to 0.838%, 0.933% and 0.45% in the descending, ascending and mixed auctions experiments.

The second simulation examines the case where participants offer bids which are uniformly distributed between 0 and their valuation, v_i. In this simulation, the participants exhibit no strategic intelligence with the exception that they never bid above their valuation. Note that the participant with the highest valuation has a distribution of bids which stochastically dominants that of other participants. Hence, we can expect that the allocation of resources will be more efficient. The welfare losses is illustrated in Figure 1. The total welfare losses corresponds to 30 currency units, less then half of the expected value of the last unit sold. Again the inefficiencies are more likely to arise at the end when efficiency requires that the last unit be given to the one participant with the highest evaluation. This simple constraint on the bidding by the virtual participants decreases the average inefficiency ratio substantially to a level of 2.31%. These findings duplicate those of Gode and Sunder (1993) on the market rationality even when composed of irrational economic unit and tend to support Evans 's (1997, p.623) questioning "if efficiency is derived mainly from the structure of the market itself rather than from the advantages typically ascribed to human economic agents ".

The other simulations exploits the estimates of the structural models reported in Tables 1, 2 and 3. For each auctions and individual i, we generate bids according to the beta distribution with cumulative $I(\frac{b^*}{v_i}, \hat{p}_i, \hat{q}_i)$. We then select the winner and the winning bid. For each simulations, we calculate the average welfare losses as they occur dynamically. The results are plotted in Figures 2 ,3 and 4. Simulations I are realized with the parameters of the regressions without the unit effects while simulations II take into account those unit effects. We observe that the dynamic of the inefficiencies from these simulations are closely related to the observed ones. The average inefficiency ratios are given in Table 4.

Table 4
Inefficiency Ratios

	Descending-Price	Ascending-Price	Mixed
Observed	0.838%	0.939%	0.45%
Random Allocation	14.3%	14.3%	14.3%
Uniform Bids	2.31%	2.31%	2.31%
Simulation I	1.35%	0.345%	0.285%
Simulation II	0.911%	0.185%	0.247%
Theoretical	0.0%	0.0%	0.0%

Table 4 summarizes the inefficiency ratios, observed and simulated. We note the importance of the game rule or the setting of an institution to enhance the rational behavior of individuals. Uniform bids with participants not allowed to bid over their valuations cut drastically the inefficiencies of a purely random allocation. The inefficiency ratios based on our simulations and structural models indicate that the participants follow in part the equilibrium strategy. This is reinforced when we take into account the units effects in our estimations.

In the Figures 5 to 7, we compare the observed and the simulated winning bids with the winning bids derived from the theory. For the descending-price auctions (Figure 5), the well documented paradox of decreasing winning prices is observed. It is captured by our structural parameter models. For the ascending-price and mixed auctions in Figures 6 and 7, the same paradox is also observed. The structural models predict the decreasing winning prices. Standard-deviations of the simulated bids presented in Figure 8 might offer an explanation for the decreasing-price anomalies. At the start of each round of auctions, the standard deviation associated with the distribution of the winning bids are fairly large. This initiates a bias pushing the winning bids to a higher level at the beginning of the round forcing an adjustment along the way.This explanation is based on the complexity associated with the first units to be auctioned in presence of many units available. It differs from the idea that the risk aversion is at the root of the declining price anomaly, an explanation first suggested by Ashenfelter (1989) and then McAfee and Vincent (1993), and from the absentee bidders of Ginsburgh (1998) In our model the introduction of a risk aversion parameter has little influence on the theoretical bid and there are no absentee bidders

VI Conclusion

The issue of market design has generated considerable interest among the academic economist profession. New innovative market and auction designs have been proposed. This impetus has been inspired by a wave of deregulation and a growing interest by regulator for auctions as allocation mechanism. Game theory has played an important role in the conduct of the large spectrum auctions organized by the Federal Communication Commission, both in the initial design of the auctioning rules and for providing advice to the participants. Experimental economic has provided a similar complementary role. Experiments were used both to test the design of the FCC auction and prepared participants.

This paper had a more limited ambition. It has provided, however, an indication of what can be achieved by exploiting together game theory, experimental data, econometric estimations and simulations. We were able to show that theory matters to explain the behavior of participants in experimental descending and ascending and mixed auctions. The results yield also a strong support in the role of mechanism designs and the definition of the rules of games to help individuals to get closer to full rationality. Much remain to be done nevertheless. We need a better explanation of the declining-price anomaly observed in our results. Other specifications for the estimation of the structural models will have to be explored and replications of auction experiments will be required to confirm our results.

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VIII Appendix

Proof of Property C: We need to show that for all $j>1, x \leq y \leq z$, we have : $\Pr ob\left[w_k^i = x | w_j^i \leq y, w_{j-1}^i = z\right] = \Pr ob\left[w_{k-j+1}^i = x | w_1^i\right]$. For $x \leq y$, we have:

$$\begin{aligned}
\Pr ob\left[w_k = x | w_1 \leq y\right] &= \Pr ob\left[w_k = x, w_1 \leq y\right] / \Pr ob\left[w_1 \leq y\right] \\
&= \sum_{m=k}^{\infty} \frac{\lambda^m \cdot e^{-\lambda}}{m!} \cdot \frac{m!}{m-k!k-1!} F(x)^{m-k} \left[F(y) - F(x)\right]^{k-1} f(x) / e^{-\lambda[1-F(y)]} \\
&= \left[\frac{\lambda^k \left[F(y) - F(x)\right]^{k-1} f(x)}{k-1!}\right] \cdot e^{-\lambda[F(y)-F(x)]} \qquad (15)
\end{aligned}$$

and for $j>1$ and $y \leq z$, we have:

$$\Pr ob\left[w_j \leq y, w_{j-1} = z\right] = \left[\frac{\lambda^{j-1} \left[1 - F(z)\right]^{j-2} f(x)}{j-2!}\right] \cdot e^{-\lambda[1-F(y)]} \qquad (16)$$

$$\begin{aligned}
&\Pr ob\left[w_k = x, w_j \leq y, w_{j-1} = z\right] \qquad (17) \\
&= \sum_{m=k}^{\infty} \frac{\lambda^m \cdot e^{-\lambda}}{m!} \cdot \frac{m!}{m-k!k-1!} F(x)^{m-k} f(x) \frac{k-1!}{k-j!j-2!} \left[F(y) - F(x)\right]^{k-1} f(z) \left[1 - F(z)\right]^{j-2} \\
&= \frac{\lambda^k e^{-\lambda[1-F(x)]}}{k-j!} \cdot \left[F(y) - F(x)\right]^{k-1} f(z) \frac{\left[1 - F(z)\right]^{j-2}}{j-2!}
\end{aligned}$$

Dividing (17) by (16), we obtain:

$$\begin{aligned}
&\Pr ob\left[w_k = x | w_j \leq y, w_{j-1} = z\right] \qquad (18) \\
&= \frac{\lambda^{k-j+1} e^{-\lambda[F(y)-F(x)]}}{k-j!} \cdot \left[F(y) - F(x)\right]^{k-j} \\
&= \Pr ob\left[w_{k-j+1}^i = x | w_1^i \leq y\right]
\end{aligned}$$

Q.E.D.

Lemma 1: *Under the assumptions of the model, we have:*

$$E\left[v_t^{-i} | v_1^j \leq v \text{ for all } j\right] = \int_0^v x \left(\lambda(n-1)\right)^t \frac{\left[F(v) - F(x)\right]^{t-1}}{(t-1)!} f(x) e^{-\lambda(n-1)[F(v)-F(x)]} dx \qquad (19)$$

Proof of Lemma 1: Let m_j be the number of positive valuations for j. By assumption, m_j follows a Poisson process with average λ. The sum of positive valuations for all bidders but i, $\sum_{j \neq i} m_j$, follows a Poisson process with average $(n-1)\lambda$. Hence, we have:

$$\Pr ob\left[v_t^{-i} = x | v_1^{-i} \leq v\right] = \left(\lambda(n-1)\right)^t \frac{\left[F(v) - F(x)\right]^{t-1}}{(t-1)!} f(x) e^{-\lambda(n-1)[F(v)-F(x)]} \qquad (20)$$

which yields the above expected value. Note that the above is strictly increasing in v. One can verify that $db_t(v)/dv = (n-1)\lambda\left[b_{t-1}(v) - b_t(v)\right] > 0$.

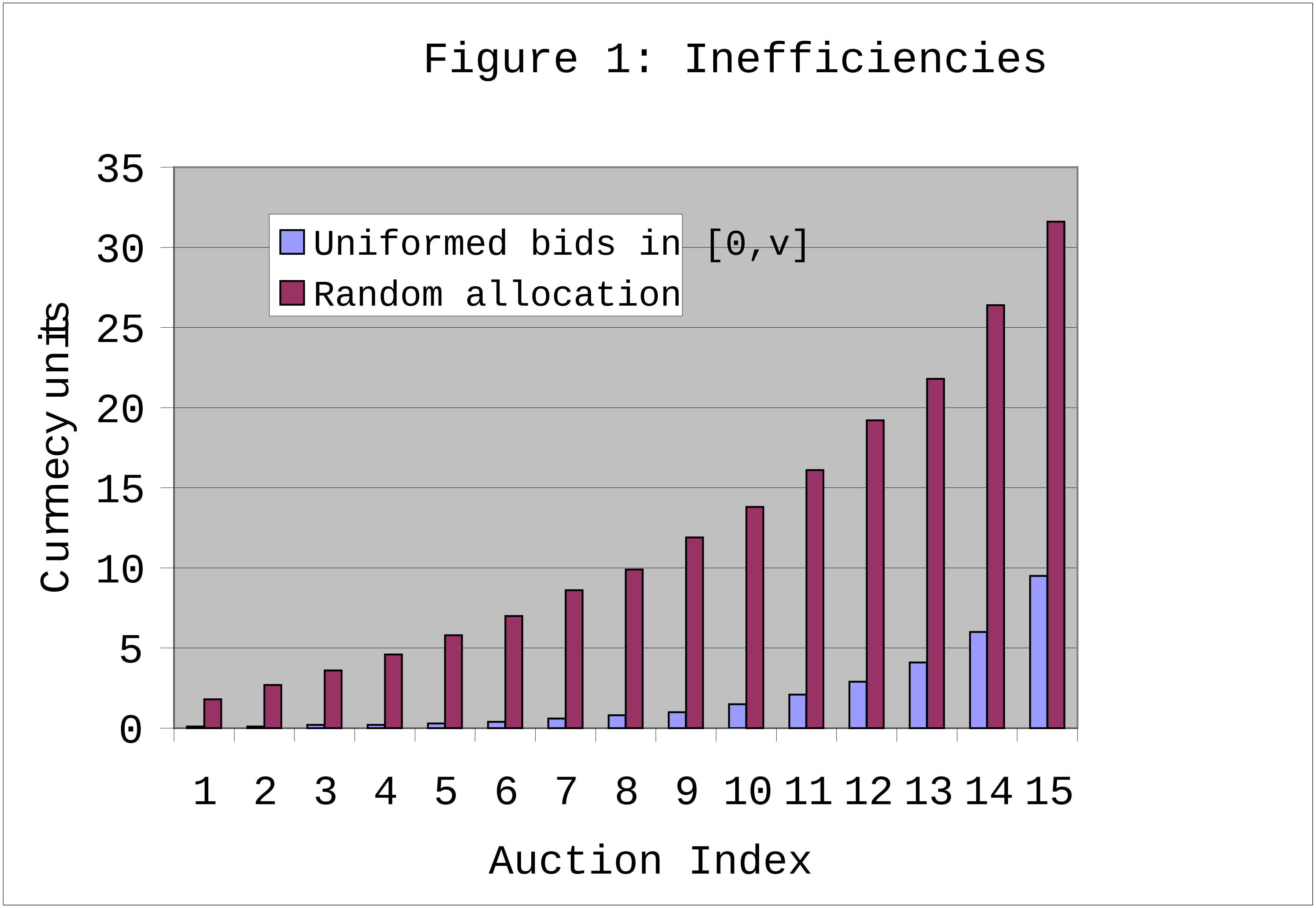


Figure 1: Inefficiencies
Uniformed bids in [0,v]
Random allocation
Currency units
0
5
10
15
20
25
30
35
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction Index

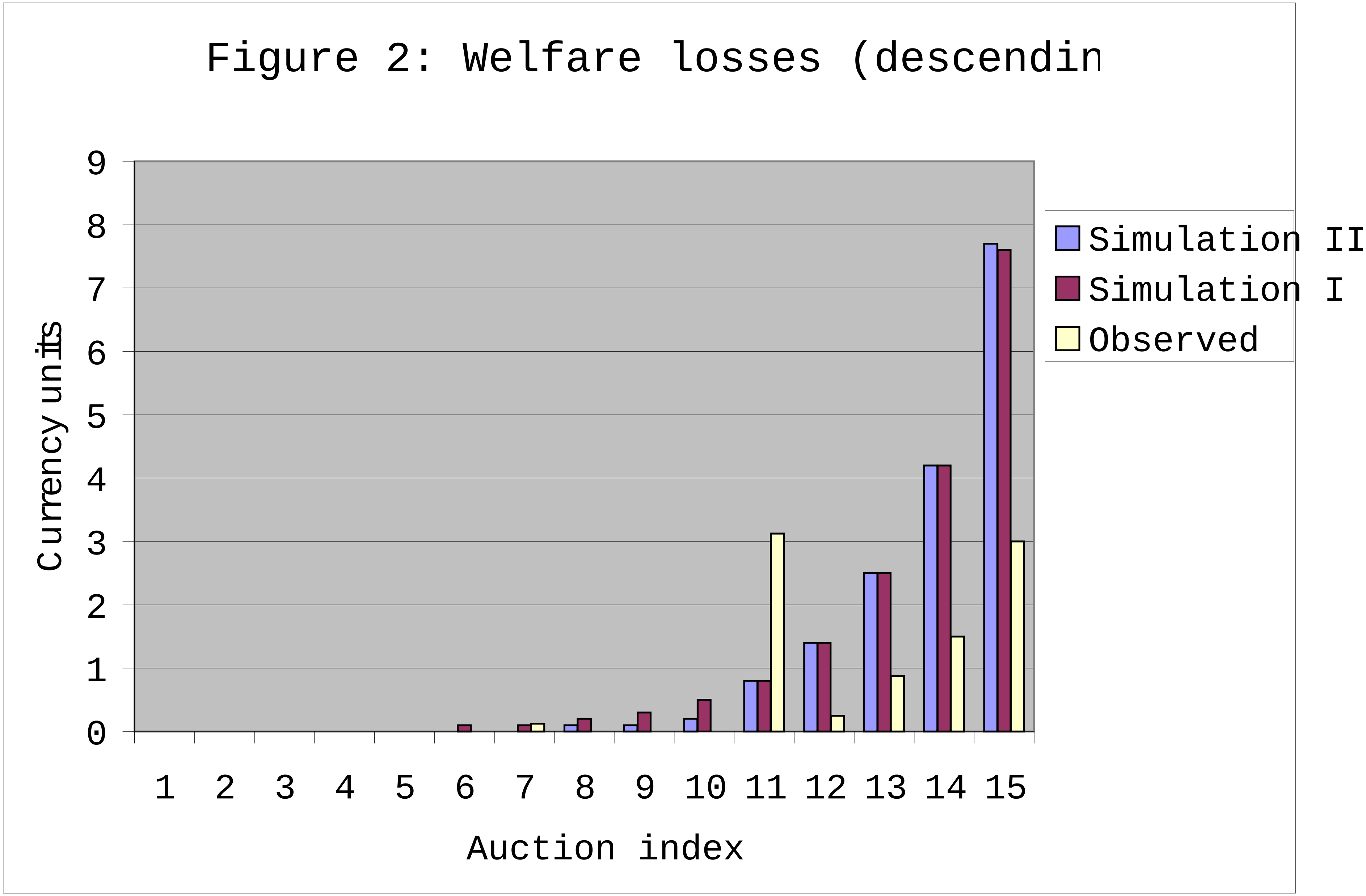


Figure 2: Welfare losses (descendin
9
8
7
6
5
4
3
2
1
0
Currency units
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction index
Simulation II
Simulation I
Observed

Figure 3: Welfare losses (ascending-price)

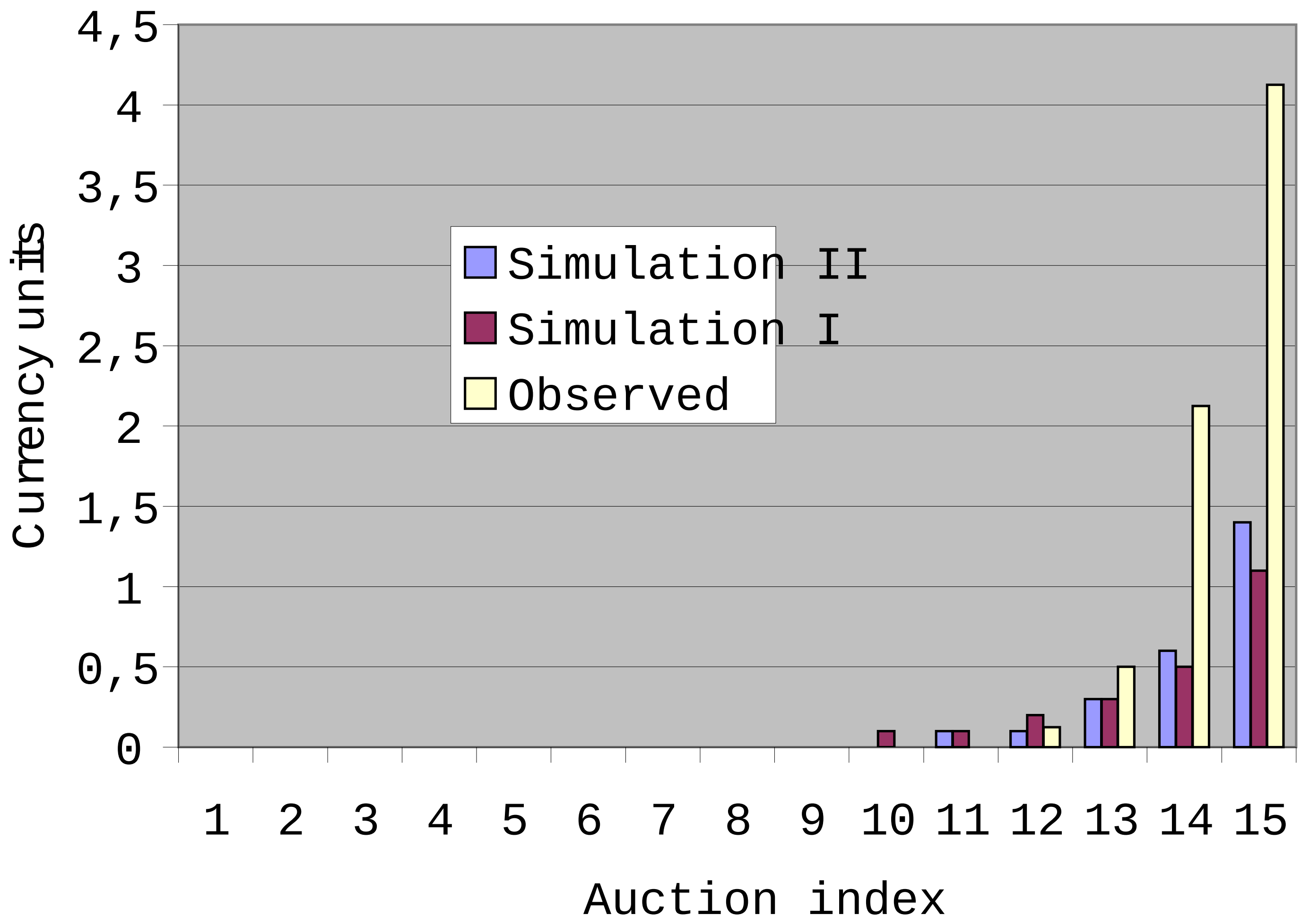
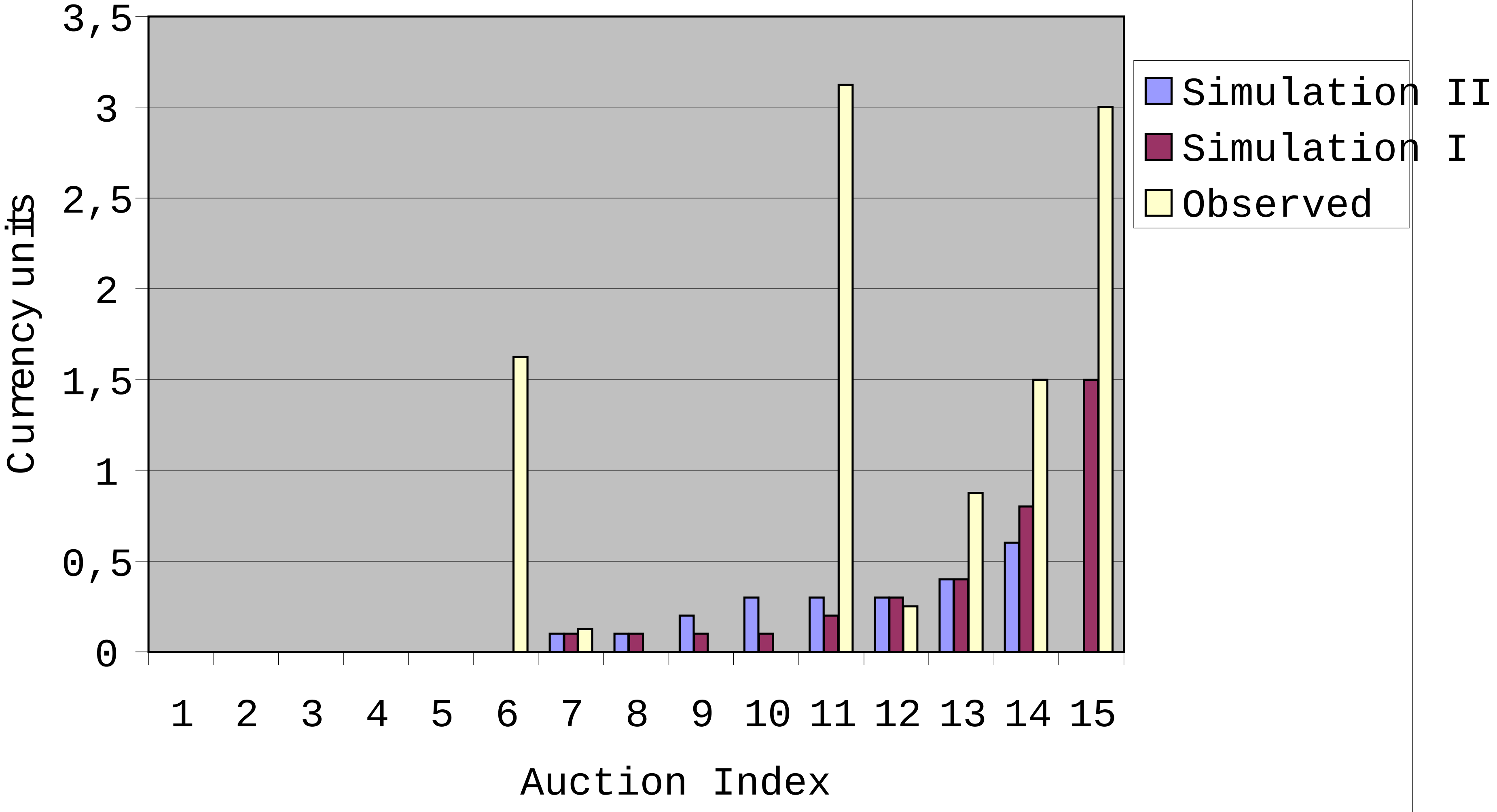


Figure 4: Welfare losses (Mixed auction)



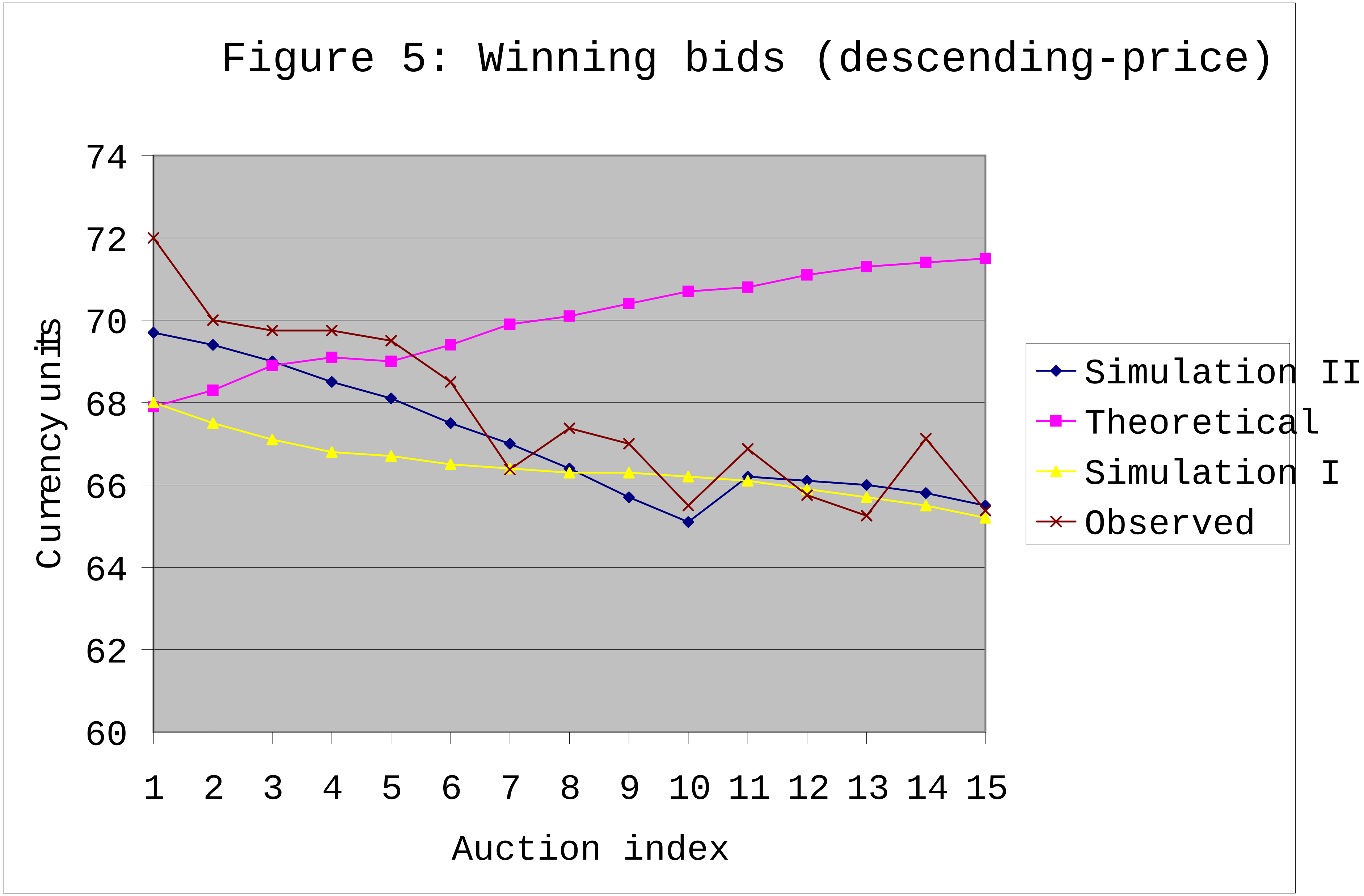


Figure 5: Winning bids (descending-price)
74
72
70
68
66
64
62
60
Currency units
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction index
Simulation II
Theoretical
Simulation I
Observed

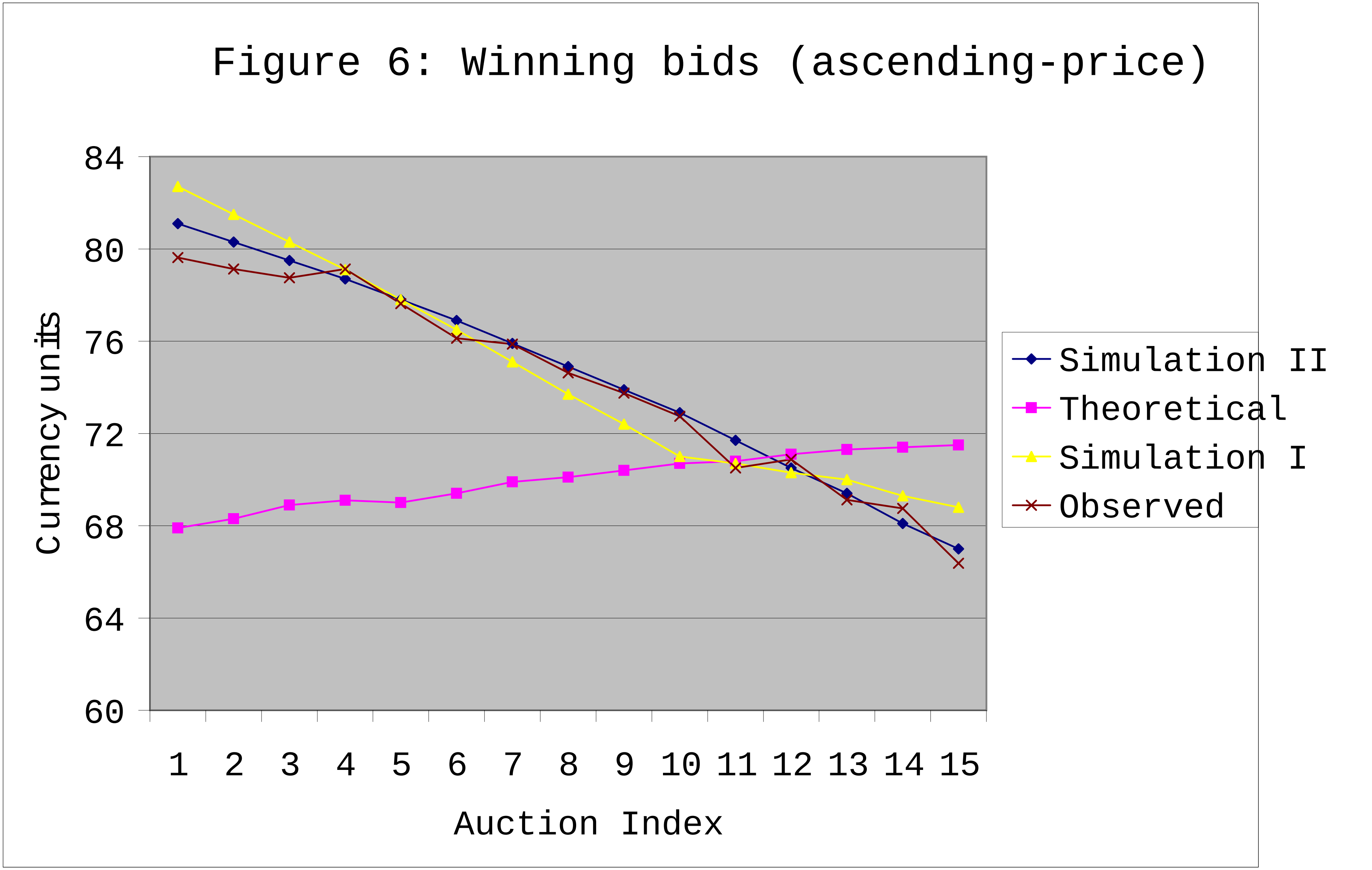


Figure 6: Winning bids (ascending-price)
84
80
76
72
68
64
60
Currency units
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction Index
Simulation II
Theoretical
Simulation I
Observed

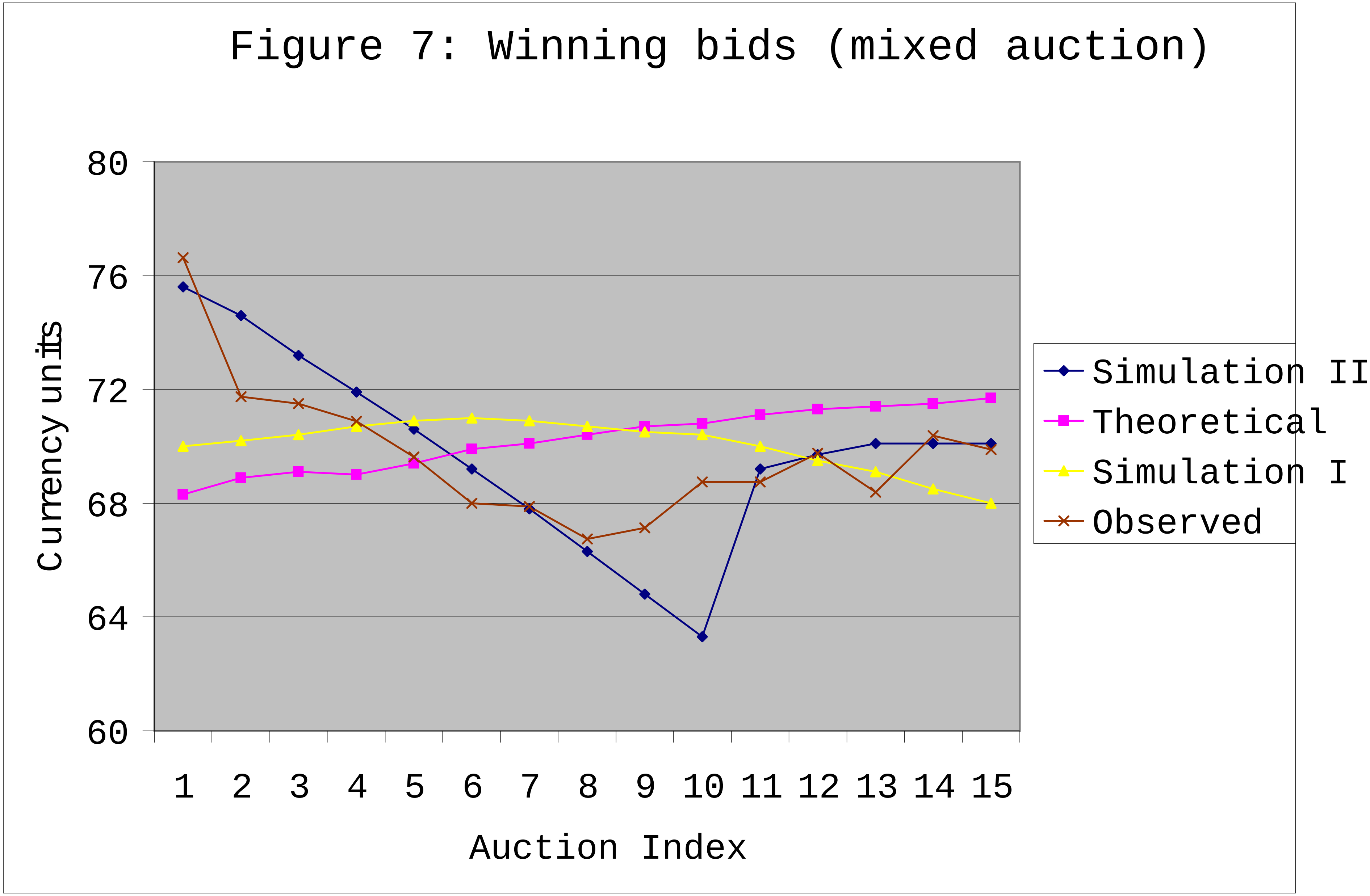


Figure 7: Winning bids (mixed auction)
80
76
72
68
64
60
Currency units
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction Index
Simulation II
Theoretical
Simulation I
Observed

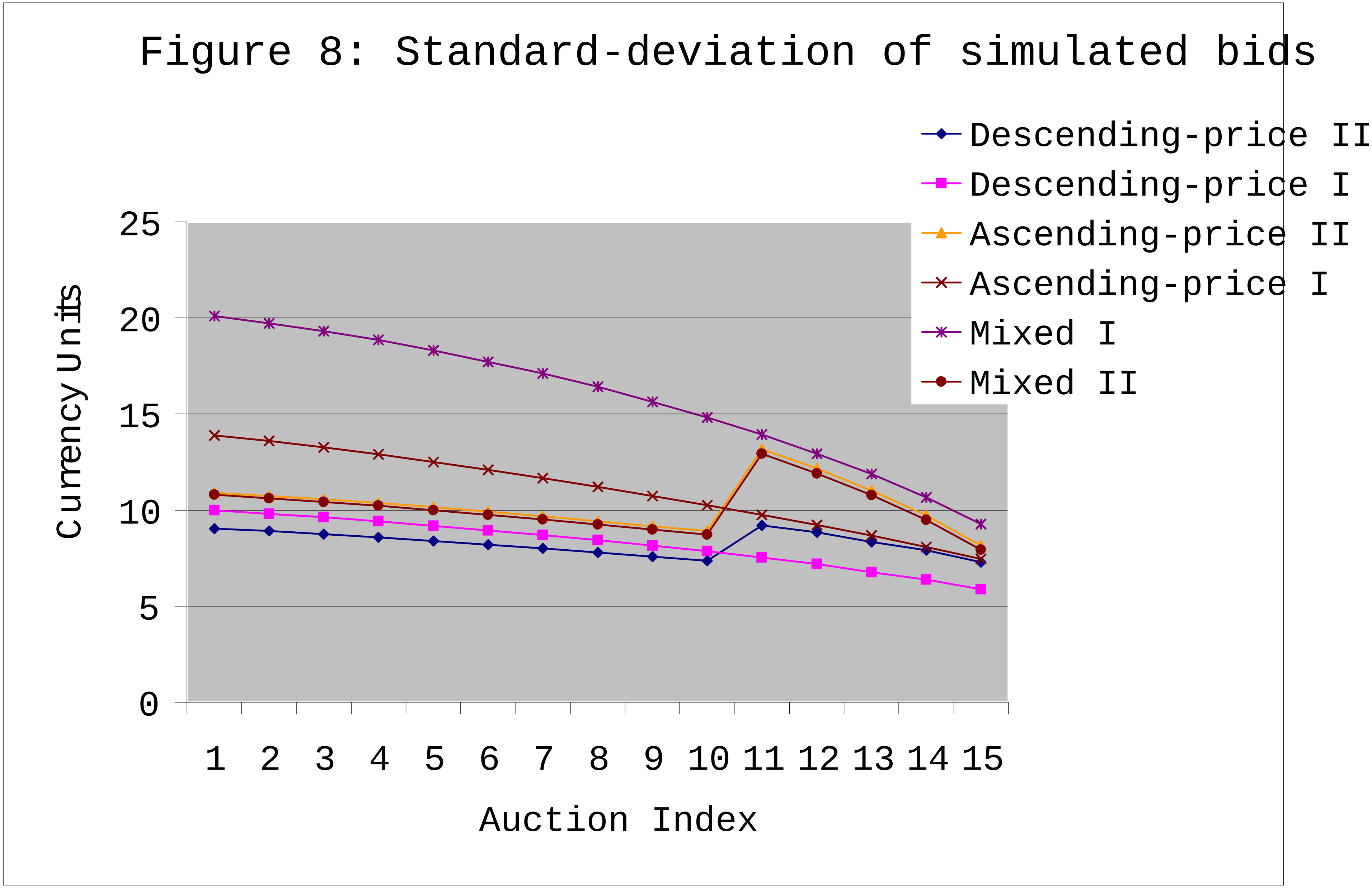


Figure 8: Standard-deviation of simulated bids
Descending-price II
Descending-price I
Ascending-price II
Ascending-price I
Mixed I
Mixed II
25
20
15
10
5
0
Currency Units
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Auction Index

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