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**Fixed Wages, Piece Rates, and  
Intertemporal Productivity: A  
Study of Tree Planters in  
British Columbia**

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# Fixed Wages, Piece rates, and Intertemporal Productivity: A Study of Tree Planters in British Columbia\*

Harry J. Paarsch<sup>†</sup>, Bruce S. Shearer<sup>‡</sup>

## Résumé / Abstract

Nous considérons les effets de différentes séquences de travail et de repos sur la productivité quotidienne des travailleurs qui s'occupent de plantation d'arbres en Colombie-Britannique. Nous faisons une comparaison des profils intertemporels de la productivité des planteurs qui sont payés à taux fixe avec celle des planteurs qui sont payés à la pièce. Nos résultats suggèrent que les planteurs qui sont payés à la pièce sont plus productifs que ceux qui sont payés un taux fixe. Pourtant, la productivité des planteurs qui sont payés à la pièce diminue avec le nombre de journées consécutives travaillées; la baisse de productivité est entre trois et cinq pour cent par jour. Les travailleurs qui sont payés un taux fixe ne démontrent aucune réduction de productivité.

*We examine the effects of different sequences of work and rest on the daily productivity of workers who planted trees in the province of British Columbia, Canada, comparing the intertemporal productivity profiles of planters who were paid either fixed wages or piece rates. We find that planters who are paid piece rates produce more, on average, than those who are paid fixed wages, but that the productivity of piece-rate planters falls with the number of consecutive days worked; the fall in productivity is between three and five percent per day. Fixed-wage planters, on the other hand, showed no such decreases.*

**Mots Clés :** Systèmes de compensation, productivité, repos, récupération

**Keywords :** Compensation Systems, Productivity, Rest, Recuperation

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## 1. Introduction and Motivation

Economists have long been interested in the effects that alternative payment schemes have on performance. Cheung (1969), who focussed on risk sharing under two different payment schemes or contractual arrangements in agriculture (*viz.*, wage labor and sharecropping), is an early example in this line of inquiry. Other examples in the modern contract literature are surveyed by Hart and Holmstrom (1985) as well as by Milgrom and Roberts (1992). In labor economics, authors such as Lazear (1986,1995,1996), Blinder (1990), Brown (1990,1992), and Ehrenberg (1990) as well as Paarsch and Shearer (1996) have focussed on two common payment schemes for workers, fixed wages and piece rates, as important examples of contracts in personnel policy. Under fixed wages, the firm pays the worker a fixed sum of money for each period of time worked (*e.g.*, \$225 per day), while under piece rates the firm pays the worker a fixed sum of money for each task completed (*e.g.*, \$0.25 per tree planted).

Most researchers in the economics and personnel literature have focussed on the notion that, with costly monitoring, workers will expend more effort under piece-rate payment schemes than under fixed-wage payment schemes. Therefore, piece rates are considered more economically efficient than fixed wages. In this paper, we examine the corollarial notion that humans are not machines and that they tire. Thus, the amount of effort that a worker expends in conjunction with the amount of rest that that worker takes can affect the ability of his body to regenerate. Today's choices of effort and rest can affect tomorrow's productivity because expending more effort today increases fatigue tomorrow.

To answer the question "What are the intertemporal effects on ef-

fort and productivity when workers are paid either fixed wages or piece rates?”, we develop a simple, intertemporal model of effort choices under fixed wages and piece rates. We then trace through the effects that these effort choices have on fatigue and productivity, under different assumptions. Finally, we investigate empirically the effects of different sequences of work and rest on the daily productivity of workers who were paid either fixed wages or piece rates for planting coniferous seedlings in the province of British Columbia, Canada.

Tree planting is a simple task. It involves digging a hole with a special shovel, placing a seedling in this hole, and then covering its roots with soil, ensuring that the tree is upright and that the roots are fully covered. While the task may be simple, it can be quite demanding physically. For example, some planters have been known to expend 6,000 calories of energy per day. Working this hard for one day and then resting is one thing, but planters typically work five to six consecutive days without rest days. In some cases, planters have been known to work as many as eleven consecutive days without rest days.

What toll do such sequences of planting and rest take on productivity, and how do the effects differ between fixed-wage and piece-rate payment schemes? The answers to these questions are not only of academic interest, but also of policy relevance in British Columbia where some 170 to 200 million seedlings planted per year: small improvements in personnel policy could result in large savings because of the enormous scale of planting.

From the personnel and payroll records of a medium-sized, tree-planting firm in the southeastern corner of British Columbia, we have obtained daily production records for 51 individuals who were involved in a total of 570 different work-and-rest cycles, 67 involving fixed-wage

planters and 503 involving piece-rate planters. In all, the data set contains 1,928 daily productivity observations, of which 223 concern planters paid fixed wages and 1,705 concern planters paid piece rates. We find that planters paid piece rates produce more, on average, per day than those paid fixed wages. We also find that average daily production for piece-rate planters falls at a rate between three and five percent per day, while that for fixed-wage planters appears unaffected by the number of consecutive days worked. Even after ten or eleven consecutive days of work, however, piece-rate planters produce more, on average, than fixed-wage planters.

The paper is in five more parts. In the next section, we describe briefly some institutional features of the tree-planting industry in British Columbia, while in section 3, we develop a simple theoretical model which highlights the intertemporal effects that the different payment schemes have on effort, fatigue, and productivity. In section 4, we develop a measurement equation that can be estimated using existing data and present our empirical estimates of the effects of work and rest on productivity under the different payment schemes. We conclude the paper in section 5, while in an appendix to the paper, we describe the creation of the data set used.

## **2. Tree Planting in British Columbia**

British Columbia produces around twenty-five percent of the softwood lumber in North America. To ensure a steady supply of timber, the factor input necessary to produce this lumber, extensive reforestation is undertaken by both the Ministry of Forests and the major timber-harvesting firms who hold Tree Farm Licenses. <sup>2</sup>

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<sup>2</sup> In British Columbia, nearly 90 percent of all timber is on government-

The mechanics of this reforestation are relatively straightforward. Prior to the harvest of any tract of coniferous timber, random samples of cones are taken from the trees on the tract, and seedlings are grown from the seeds contained in these cones. This practice ensures that the seedlings to be replanted are compatible with the local microclimates and soil as well as representative of the historical species composition. After the tract has been harvested, the land is prepared for planting, often by burning whatever slash timber remains and sometimes by “screefing” the forest floor. Screefing involves removing the natural build-up of organic matter on the forest floor so that the soil is exposed. Screefing makes planting easier because seedlings cannot be planted in screef, the organic material, but must be planted in soil.

Tree planting is a simple task. It involves digging a hole with a special shovel, placing a seedling in this hole, and then covering its roots with soil, ensuring that the tree is upright and that the roots are fully covered with soil. The typical density of seedlings is about 1,800 stems per hectare, or an inter-tree spacing of about 2.4 metres, although this can vary substantially.<sup>3</sup> A typical work day begins at 7:00 a.m. and ends around 5:00 p.m., with an hour for lunch as well as two or three 15

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owned (Crown) land. The Crown, through the Ministry of Forests, sells the right to harvest the timber on this land in two different ways. The most common way is at administratively set prices to thirty-four firms who hold Tree Farm Licenses. These licenses have been negotiated over the last three-quarter century, and require that the licensee adopt specific harvesting as well as reforestation plans. About 90 percent of all Crown timber is harvested by firms holding Tree Farm Licenses. The second, and less common way, to sell timber is at public auction through the Small Business Forest Enterprise Program. In this case, the Ministry of Forests assumes the responsibility of reforestation.

<sup>3</sup> One hectare is an area 100 metres square, or 10,000 square metres. Thus, one hectare is approximately 2.4711 acres.



minute breaks throughout the day. An average planter can plant about 900 trees per day, about half a hectare. An average harvested tract is around 250 hectares, so planting such a tract would take about 500 man days. Because the planting season is short, except to move to a new sites, planters usually work six days a week with one day off. Moving to a new site to plant is typically considered time off, and planters are usually not paid for this time. Days of rest are not endogenous; those planters who find the work too arduous generally leave the firm. However, extremely bad weather is a cause for rest days.

Typically, tree-planting firms are chosen to plant seedlings on harvested tracts through a process of competitive bidding. Depending on the land tenure arrangement, either a timber harvesting firm or the Ministry of Forests will call for sealed-bid tenders concerning the cost per tree planted, with the lowest bidder being selected to perform the work. Typically, a list of qualified potential bidders is maintained, with new firms getting on the list by first soliciting small, relatively unimportant jobs and being successful on those contracts.

Tree-planting firms are typically quite small, usually having fewer than one hundred workers. The prevalent way in which planters are hired is as follows: The principals of the firms hire several foremen. The principals then usually delegate the task of hiring planters to the foremen. Foremen are expected to find suitable planters for the firm. Since foremen are invariably responsible administratively for their planters, they have an incentive to find good ones. Typically, a foreman is responsible for between ten and fifteen planters.

Planters are paid using a variety of schemes. The most common one is a piece-rate payment scheme. In this case, the planter is paid a specific amount for each tree planted; *e.g.*, \$0.25 per tree. The next most common

payment scheme is a fixed-wage scheme. In this case, the planter is paid a specific amount for each day worked; *e.g.*, \$225 per day. A third payment scheme, not considered below, involves paying planters by the hectare for trees planted; *e.g.*, \$450 per hectare.

Under all payment schemes, planters are monitored carefully by their foremen. Foremen are mainly paid according to the output of their teams. For example, a foreman may get ten percent of his team's gross earnings. Many foremen also plant in which case they are paid according to the scheme prevailing for planters, while some foremen also get weekly or monthly salaries. In some cases, foremen are actually residual claimants, being paid whatever remains after planters, supplies, penalties, *etc.* have been paid.

Individual planters as well as their teams are penalized for poor work. Penalties can come in a variety of forms. Severe infractions, such as hiding or "stashing" trees, can result in job loss.<sup>4</sup> For infractions involving substandard planting, planters can have their pay withheld, and can be asked to replant the site free of charge. Replanting is a difficult task where all of the existing planted seedlings must first be removed before the new ones can be planted.

The tree-planting season in British Columbia, particularly in the interior of the province, is very short, typically from early May until early September, between 16 and 20 weeks. On the coast of the province, if there is no snow, some planting can occur in March and April. Planters are hired with the expectation that they will work the entire season. Labor laws in British Columbia guarantee that planters be given rest every six

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<sup>4</sup> Planters paid under piece rates will sometimes hide or "stash" seedlings, usually burying them *en masse*, claiming later to the foreman that the seedlings were planted.

days, although these laws are sometimes ignored, as we shall see in the data below.

The industry attracts three types of workers: professional planters who plant each year, students who plant as a summer job while they attend college, and novices. Some of the novices are also college students, but other novices will choose planting as their occupation. The bulk of labor turnover typically occurs within the month of May. Several reasons for this exist. First, many novices find the job too arduous, and quit. Second, some novices are not productive enough. Labor laws in British Columbia dictate that planters must earn at least the minimum wage. Some planters do not attain this standard within the first two pay periods (four weeks), and are fired. Turnover among seasoned planters and those who make it through May is typically quite low because bonuses (which can be as much as 10 percent of gross salary) are typically only paid to those who work the entire planting season.

### 3. Simple Theoretical Model

In order to get some notion concerning how fixed wages and piece rates can affect the effort decisions of workers, we consider the following simple, theoretical model: we assume that  $q(t)$ , output in period  $t$ , depends on a control variable, effort  $e(t)$ , and a state variable, “fatigue”  $x(t)$ . One could think of  $x(t)$  as a vector containing a number of measures of fatigue such as the concentration of lactic acid in the bloodstream, *etc.*, but we shall treat it as a scalar. Thus,

$$q(t) = f(e(t), x(t)) \tag{3.1}$$

with  $f(0, x)$  being zero; *i.e.*, when no effort is expended, no output

obtains. Later, we shall assume that the firm can impose a minimum level of effort  $\bar{e}$ .

Introducing the shorthand notation of  $f_z$  for  $\partial f/\partial z$  where, in this case,  $z$  can be either  $e$  or  $x$ , we assume that  $f_e$  is positive, so more effort increases output, but that  $f_{ee}$  is non-positive, extra effort has weakly diminishing marginal productivity. We also assume that  $f_x$  is non-positive, so increases in fatigue can reduce productivity, and that  $f_{xx}$  is non-positive, the effects of fatigue are increasingly negative on output. We assume no technological progress or learning, so the production function does not depend explicitly on  $t$ .

Under fixed wages, workers get a wage  $w$  for each period worked. Thus,  $y(t)$ , earnings in period  $t$ , are simply

$$y(t) = \begin{cases} w & \text{when working;} \\ 0 & \text{at rest.} \end{cases}$$

Under piece rates, workers are paid a price  $p$  for each unit of output  $q(t)$  produced, so

$$y(t) = \begin{cases} pq(t) & \text{when working;} \\ 0 & \text{at rest.} \end{cases}$$

We assume that workers gain utility from earnings, but disutility from effort and fatigue. We assume further a time-separable utility function over earnings, effort, and fatigue in each period

$$U(y(t), e(t), x(t)) \tag{3.2}$$

where  $U_y$  is positive, representing the positive marginal utility of earnings, and  $U_{yy}$  is non-positive, representing the weakly decreasing marginal utility of earnings. We also assume that  $U_e$  is negative, representing the marginal disutility of effort; that  $U_{ee}$  is non-negative, representing the increasing marginal disutility of extra effort; that  $U_x$  is non-positive,

representing the potentially negative effect of fatigue on utility; and that  $U_{xx}$  is non-positive, representing the potentially increasingly negative effect which fatigue has on utility.

Note that by allowing fatigue to enter directly into the production and utility functions, we are considering a more general class of models than is typically used. Most researchers (including ourselves in Paarsch and Shearer [1996]) employ a production function which is independent (at least directly) of fatigue. In general, fatigue can be interpreted as a state variable that affects the efficiency with which effort is transformed into output; *i.e.*, a fatigued individual will plant fewer trees per unit of effort than will a non-fatigued individual. In the more restricted model, fatigue is interpreted as the cost of effort that enters only the utility function. Below, we show how panel data on fixed-wage planters allows us to identify and to test for the presence of fatigue effects directly in the production function.

As a first approximation, we consider the case where the number of consecutive days that a planter can work without rest days is fixed at  $T_1$  by either the firm or the government and that the length of the rest period  $r$  is also fixed, so the total horizon  $T_2$  equals  $T_1$  plus  $r$ . Initially, we consider the problem of but one spell of work and rest. Later, we incorporate the complication of repeated spells.<sup>5</sup>

Since we consider very short time horizons, discounting is ignored.

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<sup>5</sup> We assume that  $T_1$  and  $r$  are fixed either by law or by the firm through some process of scientific management; see, for example, Smith's (1987) attempt to use the methodological approach first proposed by Taylor (1947) to the case of tree planters.

Thus, total utility over the time horizon  $[0, T_2]$  is

$$\int_0^{T_2} U(y(t), e(t), x(t)) dt = \int_0^{T_1} U(y(t), e(t), x(t)) dt + \int_{T_1}^{T_2} U(0, 0, x(t)) dt.$$

We assume that the equation of motion which describes how fatigue evolves is a homogeneous first-order differential equation, so improved conditioning (*e.g.*, which implies an improvement in the body's ability to absorb lactic acid), is ruled out. Thus,

$$\dot{x}(t) = \frac{dx(t)}{dt} = g(e(t), x(t)) \quad (3.3)$$

where  $g_e$  is positive, the more effort expended, the faster the build-up of fatigue, with  $g_{ee}$  being non-negative, extra effort increases fatigue build-up faster. We also assume that the effect of extra effort depends on the level of fatigue; *viz.*,  $f_{ex}$  is non-negative because effort can increase fatigue at a faster rate when one is tired than when one is fresh.

In the next two subsections, we describe the optimal choices of a planter who works one spell of  $T_1$  periods with a rest spell of  $r$  periods under fixed wages and piece rates.

### 3.1. Fixed Wages

Under fixed wages, the planter wants to supply the minimum possible level of effort in each period. Extra effort implies costs without any benefits since his wage is independent of effort. We assume further that the foreman can and does impose a minimum effort level of  $\bar{e}$  in each period. In this case, output in each period  $t$  on  $[0, T_1]$  conditional on effort  $\bar{e}$  is

$$q(t|\bar{e}) = f(\bar{e}, x(t))$$

and zero on  $[T_1, T_2]$ , while earnings on  $[0, T_1]$  are

$$y(t) = w$$

and zero on  $[T_1, T_2]$ , so total utility over the interval  $[0, T_2]$  is

$$\int_0^{T_1} U(w, \bar{e}, x(t|\bar{e})) dt + \int_{T_1}^{T_2} U(0, 0, x(t|0)) dt$$

where  $x(t|\bar{e})$ , the path of fatigue conditional on fixed effort  $\bar{e}$ , is determined by the solution to

$$\dot{x}(t|\bar{e}) = g(\bar{e}, x(t)) \quad t \in [0, T_1]$$

with initial condition  $x(0)$  equalling  $x_0$ , the value of the state variable for a fully rested worker, while  $x(t|0)$  on  $[T_1, T_2]$  is the solution to

$$\dot{x}(t|0) = g(0, x(t)) \quad t \in [T_1, T_2]$$

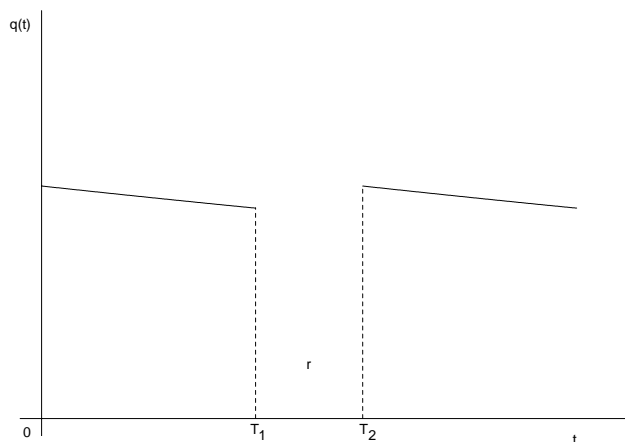
with initial condition  $x(T_1)$  equalling  $x(T_1|\bar{e})$ .

The effect of elapsed work time on output can be determined by differentiating the production function conditional on effort  $\bar{e}$ . Hence,

$$\begin{aligned} \dot{q}(t|\bar{e}) &= \frac{dq(t|\bar{e})}{dt} = f_e \times \frac{d\bar{e}}{dt} + f_x \times \frac{dx(t|\bar{e})}{dt} \\ &= f_e \times (0) + f_x \times \dot{x}(t|\bar{e}) \\ &= f_x \times \dot{x}(t|\bar{e}). \end{aligned} \tag{3.4}$$

With fatigue entering the production function, the sign of  $\dot{q}(t|\bar{e})$  is indeterminate. For while the sign of  $f_x$  is assumed to be negative, the sign of  $\dot{x}(t|\bar{e})$  is indeterminate, depending on  $\bar{e}$  and  $x_0$  as well as the specific structure of the “fatigue function”  $g(e, x)$  not just its partial derivatives. In Figure 1, we show one possible time path of  $\dot{q}(t|\bar{e})$  in which workers are becoming more fatigued as time progresses.

**Figure 1**  
**Productivity over Time under Fixed Wages**



It is clear how panel data on worker productivity under fixed wages will identify fatigue effects in the production function. Since planters always supply  $\bar{e}$ , the minimum possible effort level under fixed wages, changes in productivity over time identify the effects of fatigue. Note that if fatigue does not enter directly into the production function then (3.4) is equal to zero; no change in productivity occurs over time for fixed-wage planters. In the empirical work below, we will test whether fatigue enters directly into the production function by measuring the change in output over time under fixed wages.

### 3.2. Piece Rates

How much effort should a piece-rate planter put forth? The simplest way to investigate this question is to couch it in terms of an optimal control problem. The planter seeks a path for effort to maximize total utility



subject to the constraints imposed by technology. Thus,

$$\max_{\{e(t)\}} \int_0^{T_1} U(pf(e(t), x(t)), e(t), x(t)) dt + \int_{T_1}^{T_2} U(0, 0, x(t)) dt.$$

subject to

$$\begin{aligned} e(t) &\geq 0 \\ x(t) &\geq x_0 \\ x(0) &= x_0 \\ \dot{x} &= g(e, x). \end{aligned}$$

Introducing the costate variable  $\lambda(t)$  as well as  $\mu(t)$  and  $\psi(t)$ , the Kuhn-Tucker multipliers associated with non-negative effort and the lower bound on fatigue, the Hamiltonian  $\mathcal{H}$  for this problem on  $[0, T_1]$  is

$$\begin{aligned} \mathcal{H} = &U(pf(e(t), x(t)), e(t), x(t)) + \lambda(t)g(e(t), x(t)) + \\ &\mu(t)e(t) + \psi(t)(x(t) - x_0) \end{aligned}$$

whence come the necessary first-order conditions:

$$\begin{aligned} -\mathcal{H}_x &= \dot{\lambda} = -pU_y f_x - U_x - \lambda g_x - \psi \\ \mathcal{H}_e &= 0 = pU_y f_e + U_e + \lambda g_e + \mu \\ \mu(t)e(t) &= 0 \\ \psi(t)(x(t) - x_0) &= 0 \\ x(0) &= x_0 \\ \lambda(T_2) &= 0. \end{aligned}$$

To simplify the analysis, we make an ‘‘Inada-type’’ assumption concerning the marginal utility of earnings at zero, which means that  $e(t)$

will be strictly positive, so  $\mu(t)$  will equal zero for all  $t$  on  $[0, T_1]$ . We also assume that the differential equation (3.3) has structure such that  $x(t)$  cannot pass to  $x_0$  (*i.e.*, a fully rested person having  $x(t)$  equal  $x_0$  cannot become any more rested) when effort  $e(t)$  is positive, so  $\psi(t)$  equals zero for all  $t$  on  $[0, T_1]$ .

Without additional structure, it is still difficult to make any precise statements concerning the effect of piece rates on effort. In Figure 2, we present some sample paths for effort and fatigue derived for “reasonable”  $f(e, x)$ ,  $g(e, x)$ , and  $U(y, e, x)$  functions.<sup>6</sup> These translate into sample paths for output in Figure 3.

### 3.3. More than One Work-and-Rest Cycle

In previous subsections of this section, we analyzed the case of a single work-and-rest cycle. In reality, planters are hired for a sequence of these cycles (as many as twenty). The solution to the fixed-wage problem is not substantially different, but that to the piece-rate problem is quite

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<sup>6</sup> Specifically, when the production function is

$$q(t) = f(e(t), x(t)) = \alpha e(t)^\beta - \gamma x(t),$$

the utility function is

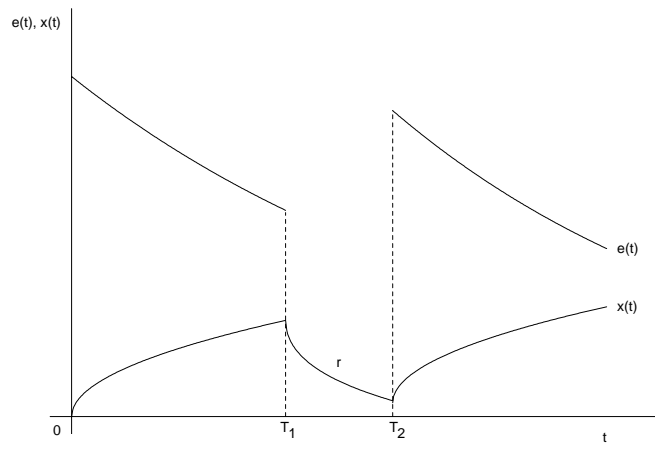
$$U(y(t), e(t), x(t)) = y(t) - e(t) = pq(t) - e(t) = p\alpha e(t)^\beta - p\gamma x(t) - e(t),$$

and fatigue is governed by

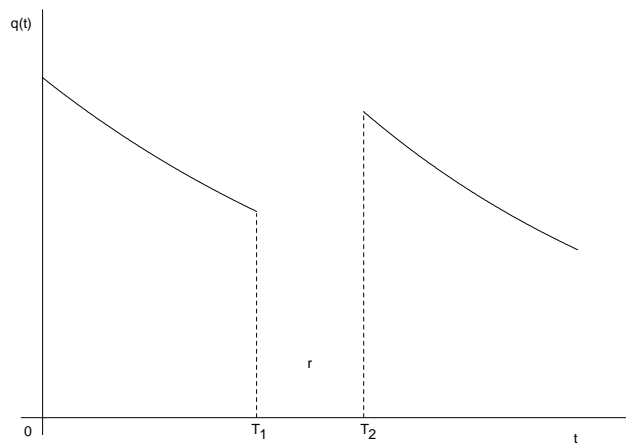
$$\dot{x}(t) = g(e(t), x(t)) = \eta e(t)x(t) - \delta x(t) \exp(-\rho x(t)),$$

no closed-form solution can be obtained for the pair of differential equations defining the equilibrium paths of the costate and state variables, but for explicit values of the parameters, one can solve this two-point, boundary-value problem using the method of Runge-Kutta; see Judd (1995) for more on this.

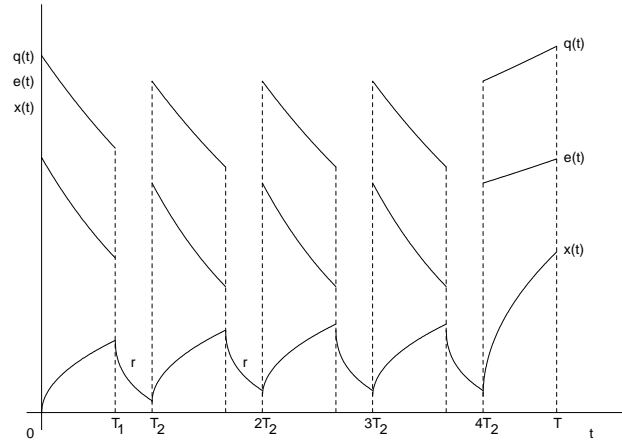
**Figure 2**  
**Effort and Fatigue over the Work Cycle**



**Figure 3**  
**Productivity over Time under Piece Rates**



**Figure 4**  
**Multiple Spells of Work and Rest**



different. Before introducing additional mathematics, we outline here the intuition behind the solution. Discussion is aided by the use of a diagram, Figure 4.

There are three “phases” to the repeated work-and-rest problem. The first, between 0 and  $T_2$  in Figure 4, is the initial planting cycle. During this phase, the planter’s fatigue rises above the fully-rested state  $x_0$ . In the second phase, between  $2T_2$  and  $4T_2$  (or, in general, between  $kT_2$  and  $\ell T_2$  for some integers  $k$  and  $\ell$ ), the planter is in a “steady-state” with regard to effort and fatigue.<sup>7</sup> In the final phase, between  $4T_2$  and  $T$  in Figure 4, the “big push”, the planter recognizes that extra effort has only implications for this last cycle, and works particularly hard.<sup>8</sup> Depending

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<sup>7</sup> This is not strictly true as a steady-state would obtain only in the infinite horizon problem.

<sup>8</sup> The big push could begin in cycles before the last, but it will be most

on the production and fatigue functions, output may rise, fall, or be flat, but both effort and fatigue rise during this last cycle.

Formally, the optimal control problem for  $(J+1)$  work sequences and  $J$  rest periods is

$$\begin{aligned} \max_{\langle e(t) \rangle} \sum_{j=1}^J & \left[ \int_{(j-1)T_2}^{jT_2-r} U\left(pf(e(t), x(t)), e(t), x(t)\right) dt + \right. \\ & \left. \int_{jT_2-r}^{jT_2} U\left(0, 0, x(t)\right) dt \right] + \\ & \int_{JT_2}^T U\left(pf(e(t), x(t)), e(t), x(t)\right) dt + V(x(T)) \end{aligned}$$

subject to

$$e(t) \geq 0$$

$$x(t) \geq x_0$$

$$x(0) = x_0$$

$$\dot{x} = g(e, x)$$

where  $V(x(T))$  represents the planter's valuation of fatigue at the endpoint.<sup>9</sup> Introducing the costate variable  $\lambda(t)$  as well as  $\mu(t)$  and  $\psi(t)$ , the Kuhn-Tucker multipliers associated with non-negative effort and the lower bound on fatigue, the Hamiltonian  $\mathcal{H}$  for this problem on  $[(j-1)T_2, jT_2-r]$  for  $j = 1, \dots, J$  as well as  $[JT_2, T]$  is

$$\begin{aligned} \mathcal{H} = & U\left(pf(e(t), x(t)), e(t), x(t)\right) + \lambda(t)g(e(t), x(t)) + \\ & \mu(t)e(t) + \psi(t)(x(t) - x_0) \end{aligned}$$

---

pronounced in the last cycle.

<sup>9</sup> In this problem,  $T_1$  and  $r$  are taken as given, perhaps by law. Another personnel problem, which is beyond the scope of this paper, involves the optimal choices of work-spell and rest-period durations.

whence come the necessary first-order conditions:

$$\begin{aligned}
 -\mathcal{H}_x &= \dot{\lambda} = -pU_y f_x - U_x - \lambda g_x - \psi \\
 \mathcal{H}_e &= 0 = pU_y f_e + U_e + \lambda g_e + \mu \\
 \mu(t)e(t) &= 0 \\
 \psi(t)(x(t) - x_0) &= 0 \\
 x(0) &= x_0 \\
 \lambda(T) &= V_x(x(T)).
 \end{aligned}$$

As one might expect, comparative dynamic analysis in this problem is very sensitive to functional forms. Ambiguous predictions are the best that this structure can provide in the absence of very strong (implausible) assumptions.

#### 4. Measurement Equation and Empirical Results

The theory discussed in section 3, however simple, is couched in terms of variables that are notoriously difficult to measure, effort and fatigue. Thus, implementing a structural model mapping directly from the economic theory to an empirical specification is unlikely to be successful. In addition, only under very strong (implausible) assumptions can one make very specific comparative dynamic predictions. This is not uncommon in labor economics. Consider, for example, labor supply, where the slope of the supply function is, in general, indeterminate. In this section, we develop a measurement equation that allows us to estimate the reduced-form dynamic paths of daily productivity for cycles not at the end of the planting season, under piece-rate and fixed-wages contracts.

A natural way in which to investigate the effects of work and rest on planter productivity under different payment schemes would be to undertake a controlled experiment. Essentially, under each payment scheme  $h$ , one would find a group of individuals indexed by  $i$ , and subject them to sequences of work indexed by  $j$  over a number of work cycles indexed by  $k$  with prior numbers of rest days indexed by  $\ell$  starting from being fully rested, and then repeat this a number of times indexed by  $s$ . Thus, introducing the dummy variables for rest

$$R_{is}^{\ell k} = \begin{cases} 1 & \text{if individual } i \text{ for sample } s \text{ rested} \\ & \ell \text{ periods prior to work cycle } k, \\ 0 & \text{otherwise;} \end{cases}$$

and the dummy variables for working consecutive days

$$W_{is}^{jk} = \begin{cases} 1 & \text{if individual } i \text{ for sample } s \text{ is} \\ & \text{working the } j^{\text{th}} \text{ consecutive day} \\ & \text{in work cycle } k, \\ 0 & \text{otherwise;} \end{cases}$$

one could then write down an empirical specification for the conditional mean of the logarithm of daily production under payment scheme  $h$  for planter  $i$  in sample  $s$  who has worked  $j$  consecutive days during work cycle  $k$  and has rested  $\ell$  days prior to this work cycle of the following form:

$$\begin{aligned} \text{E}[\log Q_{is}^h | R_{is}^{\ell k}, W_{is}^{jk}] &= \sum_{j=1}^{\mathcal{J}_h} \sum_{k=1}^{\mathcal{K}_h} \omega_{Whi}^{jk} W_{is}^{jk} + \sum_{k=1}^{\mathcal{K}_h} \sum_{\ell=1}^{\mathcal{L}_h} \omega_{Rhi}^{k\ell} R_{is}^{k\ell} + \\ &\sum_{j=1}^{\mathcal{J}_h} \sum_{k=1}^{\mathcal{K}_h} \sum_{\ell=1}^{\mathcal{L}_h} \omega_{RWhi}^{jk\ell} R_{is}^{k\ell} \cdot W_{is}^{jk} \end{aligned} \quad (4.1)$$

where  $h$  takes on the values  $\{F, P\}$  for fixed wages and piece rates,  $i$  takes on the values  $\{1, 2, \dots, \mathcal{I}_h\}$ ,  $j$  takes on the values  $\{1, 2, \dots, \mathcal{J}_h\}$ ,  $k$  takes on the values  $\{1, 2, \dots, \mathcal{K}_h\}$ ,  $\ell$  takes on the values  $\{1, 2, \dots, \mathcal{L}_h\}$ , and  $s$  takes on the values  $\{1, 2, \dots, \mathcal{S}_h\}$ .

In this specification,  $\omega_{Whi}^{jk}$  denotes the contribution to the conditional mean of the logarithm of production under payment scheme  $h$  for individual  $i$  for the  $j^{\text{th}}$  consecutive day in work cycle  $k$ . The first term in (4.1) captures the effect of consecutive days worked on productivity. Note that we allow this coefficient to vary across cycles as well as across individuals. The second term in (4.1) captures the effect of days rest, prior to the start of a cycle, on productivity. Again, these effects vary across cycles and individuals. Finally, the third term captures interactions between the number of days rested before the start of a cycle and the number of consecutive days worked. For example, this allows the effect of an additional day's rest to affect the second consecutive day worked differently from the first day worked. Cycle-specific effects do not enter into (4.1) because the coefficients on productivity vary across cycles.

In general, the effects of time and rest on productivity are ambiguous and must be determined empirically. The pattern on the  $\omega_{Whi}$ s under fixed wages provides the foundation for a test of whether fatigue enters directly into the production function. If these coefficients are equal, then a model excluding fatigue from the production function is supported by the data. However, if these coefficients are not equal, then such a model is rejected by the data. Under piece rates, we have characterized the effects of days worked and rest on productivity for a particular utility function and particular parameter values. While these results show a decreasing time profile of productivity, economic theory does not rule out contradictory results. Furthermore, while intuition suggests that increases in rest will increase productivity, these effects may be confounded in the data, particularly if planters lose fitness and sustainable effort levels. Therefore, we now turn to issues of estimating (4.1).

From the personnel and payroll records of a medium-sized, tree-



planting firm in the southeastern corner of British Columbia, we have obtained daily production rates as well as the number of consecutive days worked and the number of rest days taken between work cycles for 51 individuals who were involved in a total of 570 different work-and-rest cycles, 67 involving fixed-wage planters and 503 involving piece-rate planters. In all, the data set contains 1,928 daily productivity observations, of which 223 concern planters paid fixed wages and 1,705 concern planters paid piece rates.

The construction of the data set is discussed in detail in the appendix to the paper. Before examining our regression results, we highlight some important features of the data, discussing some interesting descriptive statistics concerning our sample. Under fixed wages, average work-spell duration was about 3.7 days, while under piece rates it was 3.4 days. Rest periods averaged 1.7 days under fixed wages and 1.3 days under piece rates. Under both payment schemes, the proportion of female workers was about 35 percent. Under fixed wages, average daily earnings were \$228.34, while under piece rates they were \$226.51. The major difference occurred in the number of trees planted: under fixed wages, an average of 609 seedlings were planted per day, while under piece rates 939 seedlings were planted each day. The average price per seedling planted under piece rates was \$0.26.

In Figure 5, one notes that the empirical distributions of work and rest are relatively similar for the two payment schemes, although under piece rates there are a few long work spells, one as long as 11 consecutive days. In Figure 6, on the other hand, one notes that both the distributions of earnings as well as the distributions of trees planted are quite different. These differences are also reflected in the distributions of the effective

price per tree planted depicted in Figure 7.<sup>10</sup>

Given these facts, why would a tree-planting firm ever use fixed wages? Firms in the tree-planting industry often vary their payment system in response to planting conditions in an effort to ensure that planted trees meet quality standards. The quality of planted trees is very important since poorly planted trees will not survive, exposing the firm to fines from the government and a loss of future reputation. When conditions are poor, thus rendering planting slow, workers who are paid piece rates may not take the time and effort that is necessary to ensure their trees are planted well, so fixed wages are used. In related work, Paarsch and Shearer (1996), we develop these ideas formally and apply them to a subset of the data examined here.

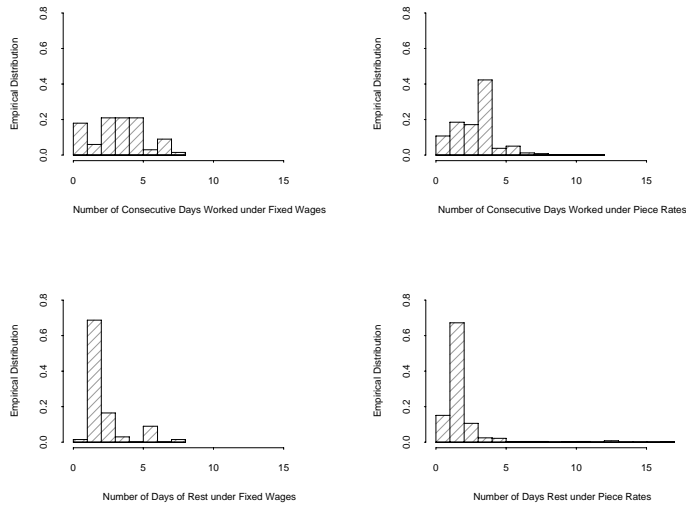
There remains an issue concerning the fact that sites to be planted are not randomly assigned to one of the two payment schemes, but rather are often assigned by the firm according to the level of planting difficulty. We assume that heterogeneity among planters is orthogonal to site characteristics.<sup>11</sup> Thus, even though selection is non-random, it is uncorrelated with the individual-specific or cycle-specific covariates used in the analysis; only estimates of the intercept coefficients will be affected. We also assume that planters are more likely to tire on sites which are difficult to plant than on easy sites which are easy to plant, even though effort levels may be different across sites. Thus, given fixed-wage contracts are paid

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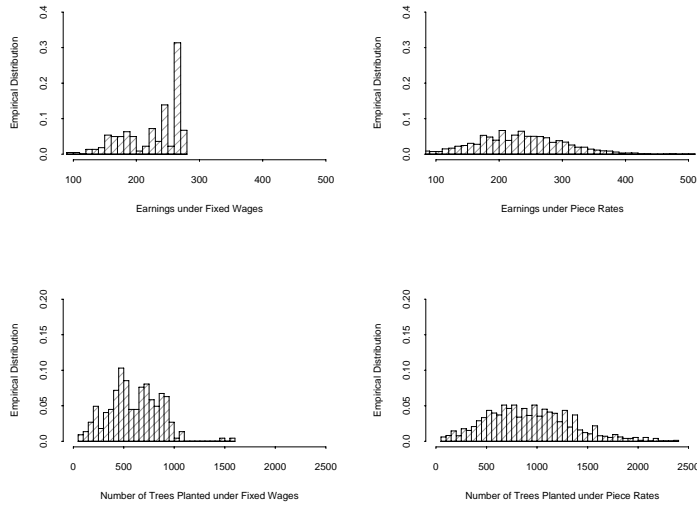
<sup>10</sup> Effective price is earnings per day divided by number of trees planted. Under piece rates, this is simply the piece-rate price, but under fixed wages it is the “average” price per tree planted, which we call the “effective” price per tree planted.

<sup>11</sup> Anecdotal evidence gathered from interviews suggests that planter personal characteristics are not used to select who is paid under the different payment schemes.

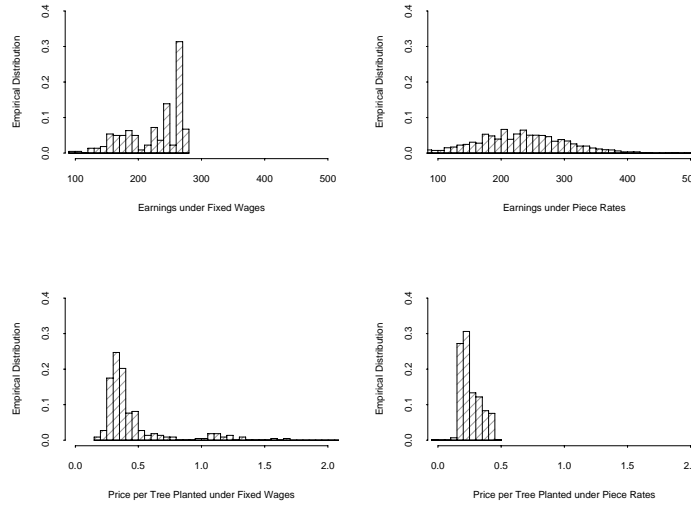
**Figure 5**  
**Histograms of Work and Rest Duration**



**Figure 6**  
**Histograms of Earnings and Trees Planted**



**Figure 7**  
**Histograms of Earnings and Price per Tree Planted**



on sites for which planting is more difficult and planters will tire more quickly on such ground, the observed profile under piece-rate contracts will be flatter than the one calculated under identical conditions to the fixed-wage contracts. The non-random selection biases the piece-rate profile to be flatter and the fixed-wage profile to be steeper than they would be had they been calculated under identical conditions.

Having reviewed some salient descriptive statistics concerning our data, we would now like to examine the main reduced-form results. Unfortunately, a specification like (4.1) is too rich for the data we have at hand. Thus, we need to impose a number of restrictions on (4.1) to reduce the number of parameters to be estimated. These restrictions will imply particular interpretations for the estimated coefficients in the resulting measurement equation.

First, we assume that the interaction effects between rest and days

worked are zero. Second, we assume the same cycle effects across time and individuals. Finally, rather than letting all coefficients vary with individuals, we introduce individual-specific dummy variables, so only the means vary across individuals, not any of the “slope” coefficients. The resulting specification

$$\mathbb{E}[\log Q_{is}^h | R_{is}^{\ell k}, W_{is}^{jk}] = \omega_{0hi} + \sum_{j=1}^{\mathcal{J}_h} \omega_{Wh}^j W_{is}^{jk} + \sum_{\ell=1}^{\mathcal{L}_h} \omega_{Rh}^{\ell} R_{is}^{\ell k} \quad (4.2)$$

still has over one hundred and twenty parameters. We make two other simplifying assumptions. First, that work effects are linear in consecutive days worked, so

$$\omega_{Wh}^j = j\omega_{Wh} \quad j = 1, \dots, \mathcal{J}_h \quad (4.3)$$

and second, that the rest effects are linear in the number of days rested, so

$$\omega_{Rh}^{\ell} = \ell\omega_{Rh} \quad \ell = 1, \dots, \mathcal{L}_h. \quad (4.4)$$

Letting  $Q_{it}^h$  denote the number of trees planted by worker  $i$  in period  $t$  under payment scheme  $h$ ,  $N_{it}^h$  denote the number of consecutive days worked without a rest day by worker  $i$  in period  $t$  under payment scheme  $h$ , the basic measurement equation that we use is

$$\log Q_{it}^h = \tau_{it}^h + \theta^h N_{it}^h + U_{it}^h. \quad (4.5)$$

Here, we allow  $\tau_{it}^h$  to vary with the planter, the cycle being planted, as well as with the number of days rest taken prior to the current cycle.

We estimated a number of equations like (4.5) separately using data for fixed wages and piece rates. The initial results are presented in the first pair of rows of Tables 1 and 2. The estimate for  $\theta^P$  is  $-0.0380$ ; average productivity fell by about 3.8 percent per consecutive day worked. The

estimate for  $\theta^F$ , on the other hand, is 0.0126, but the standard error is so large as to make it insignificantly different from zero.<sup>12</sup> In the second pair of rows of Table 1 and 2, we present results for the specification where the number of days rest *REST* prior to the current cycle is included as a regressor. In the third pair of rows of Table 1 and 2, we present results for the specification where dummy variables for each planting cycle are included as regressors. In the final pair of rows of Table 1 and 2, we present results for the specification where individual-specific dummy variables for each planter are included as regressors.

Perhaps the most remarkable feature of the results is how stable the estimates of both  $\theta^P$  and  $\theta^F$  are across the different specifications as well as the fact that estimates of  $\theta^P$  are significantly negative, while those of  $\theta^F$  are small and not significantly different from zero.

When the distributions of the regressors includes outliers, such observations can be “influential” in the sense made popular by Belsley, Kuh, and Welch (1980). To be more precise concerning the notion of influential, consider the scatterplot of twelve observations in Figure 8. The observation on the far right of the scatterplot is called a “leverage point” and it is influential in the regression analysis because the realization of  $Y$  for this observation will have extra weight by virtue of the fact that it is far from the point cloud of the regressors on the left. Belsley, Kuh, and Welch (1980) propose examining the diagonal of the “hat matrix” (more commonly known as the “projection matrix”) which, in the regression model

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

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<sup>12</sup> Note that the standard errors for all of the estimates derived using the method of least squares are robust to arbitrary forms of heteroskedasticity and were calculated according to the formulae in White (1980).

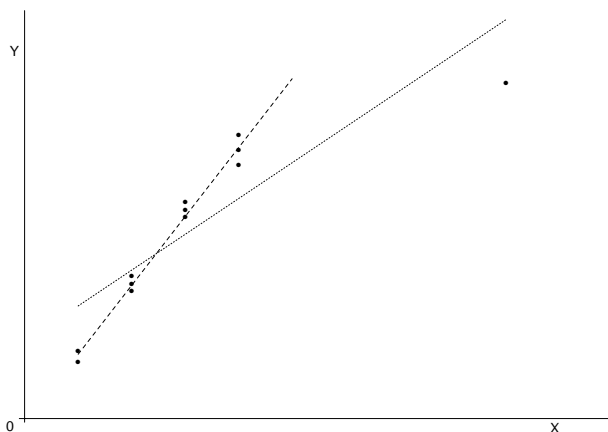
**Table 1**  
**Regression Results for Piece Rates**  
**Sample Size = 1,705**

Variable	<i>N</i>	<i>REST</i>	Cycle	Ind.	SSR	<i>R</i> <sup>2</sup>
Estimate	-0.038	-	-	-	501.845	0.010
St. Error	(0.009)	-	-	-		
Estimate	-0.039	0.010	-	-	501.177	0.012
St. Error	(0.009)	(0.005)	-	-		
Estimate	-0.042	0.004	Included	-	462.391	0.088
St. Error	(0.009)	(0.005)		-		
Estimate	-0.040	0.002	Included	Included	285.246	0.412
St. Error	(0.008)	(0.005)				

**Table 2**  
**Regression Results for Fixed Wages**  
**Sample Size = 223**

Variable	<i>N</i>	<i>REST</i>	Cycle	Ind.	SSR	<i>R</i> <sup>2</sup>
Estimate	0.013	-	-	-	54.984	0.002
St. Error	(0.024)	-	-	-		
Estimate	-0.006	0.059	-	-	53.852	0.002
St. Error	(0.026)	(0.022)	-	-		
Estimate	0.003	0.062	Included	-	50.499	0.083
St. Error	(0.025)	(0.023)		-		
Estimate	0.004	0.047	Included	Included	29.630	0.462
St. Error	(0.018)	(0.027)				

**Figure 8**  
**Example of a Leverage Point**



where  $\mathbf{Y}$  is a  $(\mathcal{T} \times 1)$  vector,  $\mathbf{X}$  is a  $(\mathcal{T} \times k)$  matrix,  $\boldsymbol{\theta}$  is a  $(k \times 1)$  vector, and  $\underline{\boldsymbol{\xi}}$  is a  $(\mathcal{T} \times 1)$  vector, is the diagonal of

$$\mathbf{H} = \mathbf{X}(\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top$$

to find influential observations. Now, the average of the diagonal values of  $\mathbf{H}$  should be  $(k/\mathcal{T})$  because the trace of  $\mathbf{H}$ , the sum of its diagonal elements, is  $k$ , since  $\mathbf{H}$  is symmetric and idempotent, having  $k$  eigenvalues equal to one and  $(\mathcal{T} - k)$  eigenvalues equal to zero. In Figure 8, this would mean an average of  $(2/12)$  or  $0.1667$ . The value for the diagonal of  $\mathbf{H}$  corresponding to the leverage point in this example is  $0.7864$ . Note that the dashed line with the smaller slope illustrates how much effect the leverage observation has on the regression line. The other dashed line is the regression line excluding the leverage point.

Since the regressors contain outliers in the variable consecutive days



worked  $N$ , especially under piece rates where there are three observations with either ten or eleven consecutive days worked, we were concerned that a few outliers could be driving the results. We calculated the diagonal of the hat matrix for the most general specification in Tables 1 and 2. We then sorted the variable  $N$  and plotted it along with the corresponding values for the diagonal of  $\mathbf{H}$  to see if large values of  $N$ , especially under piece rates, were associated with large values of the diagonal of  $\mathbf{H}$ , but this did not turn out to be the case.

There are always concerns that regression results are sensitive to outliers or contaminated data in the regressand. This is because the breakdown point of the least squares estimator is  $(1/\mathcal{T})$ . The notion of breakdown point is as follows: consider estimating the population mean of a normal distribution using the sample mean from  $\mathcal{T}$  observations. Now suppose that some observations are mis-measured or contaminated. Suppose that these observations come from the worst realizations imaginable. What is the least number of contaminated observations that one can have before the sample mean loses the property that it is a consistent estimator for the population mean? For the sample mean, it is one observation, or  $(1/\mathcal{T})$  of the sample, so the breakdown point is said to be  $(1/\mathcal{T})$ . Now consider the sample median as an estimator of the population mean of a normal family in the presence of contaminated observations. In this case, more than half the sample would have to be contaminated before the estimator would breakdown. For more on breakdown points, see Huber (1981).

Estimating the coefficients (4.5) by the method of least absolute deviations when  $U_{it}^h$  is symmetric about zero yields more robust estimates, those having a breakdown point of one half. In Tables 3 and 4, we present

**Table 3**  
**Least Absolute Deviations Results for Piece Rates**  
**Sample Size = 1,705**

Variable	$N$	$REST$	Cycle	Ind.	SSR	$R^2$
Estimate	-0.049	-	-	-	518.905	0.010
St. Error	(0.007)	-	-	-		
Estimate	-0.049	0.003	-	-	518.350	0.011
St. Error	(0.006)	(0.005)	-	-		
Estimate	-0.044	-0.013	Included	-	473.110	0.080
St. Error	(0.006)	(0.005)		-		
Estimate	-0.033	-0.005	Included	Included	298.011	0.428
St. Error	(0.005)	(0.004)				

**Table 4**  
**Least Absolute Deviations Results for Fixed Wages**  
**Sample Size = 223**

Variable	$N$	$REST$	Cycle	Ind.	SSR	$R^2$
Estimate	0.020	-	-	-	56.556	0.002
St. Error	(0.016)	-	-	-		
Estimate	-0.004	0.056	-	-	54.576	0.002
St. Error	(0.013)	(0.021)	-	-		
Estimate	-0.014	0.035	Included	-	51.113	0.026
St. Error	(0.013)	(0.022)		-		
Estimate	0.000	0.038	Included	Included	31.812	0.435
St. Error	(0.010)	(0.019)				

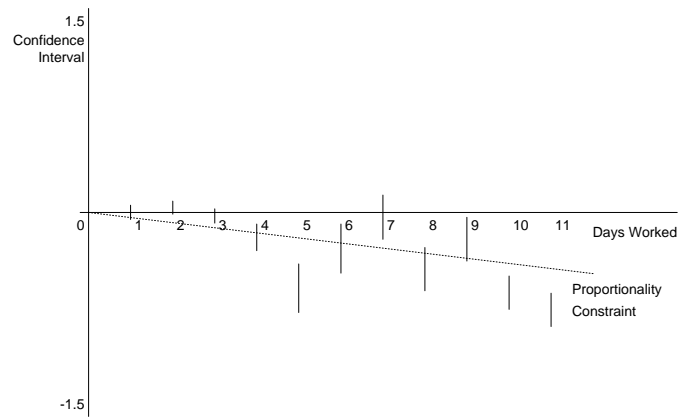
the least absolute deviations estimates.<sup>13</sup> The most notable feature of these two tables is how similar the entries are to those in Tables 1 and 2. Note too that the estimated coefficients for piece-rate payment schemes are, in general, systematically smaller (more negative) than the regression coefficients, while for fixed-wage payment schemes they are basically zero. In any case, these estimates suggest that average productivity under piece rates falls by at least 3 percent and by as much as 5 percent for each additional consecutive day worked, while average productivity under fixed wages does not vary with consecutive days worked. The fact that productivity under fixed wages is constant throughout the cycle suggests that a restricted model in which fatigue is not included in the production function is a reasonable approximation for these data.

To see if the strongly negative and precise results concerning the effect of  $N$  on expected  $\log Q$  in the case of piece rates are a result of the proportionality assumption (4.3), we relaxed this assumption by introducing eleven dummy variables, one for each of the possible cumulative

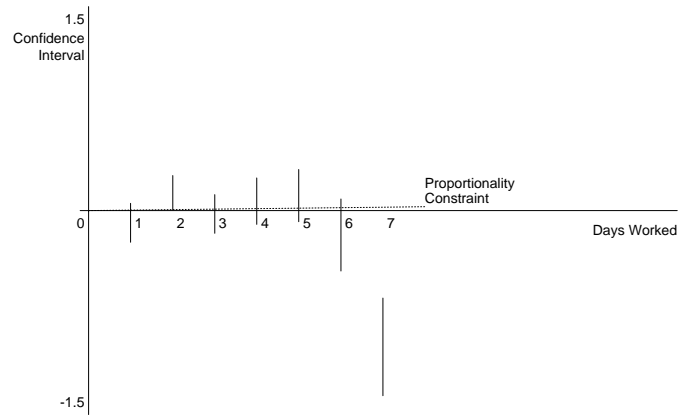
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<sup>13</sup> Note that unlike the least squares estimates, the least absolute deviations estimates need not be unique. This is a common property of quantile estimates like the median. Basically, the algorithm used to calculate the least absolute deviations estimates involves casting the problem in terms of a linear programming problem and solving it using the simplex algorithm; for more on this, see Barrodale and Roberts (1980). Sometimes the solution to the linear programme will obtain on the face of the constraints, which implies a set of optima yielding a global minimum. Note that some of the entries in Tables 3 and 4 are non-unique global minima. Note too that because the optimization problem involves constraints that are non-differential at potential optima, alternative forms of analysis are required to find the asymptotic distribution of the estimators. For details concerning the asymptotic properties of the estimators, see Koenker and Bassett (1978). Note that the standard errors presented in Tables 3 and 4 were calculated assuming that  $U_{it}^h$  is from the Laplace family of distributions, the most favorable distribution.

**Figure 9**  
**Confidence Intervals of Work Effect under Piece Rates**



**Figure 10**  
**Confidence Intervals of Work Effect under Fixed Wages**



days worked. We also introduced a similar set of seven dummy variables into the fixed-wage specification. Given the data available, this structure permits the most non-linear relationship possible. In Figures 9 and 10, we present the 95 percent confidence intervals for the estimated coefficients on the dummy variables for each cumulated day worked under both piece rates and fixed wages as well as the estimated predicted effects under the proportionality assumption. The vertical lines at each integer on the abscissa are the width of the 95 percent confidence interval, with the center of the interval being the point estimate, while the dotted line is the predicted value under the proportionality assumption. In both cases, the restrictions implied by the proportionality assumption are rejected with p-values less than 0.01, but the important conclusion to glean from this sort of sensitivity analysis is that, under piece-rate contracts, the estimated coefficients on the dummy variables for consecutive days worked grow more negative with time, while those under fixed-wage contracts remain close to zero, except for the one associated with seven consecutive days worked. We note that the leverage point in the data for the fixed-wage specification corresponds to the single observation having  $N$  equal to seven, and an unusually low realization for  $\log Q$ , some four standard deviations from the mean. We conjecture that the planter corresponding to this observation may have not worked a full day.

## 5. Summary and Conclusions

In this paper, we have examined the intertemporal effects on effort and productivity when workers are paid either fixed wages or piece rates. For the data concerning tree planters in British Columbia, we find that planters paid under piece rates produce more, on average, per day than those paid fixed wages. We also find that the average daily production

rate for piece-rate planters falls at a rate between three and five percent per day, depending on the model specification and the method of parameter estimation, while that for fixed-wage planters appears unaffected by the number of consecutive days worked, regardless of the model specification or the method of parameter estimation. These results highlight the importance of considering the long-term effects of incentive contracts as well as the short-term ones.

## A. Appendix

In this appendix, we document the development of the data set used, describing the sources from which the data were taken as well as the transformations used in obtaining the final data set.

The data used in this paper are derived from the personnel records of a medium-sized tree-planting company located in the southeastern corner of British Columbia. This firm won contracts throughout British Columbia.

We keypunched information from copies of the firm's records. For some days, some of these records were incomplete. In such cases, those days were eliminated. Reasons for a record being incomplete were worker sickness or worker leaves the firm. For fixed-wage planting, the foremen sometimes did not count the number of trees that a worker planted. Thus, the number of fixed-wage planting records is under-represented. We do not believe that this non-reporting is systematic. Specifically, we do not believe that it is related to the number of trees planted, worker effort, or the worker's personal characteristics.

We have data concerning 51 workers, who were involved in a total of 570 different work-and-rest cycles, 67 involving fixed-wage planters and 503 involving piece-rate planters. In all, the data set contains 1,928 daily productivity observations, of which 223 concern planters paid fixed wages, and 1,705 concern planters paid piece rates. Our data set includes the following variables:

*ID*: the identification number for the planter.

*SEX*: a dummy variable indicating the sex of the planter.

*WORK*: the current day worked of this particular cycle (starting at 0, and then starting again at 0 when the next cycle starts).

*CWP*: cumulative days worked, not including the current day, under a piece rate. If the planter has only used a piece rate, then adding *CWP* to *WORK* will give the total number of days worked.

*CP2*: sometimes, a planter takes a long time off between cycles. Instead of giving the planter a month of rest, we have restarted the calculations as if at cycle zero. In general, we did this whenever a planter went more than ten days between cycles (unless the intervening days were specifically entered as days off). Once a planter starts over, *CWP* refers to cumulative days worked for a piece rate for that particular stretch of work. If one adds *CP2* and *CWP*, then this will give the overall total days worked for a planter. For example, if a planter had 23 cumulative days worked before taking a 2 month break, then, when he returned, *CWP* would start over from zero, but *CP2* would be equal to 23 for all subsequent entries. Long absences were unusual in this data set, so these distinctions are relatively unimportant.

*CWD*: cumulative days worked, not including the current day, under a day wage.

*CD2*: like *CP2*, but applies to cumulative days worked under a day wage regime in previous stretches of work.

*CYCLE*: the cycle number, starting with 0. A cycle is defined as any number of consecutive days worked; a new cycle starts when at least one day of non-work occurs. Note that some planters may seem to be missing some cycles; *e.g.*, jumps from the third to sixth cycle. This occurs because some entries were deleted; *e.g.*, no number of trees entered. If it was known that a person worked, then the relevant variables were adjusted accordingly.



*CYCLE2*: if a planter was restarted at cycle zero after a long break, then *CYCLE2* refers to the number of cycles worked in the previous stretch. Adding *CUM2* and *CYCLE* gives the total number of cycles worked during 1991.

*REST*: the number of days rested between the end of the last cycle and this one.

*CUMR*: cumulative days of rest to this point.

*PIECE*: the piece rate, if it applies.

*DAYRATE*: the day rate, if it applies.

*TREES*: the number of trees planted.

*DWAGE*: a dummy variable indicating whether a day wage was paid.

*PAY*: total earnings.

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