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REAL INVESTMENT DECISIONS UNDER INFORMATION CONSTRAINTS

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Real Investment Decisions Under Information Constraints*

Gérard Gaudet[†], *Pierre Lasserre*[‡], *Ngo Van Long*[⁜]

Résumé / Abstract

En présence d'asymétrie d'information entre propriétaires (le principal) et administrateurs (l'agent), la relation investissement coût du capital et la relation investissement valeur implicite du capital subissent une distorsion pour tous les types. Dans un modèle avec coût d'ajustement par ailleurs standard, il apparaît notamment un régime d'inaction pour une certaine gamme de coûts. Ce phénomène se présente sous une forme différente de ce qu'implique la présence de coûts fixes ou d'irréversibilités, mais ressemble à ce qui survient lorsque le coût du capital est différent pour une hausse que pour une baisse du stock. L'incertitude, qui prend la forme d'un élargissement de la distribution des types, tend cependant à réduire l'investissement. Le modèle clarifie l'interprétation du q de Tobin sous asymétrie d'information et explique certains résultats de la littérature sur les fusions et acquisitions.

We investigate investment behavior when there is asymmetry of information between owners (the principal) and managers (the agent). The model accepts the standard cost-of-adjustment model as a particular case and is directly compared with it. For all types, information asymmetry distorts the relationship between investment and the cost of capital, and the relationship between investment and the shadow value of capital. In particular, a regime of inaction appears over a certain cost range, in an observationnally different way than when fixed adjustment costs, or irreversibilities, cause a similar phenomenon. Uncertainty, in the form of an increase in the spread of agents' types, tends to reduce investment despite symmetric adjustment cost and perfect competition. The model clarifies the interpretation of Tobin's q *under asymmetric information and explains some results of the mergers and acquisition literature.*

Mots clés : Contrats incitatifs ; Dynamique ; Information asymétrique ; Relation principal-agent ; Investissement ; Incertitude.

Key words: Incentive contracts; Dynamic; Asymmetric information; Principal-agent relationship; Investment; Cost of adjustment; Uncertainty.

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1 Introduction

There is a huge literature on real business investment. It reflects the importance of investment both as a componentΓ and as a determinantΓ of economic activity. It also reflects the considerable empirical and theoretical difficulties associated with the analysis of capital spending. This literature is exemplified in the survey by Robert Chirinko (1993)Γ or in the recent paper by Abel and Eberly (1994). In its branch evolving from the neoclassical traditionΓ it still largely neglects important issues of agency and information.

YetΓ investmentΓ as perhaps the most important business decisionΓ is at the core of the corporate governance challenge. In the introduction of its 1994 survey of that subjectΓ The Economist wrote: '... managers have become insufficiently accountable to shareholders. From blatant thieveryΓ such as that by Robert MaxwellΓ to lousy investments (Japanese firms' purchases of overpriced American property or American Express's attempt to become a financial conglomerate)Γ to failures to tackle looming problems quickly enoughΓ as at IBMΓ there is ample evidence of waste that might have been avoided had bosses been on a tighter rein.'

To be fairΓ it should be noted that agency costsΓ asymmetric informationΓ and corporate control considerations have been extensively studied on the financial side of the investment equation (see Harris and RavivΓ 1991). HoweverΓ because of its emphasis on capital structure and financial marketsΓ the finance literature has not contributed directly to investigating the role of such considerations in the determination of real investment.

Our paper is a contribution toward filling this gap. We introduce asymmetric information into a standard cost-of-adjustment model of investmentΓ and we derive the resulting behavior of the investment function. Our model describes a principal-agent relationship with investors (inside or outside shareholders) as principalΓ and managers as the agent. It could also apply to situations involving a regulator as principalΓ and the firm as agent.

Unlike what might have been expected, the incentive compatible investment function is not merely an attenuation of its full information counterpart, but may involve a qualitatively different behavior for some types. Several results appear surprising at first glance, both because they add some hitherto ignored considerations to real investment decisions and their determinants, and because they illustrate the implications of introducing dynamic considerations to standard principal-agent models. Thus we find that situations may occur where both the low cost, and the high cost, types are asked to carry out the same investment as they would under full information, if the principal faced the same shadow price of capital. However, because the shadow price of capital is affected by informational asymmetry, we find that the investment behavior of all types must in fact be modified relative to the full information situation. We also show that investment behavior under asymmetric information exhibits hysteresis, although for reasons quite different from those described by Dixit and Pindyck (1994). Of course, these results depend on the seriousness of informational asymmetries; in contrast with otherwise similar adjustment cost models (Abel, 1983; Caballero, 1991) an increase in uncertainty, taking the form of an increase in the spread of types, has a depressing effect on investment. Our model also casts a new shadow on the q theory of investment. We show that, if informational asymmetries matter, evaluating the shadow price of capital by market mechanisms raises a much more serious issue than the well understood problem of the identity between average and marginal capital value. The relevant rent actually is split between shareholders and managers, and the stock exchange only provides information on the share accruing to the formers.

We describe and further motivate the model in Section 2. Section 3 is devoted to solving the problem of the principal, which is to tell managers what investment to choose, given their informational advantage and conflicting objective. The similarity of our basic model with the standard cost of adjustment model allows easy comparisons with the latterΓ in particular with the extended model of Abel and Eberly In Section 4Γ we identify and explain the qualitative differences and similarities in the predictions of our model and the standard cost of adjustment model. We also discuss the implications of information asymmetry on the q theory of investment andΓ more generallyΓ the additional contribution of our model to major existing models of real business investment.

2 The model

As mentioned earlierΓ there are several reasons why the objectives of managers and of shareholders may not coincideΓ and why managers may have an informational advantage over shareholders. Harris and Raviv (1991)Γ although with a focus on implications for capital structureΓ survey most of them. Since many capital structure decisions aim at improving real investment decisionsΓ they are relevant to our discussion. The reader is referred to their paper for details and references.

Conflicts between shareholders and managers may arise because managers hold only part of the residual claim. Rather than devoting themselves entirely to profit enhancing objectivesΓ they may withhold information about the best investment prospects in order to promote decisions more favorable to their own personal benefits. Another type of conflict may arise because shareholders have an incentive to invest suboptimally under limited liability: if a project is successfulΓ they capture most of the gainΓ but if the project failsΓ they cannot lose more than their investment. On the other handΓ managers may lose their reputation in a bankruptcy (DiamondΓ 1989; Hirshleifer and ThakorΓ 1989). SimilarlyΓ it has been shown by Myers and Majluf (1984) thatΓ if outside investors are less well-informed than managers about the value of the firm's assetsΓ 'then equity may be mispriced by the market. If an investment is to be financed by issuing new equityΓ underpricing may be so severe that new investors capture more than the net present value of the project, resulting in a net loss' (Harris and Raviv, p.306) to insiders, including managers if they own equity.

These are some of the reasons why shareholders' and managers' objectives may conflict. Our model will most directly reflect those where managers try to enhance their own well-being while shareholders try to keep the highest possible expected surplus to themselves. While the model would have to be altered if it was to focus on the other types of conflicts listed above, the spirit of the exercise would remain the same. Several avenues may be available to (partially) resolve this conflict. We focus on one of them, by modelling a principal-agent relationship, where the shareholders act as a principal while managers, as a group, are their agent

The objective of the agent is the maximization of managers' residual claim, expected cumulative discounted revenues net of relevant costs, minus transfers to the principal. The objective of the principal is to maximize a weighted sum of what is left to the agent and what is transferred to her. It is not crucial for the principal to assign a positive weight to the objective of the agent. In a crude way, a positive weight may reflect the fact that shareholders often hold administrative positions themselves in other firms, so that they may show concern for the well-being of managers as a profession, and the fact that managers usually hold shares in their own company. Both the principal and the agent have the same discount rate.

Consider the simplest possible technology, giving output at date t as a concave, differentiable, positively monotonic function of the stock of capital (time subscripts will be omitted where no ambiguity arises)

$$q_t = g(k_t)$$

Capital may be interpreted to include plant and equipment, but also goodwill acquired through advertising or other marketing expenses, knowhow, the size and training level of the labor force, etc.. We assume that $g'(k)$ is finite and $g(k)$ is non negative for any non negative k. Capital evolves according to investment i_t. For simplicity, assume that there is no depreciation, so that

$$k_{t+1} = k_t + i_t \ , \quad k_t \geq 0 \tag{1}$$

The cost of investment is assumed to consist of the asset cost of equipment vi, plus a cost of adjustment which we take to be quadratic: $ai + \frac{1}{2}bi^2$. Thus, in total, the cost of an investment i is

$$[v + a]\, i + \frac{1}{2}bi^2$$

As argued above, there are several reasons why the agent might prefer the principal not to know this cost with certainty. There are also several reasons why this cost may be private information. Consider v. Whether equipment is being sold or purchased, the transaction price may differ from the posted price by an amount which depends on the relationship between the parties to the transaction. An extreme case would be transactions involving kickbacks. Such cost components are likely to be unknown to the principal, whether it is a board of shareholders or a government. Also, access to financing may depend on the relationship between managers and lenders; once financing costs are capitalized into the asset price, personal differences imply that the asset cost component of the cost of investment is firm specific, possibly unknown to shareholders. Similarly a may reflect firm, or manager, specific inconveniences associated with changes in size and organization; this is likely to be private information and, as we argue further below, of interest to the principal. More generally, since expected investment returns determine the relative cost of investment, assuming that the cost of investment is private information will serve to model the idea that managers are better informed about both returns and costs than shareholders.

Assuming for simplicity that the public information component of $v + a$ is zeroΓ and that b is common knowledgeΓ the cost of investment may be written

$$c(i_t, \theta_t) = \theta_t i_t + \frac{1}{2} b i_t^2 \tag{2}$$

where $\theta_t > 0$ is a privately observed parameter that varies from period to period. This formulation is also compatible with a focus on the cost of intermediationΓ as in Bernanke and Gertler (1989)Γ or Cooper (1994)Γ where variations in the cost of capital accumulation reflect fluctuations in the frequency of monitoring a representative project. Managers may be better aware of such costs than shareholders[1].

In order to focus on the effect of asymmetric information on investment decisionsΓ we eliminate also any common uncertainty: future prices p_t are known with certainty and technology does not change over time. These assumptions are inconsequential for our purposes and have the advantage of simplicity[2]. We also assume that the θ's are uncorrelated over time so that θ has the same continuous distribution $f(\theta) > 0$ over the same interval $\left[\theta^L, \theta^H\right]$ at all dates. Thus knowing θ at any date t does not provide any information about its subsequent values. This assumption may not be very realistic; it clarifies the role of asymmetric information by ensuring that the sole source of dynamics in the model is the investment process. As in the standard cost of adjustment model of investmentΓ the sole reason why the decision maker must be forward looking is that it is infinitely costly to change the level of capital instantaneouslyΓ and that current

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[1] Altering Cooper's formulation slightly in order to allow for capital to be durableΓ we may write $k_{t+1} = k_t + \tilde{\theta} i_t$ where $\tilde{\theta}$ reflects the cost of intermediation: a given investment effort may affect the stock of capital differentlyΓ according to the value of $\tilde{\theta}$. Allowing for costs of adjustmentΓ the corresponding cost is $v i_t + \frac{1}{2} b \left[k_{t+1} - k_t\right]^2$. ConsequentlyΓ the cost of obtaining k_{t+1} given that the current stock is k_t is $\frac{v}{\tilde{\theta}} \left[k_{t+1} - k_t\right] + \frac{1}{2} b \left[k_{t+1} - k_t\right]^2$ which is analogous to (2).

[2] In the problem presented belowΓ it may alternatively be assumed that future output prices and technology are stochastic and combine in such a way that net revenues at date t are $R(k_t, \epsilon_t)$Γ where ϵ_t follows a Brownian motion. As long as there is no asymmetry in the observation of ϵ_tΓ the standard dynamic programming solution approach will apply in that case and our qualitative results will not be affected.

investment decisions affect future capital levels.

The objective of the agent Π_t may be decomposed into current net profit π_t and expected cumulative discounted net future profit ψ_t from $t+1$ on as evaluated at t. For the technology just described this means maximizing

$$\begin{aligned} \Pi_t &= \pi_t + \psi_t \\ &= p_t g\left(k_t\right) - \left[\theta_t i_t + \tfrac{1}{2} b i_t^2\right] - R_t + \psi_t\left(k_{t+1}\right) \end{aligned} \tag{3}$$

subject to (1) where R_t is the amount transferred to the principal. R may be thought of as dividends demanded by the principal.

As explained earlier we assume that one dollar left to the agent is worth α, $0 \leq \alpha < 1$ to the principal. Thus the principal maximizes

$$\alpha\left\{p_t g\left(k_t\right) - \left[\theta_t i_t + \frac{1}{2} b i_t^2\right] - R_t + \psi_t\right\} + R_t + \Gamma\left(k_{t+1}\right) \tag{4}$$

subject to (1) and the rationality constraint $\Pi_t \geq 0$ where $\Gamma\left(k_{t+1}\right)$ is cumulative discounted transfers to the principal from $t+1$ on as expected at t.

In the rest of this Section we solve the symmetric information version of this problem. This solution will serve as a benchmark against which we will compare the solution of the asymmetric information problem. We will add the superscript s to refer to variables or functions that are defined or evaluated under symmetric information. Thus the use of Γ^s and ψ^s will indicate that the principal is aware that future decisions will be made in a symmetric information setup.

Under symmetric information θ is observed by both the principal and the agent upon its realization. Since the principal has the power to set R it is obvious that her best choice at all dates is to set R in such a way that $\Pi = 0$ leaving the agent indifferent between participating in the relationship or not. Consequently problem (4) is equivalent to maximizing by choice of i

$$p_t g(k_t) - \left[\theta_t i_t + \frac{1}{2} b i_t^2\right] + \Gamma_t^s(k_{t+1})$$

subject to (1). This is a simplified but standard version of the cost of adjustment investment model. The first-order condition for an interior solution $(i_t > -k_t)$ is

$$i_t^s = \frac{\Gamma_t^{s\prime} - \theta_t}{b} \tag{5}$$

where $\Gamma_t^{s\prime}$ is the shadow price of capital the discounted sum of expected future marginal revenue products. Thus if t happens to be the last period Γ^s and its derivatives vanish so that i is negative: since there is no use keeping any capital for future periods it is desirable to sell as much of the remaining stock as possible while keeping adjustment costs to an acceptable level. If instead t is the second last period then keeping a marginal unit of capital for period $t+1$ yields an advantage of $\Gamma^{s\prime} = \delta p_{t+1} g'(k_{t+1})$ in terms of increased future production where δ is the discount factor $0 < \delta < 1$. This marginal value product of capital is non stochastic. However suppose now that there might be yet an extra period $t+2$; then since θ_{t+1} is unknown i_{t+1} which will be given by (5) at $t+1$ is unknown at t; as a result the marginal product of k_{t+2}, $g'(k_{t+2})$ is unknown at t so that the marginal impact on future revenues of increasing the stock of capital at t is stochastic.

Define T as the first interruption in production or equivalently the first date at which k is zero[3]. In general T if finite is unknown and stochastic. Then the shadow

[3]Depending on the trajectory of $p\Gamma$ production may start again after an interruption. However marginal products in the new production phase will be independent of capital levels before the interruptionΓ which justifies defining T as the first interruption.

price of capital at t is under symmetric information

$$\Gamma_t^{s\prime} = E_t \left\{ \sum_{\tau=1}^{T^s - t} \delta^\tau p_{t+\tau} g' \left(k_{t+\tau}^s \right) \right\}$$

As (5) indicates when θ is high relative to $\Gamma^{s\prime}$ it is desirable to sell capital; otherwise it is desirable to buy. This depends on whether future output prices are high and on expected future levels of k as they develop when (5) is applied after successive realizations of θ. To avoid technical difficulties we rule out bubbles that is price trajectories that would cause $\Gamma^{s\prime}$ (and i^s) to be infinite.

3 The model under asymmetric information

3.1 Preliminary remarks

Consider now the asymmetric information case. θ is observed upon its realization by the agent. Consequently the principal must rely on the information given to her by the agent in order to pursue her objective. Since there is no intertemporal correlation between the θ's the agent does not lose any of his future informational advantage when he reveals current information to the principal. As a consequence if the principal chooses to use an incentive mechanism there is no possibility of a ratchet effect as in Laffont and Tirole (1988) and the revelation principle applies as in static setups.

We assume that the principal cannot credibly give up her claim to a share in any future rents in exchange for a lump-sum payment whose amount would be agreed upon before future cost conditions are revealed to the agent. This assumption is justified by wealth constraints on managers. Indeed almost by definition the existence of a publicly-held firm implies that managerial skills and wealth are held by separate groups of individuals. We do assume however that the principal is able to commit to one-period contracts. Thus, during any period, shareholders will keep the managers if they receive the dividends and see the action (investment) that were agreed upon.

Under such circumstances, the best the principal can do is to design a succession of one-period contracts or mechanisms in such a way as to maximize her objective subject to her informational disadvantage. By the revelation principle, whenever the contracts discriminate between types, they must induce the agent to reveal his private information, which requires the properties described below.

3.2 The incentive contract

The incentive contract consists of a menu of investment-transfer pairs $\left\{\left(i_t\left(\hat{\theta}_t\right), R_t\left(\hat{\theta}_t\right)\right)\right\}$, where $\hat{\theta}_t$ represents the level of θ_t announced by the agent when selecting a pair from the menu. In order to induce truthful revelation at t, the menu must be such that it is in the interest of the agent to choose $\hat{\theta} = \theta \ \forall \theta \ \in \left[\theta^L, \theta^H\right]$. The objective of the agent is to maximize, by choice of $\hat{\theta}$

$$\phi\left(\theta, \hat{\theta}\right) = pg\left(k\right) - \left[\theta i\left(\hat{\theta}\right) + \frac{1}{2} b i\left(\hat{\theta}\right)^2\right] - R\left(\hat{\theta}\right) + \psi\left(k + i\left(\hat{\theta}\right)\right) \tag{6}$$

where, unless otherwise mentioned, variables (functions) are evaluated (defined) at t. Define the optimized value of ϕ as

$$\Pi\left(\theta\right) = \phi\left(\theta, \theta\right)$$

From the first and second order conditions for the maximization of ϕ, we have (Guesnérie and Laffont, 1984)

$$\frac{di}{d\theta} \leq 0 \tag{7}$$

and

$$\frac{d\Pi}{d\theta} = -i(\theta) \tag{8}$$

Furthermore as a rationality condition the contract must be such that for any participating agent

$$\Pi \geq 0 \tag{9}$$

Although it is clear from (6) that Π is made up of a current component plus a component corresponding to future profits where both components are net of transfers to shareholders (9) does not imply a commitment by the principal to keep the same manager in the future. It implies that if the agent is fired at t he gets a compensation of ψ. Our assumption is that shareholders are able to commit to one period contracts involving such golden parachutes[4].

3.3 The problem of the principal

At any date t given k and under constraints (7) (8) and (9) the principal must choose functions $i(\theta)$ and $R(\theta)$ in such a way as to maximize

$$E\left\{\alpha\Pi(\theta) + R(\theta) + \Gamma(k + i(\theta))\right\}$$

Substituting the definition of Π using (6) and rearranging this is equivalent to choosing $i(\theta)$ in such a way as to maximize

$$\int_{\theta^L}^{\theta^H}\left\{pg(k) - \theta i(\theta) - \frac{1}{2}bi(\theta)^2 - [1-\alpha]\Pi(\theta) + S(k + i(\theta))\right\}f(\theta)\,d\theta \tag{10}$$

[4]If golden parachutes were not availableΓ a second rationality constraint requiring current net cash flows π to be non negative would have to be satisfied. Since ψ is non negative and $\Pi = \pi + \psi\Gamma$ that constraint is at least as strict as (9). Thus golden parachutes allow shareholders to operate in a less constrained environment.

subject to (7), (8), and (9), where $S(k+i)$, defined as $\Gamma^a(k+i)+\Psi^a(k+i)$, is the sum of surpluses to be shared by the agent and the principal at, and beyond, $t+1$, as expected at t and discounted to t. As indicated by the superscripts a, it is understood that future decisions will be made in the same asymmetric information setup so that S reflects this knowledge.[5]

Problem (10) can be treated as an optimal control problem, where i is the control variable and where Π, the state variable, is subject to a non negativity constraint. Define

$$\begin{aligned} L(\Pi,i,\mu,\lambda;k) &= \left\{pg(k)-\theta i(\theta)-\frac{1}{2}bi(\theta)^2-[1-\alpha]\Pi(\theta)+S(k+i(\theta))\right\}f(\theta) \\ &\quad -\mu(\theta)i(\theta)+\lambda(\theta)\Pi(\theta) \end{aligned} \tag{11}$$

where $\lambda \geq 0$ and μ respectively are the Lagrangian multiplier associated with constraint (9) and the costate variable associated with Π. At this stage, constraint (7) is being ignored; we will make use of it in the process of selecting a candidate solution, and we will specify *ex post*, conditions under which it is satisfied by the unconstrained solution. One necessary condition, the maximum principle, implies that, at all θ where i is continuous and $\frac{d\mu}{d\theta}$ exists

$$i^a(\theta)=\frac{S'-\theta}{b}-\frac{\mu(\theta)}{bf(\theta)} \tag{12}$$

where S', the shadow value of capital, represents the combined value to both the principal and the agent of expected discounted future marginal products. This rule differs from its symmetric-information counterpart (5) in two important ways. First, since the share accruing to the principal under symmetric information is the total surplus, Γ^s in (5) is the analog of S in (12); however, surpluses will normally differ under symmetric and under asymmetric, information as the investment programs implied by (5) and (12) will

[5]We assume that $S(k_{+1})$ exists and is continuously differentiable and concave for any $k_{+1} \geq 0$; this implies that we rule out price trajectories that would cause S' to be infinite (as was done with $\Gamma^{s\prime}$ under symmetric information).

usually differ. Consequently, the first term on the right-hand side of (5) will normally differ from the first term on the right-hand side of (12), although they measure the same concept. Second, and more familiar, is the presence of an extra term, $-\frac{\mu(\theta)}{bf(\theta)}$, in the expression. As in static principal-agent models, this term causes a distortion to the operative decisions of the agent. As will be shown below, unlike static asymmetric information production models where agents are typically induced to produce less than under full information, this term may be positive or negative, causing i to be either higher, or lower, than under full information. Perhaps more fundamental a difference will be the fact, established further below, that the solution is not fully separating under conventional assumptions on $f(\theta)$.

Another condition, which must hold over intervals where $\frac{d\mu}{d\theta}$ exists, is

$$\frac{d\mu}{d\theta} = [1-\alpha]\, f(\theta) - \lambda(\theta) \tag{13}$$

Integrating gives

$$\mu(\theta) = [1-\alpha]\, F(\theta) + A - \Lambda(\theta) \tag{14}$$

where F is the cumulative distribution of θ, A is a constant of integration, and

$$\Lambda(\theta) \equiv \int_{\theta^L}^{\theta} \lambda(\tau)\, d\tau \tag{15}$$

measures the cumulative impact on the objective of the principal, of meeting rationality constraints (9) for all types $\tilde{\theta} \leq \theta$.

3.4 Solution

In problems with constraints on the state variables such as (10), discontinuities in costate variables may occur only at junctions between an interval where the state constraint is binding and an interval where it is not binding[6]. The continuity of μ elsewhere will be useful to characterize the solution. It is also useful to note that the first two terms on the right-hand side of (14), together, are strictly increasing in θ while the last term goes the opposite way. Thus, on intervals over which λ is positive ($\Pi = 0$), μ may be increasing or decreasing; when $\lambda = 0$, μ is strictly monotonic; at junctions μ may have a discontinuity. Similarly, $\Lambda(\theta)$ is continuous as an integral of multipliers and the control variable i is continuous except, possibly, at junctions between intervals where $\Pi = 0$ and intervals where $\Pi > 0$. Furthermore, given our assumption that S' is finite, adjustment costs imply that i is finite for any θ; it follows by (8) that Π is continuous over $\left[\theta^L, \theta^H\right]$.

Let $[\theta^-, \theta^+] \subseteq \left[\theta^L, \theta^H\right]$ be an interval over which (9) is binding; define θ^- by the condition that $\Pi(\theta^-) = 0$ and, if $\theta^- \neq \theta^L$, then $\Pi(\theta) > 0$ for any $\theta < \theta^-$ in a neighborhood of θ^-; define θ^+ by the condition that $\Pi(\theta^+) = 0$ and, if $\theta^+ \neq \theta^H$, then $\Pi(\theta) > 0$ for any $\theta > \theta^+$ in a neighborhood of θ^+. Thus, locally, $[\theta^-, \theta^+]$ is the largest possible interval over which (9) is binding; however making the definition local allows for the possibility that $\left[\theta^L, \theta^H\right]$ contains zero, one, or several (disjoint) intervals satisfying the definition of $[\theta^-, \theta^+]$. By Lemma 1 (see Appendix), one and only one such interval exists; it must be different from $\left[\theta^L, \theta^H\right]$.

Depending on the position of $[\theta^-, \theta^+]$ in $\left[\theta^L, \theta^H\right]$, the trajectories of Π and λ over $\left[\theta^L, \theta^H\right]$ must conform to one of five cases. This is described in the following proposition.

Proposition 1 *If it exists, the candidate solution must fall under one of the following five cases:*

- *Case 1:* $\theta^- = \theta^+ = \theta^L$; $\lambda(\theta) = 0\ \forall\theta$; $\Pi(\theta) > 0\ \forall\theta > \theta^L$; $\Pi\left(\theta^L\right) = 0$. *In this case:*

    1. $\mu(\theta)$ *is strictly positively monotonic;* $\mu\left(\theta^H\right) = 0$;

[6] See Seierstad and Sydsaeter (1986, chapter 5). Their sufficiency conditions for a solution may be applied in this context. An important characteristic of these conditions is the possibility of discontinuities in the solution $i(\theta)$ and in $\mu(\theta)$ when $\Pi = 0$.

2. $i(\theta) < 0 \; \forall\theta$;

3. $\Pi(\theta)$ *is strictly positively monotonic.*

- *Case 2:* $\theta^- = \theta^L < \theta^+ < \theta^H$; $\Pi(\theta) = 0 \; \forall\theta \in \left[\theta^L, \theta^+\right]$; $\lambda(\theta) = 0$ *and* $\Pi(\theta) > 0$, $\forall\theta > \theta^+$. *In this case:*

  1. $\mu(\theta)$ *is strictly positively monotonic over* $]\theta^+, \theta^H]$; $\mu\left(\theta^H\right) = 0$;

  2. $i(\theta) = 0 \; \forall\theta < \theta^+$; $i(\theta) < 0 \; \forall\theta \geq \theta^+$;

  3. $\Pi(\theta)$ *is strictly positively monotonic* $\forall\theta \geq \theta^+$.

- *Case 3:* $\theta^L < \theta^- < \theta^+ \leq \theta^H$; $\lambda(\theta) = 0$ *and* $\Pi(\theta) > 0 \; \forall\theta < \theta^-$; $\Pi(\theta) = 0 \; \forall\theta \in [\theta^-, \theta^+]$; $\lambda(\theta) = 0$ *and* $\Pi(\theta) > 0 \; \forall\theta > \theta^+$. *In this case:*

  1. $\mu(\theta)$ *is strictly positively monotonic over* $[\theta^L, \theta^-[$ *and over* $]\theta^+, \theta^H]$; $\mu\left(\theta^H\right) = \mu\left(\theta^L\right) = 0$;

  2. $i(\theta) > 0 \; \forall\theta \leq \theta^-$; $i(\theta) = 0$, $\theta^- < \theta < \theta^+$; $i(\theta) < 0 \; \forall\theta \geq \theta^+$;

  3. $\Pi(\theta)$ *is strictly positively (negatively) monotonic* $\forall\theta \geq \theta^+$ *(* $\forall\theta \leq \theta^-$ *)*;

  4. $\theta^- \neq \theta^+$.

- *Case 4:* $\theta^L < \theta^- \leq \theta^+ = \theta^H$; $\lambda(\theta) = 0$ *and* $\Pi(\theta) > 0 \; \forall\theta < \theta^-$; $\Pi(\theta) = 0$, $\forall\theta \in \left[\theta^-, \theta^H\right]$. *In this case:*

  1. $\mu(\theta)$ *is strictly positively monotonic over* $\left[\theta^L, \theta^-\right]$; $\mu\left(\theta^L\right) = 0$;

  2. $i(\theta) > 0 \; \forall\theta \leq \theta^-$; $i(\theta) = 0 \; \forall\theta > \theta^-$;

  3. $\Pi(\theta)$ *is strictly negatively monotonic* $\forall\theta \leq \theta^-$.

- *Case 5:* $\theta^L < \theta^- = \theta^+ = \theta^H$; $\lambda(\theta) = 0 \; \forall\theta$; $\Pi(\theta) > 0 \; \forall\theta < \theta^H$; $\Pi\left(\theta^H\right) = 0$. *In this case:*

1. *$\mu(\theta)$ is strictly positively monotonic; $\mu\left(\theta^L\right) = 0$;*

2. *$i(\theta) > 0 \; \forall \theta$;*

3. *$\Pi(\theta)$ is strictly negatively monotonic.*

Proof.

1. In all cases, the monotonicity of μ follows from (13) when $\lambda = 0$, and the values of $\mu\left(\theta^H\right)$ and (or) $\mu\left(\theta^L\right)$ are transversality conditions. In Case 3, $\mu(\theta^-) > 0$ follows from the fact that μ strictly rises from $\mu\left(\theta^L\right) = 0$ to $\mu(\theta^-)$; similarly, $\mu(\theta^+) < 0$ follows from the fact that μ strictly rises from $\mu(\theta^+)$ to $\mu\left(\theta^H\right) = 0$.

2. In all cases, $i(\theta) = 0$ whenever $\Pi(\theta) = 0$ because of (8). Now we prove that $i(\theta) < 0 \; \forall \theta$ in case 1. Starting from $\Pi\left(\theta^L\right) = 0$, Π is to become positive; given (8), it follows that $i\left(\theta^L\right)$ must be negative; because of (7), i will then remain negative over the rest of the interval. The sign of i in other cases is established in a similar way.

3. In all cases, the claimed monotonicity of Π is implied by (8) and the sign of i over the relevant interval.

4. To show that $\theta^- \neq \theta^+$ in case 3, suppose otherwise. Given the monotonicity of μ over $[\theta^L, \theta^-[$ and over $]\theta^+, \theta^H]$, and the fact that $\mu\left(\theta^H\right) = \mu\left(\theta^L\right) = 0$, there must be a discontinuity in μ at $\theta^+ = \theta^-$, with $\mu(\theta^-) > 0$ and $\mu(\theta^+) < 0$; since $\frac{\partial i^a}{\partial \mu} < 0$ by (12), it follows that $i(\theta^+) > 0$, a contradiction. ■

Proposition 1 describes the qualitative properties of all possible solutions. The five cases are illustrated in Figure 1. Case 1 represents a situation where the interval $[\theta^-, \theta^+]$ is squeezed to the left of $\left[\theta^L, \theta^H\right]$ and is actually reduced to θ^L. In other cases, the interval is progressively shifted to the right, so that case 5 represents a situation where the interval is reduced to θ^H. For each caseΓ the figure gives the optimal investmentΓ profitΓ and shadow price of profit satisfying the qualitative properties stated in Proposition 1.

As will be shown belowΓ these situations occur according to the magnitude of S'. To complete the characterizationΓ we pick a candidate solution that assumes the absence of any discontinuities in μ at θ^+ and θ^-Γ and we verify that it satisfies all other conditions in Seierstad and Sydsaeter's sufficiency theoremΓ allowing us to conclude that it solves problem (10). By Lemma 2 (see Appendix)Γ such a solution exists if f satisfies the following assumption.

Assumption 1 *f has the following properties:*

$$l(\theta) \equiv \frac{1 - F(\theta)}{f(\theta)} \text{ is non increasing}$$

$$h(\theta) \equiv \frac{F(\theta)}{f(\theta)} \text{ is non decreasing}$$

The solution is described in the following propositionsΓ proven in the Appendix.

Proposition 2 *If $\mu(\theta)$ is continuous over $\left[\theta^L, \theta^H\right]$, it is given, in cases 1-3 and 4-5 respectively, by*

$$\mu(\theta) = [1 - \alpha] F(\theta) - [1 - \alpha] + \Lambda\left(\theta^+\right) - \Lambda(\theta) \tag{16}$$

and

$$\mu(\theta) = [1 - \alpha] F(\theta) - \Lambda(\theta) \tag{17}$$

where $\Lambda(\theta) = 0 \; \forall\theta$ in cases 1 and 5, while, in cases 2 and 3, $\Lambda(\theta) = 0 \; \forall\theta \leq \theta^-$, $\Lambda(\theta) = \Lambda(\theta^+)$, $\theta \geq \theta^+$, and, for $\theta^- < \theta < \theta^+$

$$\Lambda(\theta) = \left[[1 - \alpha] \frac{F(\theta) - 1}{f(\theta)} - [S' - \theta] + \frac{\Lambda(\theta^+)}{f(\theta)}\right] f(\theta) \tag{18}$$

with $\Lambda(\theta^+) = \left[S' - \theta^L\right] f\left(\theta^L\right) + [1 - \alpha]$ in case 2 and $\Lambda(\theta^+) = [1 - \alpha]$ in case 3. In *case 4*

$$\Lambda(\theta) = \left[[1-\alpha] \frac{F(\theta)}{f(\theta)} - [S' - \theta] \right] f(\theta) \tag{19}$$

The function Λ in Proposition 2 is well defined. HoweverΓ by its definition (15)Γ it must also be increasing. Lemma 2 ensures that this is true.

Proposition 1 does not define θ^- and θ^+ when they are in the interior of $\left[\theta^L, \theta^H\right]$. This is done in Proposition 3.

Proposition 3 *If $\mu(\theta)$ is continuous over $\left[\theta^L, \theta^H\right]$, in cases 3 and 4, θ^- is defined by the condition*

$$0 = \left[S' - \theta^-\right] f\left(\theta^-\right) - [1-\alpha] F\left(\theta^-\right) \tag{20}$$

In cases 2 and 3, θ^+ is defined by the condition

$$0 = \left[S' - \theta^+\right] f\left(\theta^+\right) - [1-\alpha] F\left(\theta^+\right) + 1 - \alpha \tag{21}$$

Proposition 3 defines θ^- and θ^+ implicitly. By Lemma 2Γ when they exist within the interval $\left[\theta^L, \theta^H\right]$Γ they also satisfy$\theta^- < \theta^+$. The next proposition specifies the values of the shadow price of capital that cause each of the five cases to arise.

Proposition 4 *Let $S_1' < \theta^L$ and $S_4' > \theta^H$ be respectively defined by*

$$0 = \left[S_1' - \theta^L\right] f\left(\theta^L\right) + 1 - \alpha \tag{22}$$

and

$$0 = \left[S_4' - \theta^H\right] f\left(\theta^H\right) - [1-\alpha] \tag{23}$$

Then cases 1-5 arise according to the value of S' relative to S_1', θ^L, θ^H and S_4':

- *Case 1 corresponds to: $S' \leq S_1' < \theta^L$;*

- *Case 2 corresponds to:* $S'_1 < S' \leq \theta^L$;

- *Case 3 corresponds to:* $\theta^L \leq \theta^- \leq S' \leq \theta^+ \leq \theta^H$ *where the left two inequalities are strict unless* $S' = \theta^L$ *and the right two inequalities are strict unless* $S' = \theta^H$;

- *Case 4 corresponds to:* $\theta^H < S' < S'_4$;

- *Case 5 corresponds to:* $S' \geq S'_4 > \theta^H$.

The candidate solution described in propositions 1-4 exists since it was obtained by construction. It satisfies all conditions for an optimum except that (7) was not imposed that the monotonicity of Λ was not verified and that the condition $\theta^- < \theta^+$ was not verified. By Lemma 2 Assumption 1 is sufficient for these properties to be satisfied.

4 Discussion

The optimal program under asymmetric information differs notably from its full information counterpart. The most visible difference is the pooling phase (at $i = 0$) that occurs between negative and positive investment regimes under asymmetric information. Under symmetric information as (5) clearly shows investment is positive if $\Gamma^{s\prime}$ the marginal value of capital in terms of expected discounted future surpluses is higher than θ and vice versa; there is no interval of θ between the two regimes over which $i = 0$. Informational asymmetry usually introduces inefficiency under assumptions similar to those made here but without causing such pooling. What happens here is that there is a conflict between two incentives to misrepresent θ. A manager whose θ would place him in the category of capital buyers under full information has an incentive to overstate θ in order to overstate his cost of buying; but too big an overstatement might place him in a high θ group of types who should normally sell capital under full information. However for sellers the incentive to misrepresent goes in the opposite direction: they should understate θ to understate their revenues from selling. Managers whose θ is close to S', the marginal value of capital at which investment would switch from negative to positive under full information, would face such a dilemma. Precisely, all types in $[\theta^-, \theta^+]$ face that dilemma, which is resolved by asking them not to invest. This situation involving conflicting incentives is an example of 'Inflexible Rules in Incentive Problems' as analyzed by Lewis and Sappington (1989). It arises here naturally rather than being engineered by a principal in order to alleviate an incentive problem as in their case.

For types whose cost of capital is close to the S', the loss from setting $i = 0$ is low relative to the saving that such inefficiency allows in the cost of inducing more profitable types to reveal their true θ. Such profitable types may occur at both ends of the θ spectrum, with profits being generated either by buying (low θ), or selling (high θ) capital. Taking Case 3 as an example, this appears more clearly if (12) is written (using Proposition 2) in the following form

$$\theta + bi(\theta) = \begin{cases} S' + [1 - \alpha]\, l(\theta)\,, & \theta \geq \theta^+ \\ S' - [1 - \alpha]\, h(\theta)\,, & \theta \leq \theta^- \end{cases} \tag{24}$$

The left-hand side is the cost of the marginal unit of capital, inclusive of its adjustment cost component. For firms at either end of $\left[\theta^L, \theta^H\right]$, the second term on the right-hand side of the appropriate line vanishes, implying that marginal cost is set equal to S'. This is indeed the same rule as under full information, (misleadingly) suggesting that types θ^L and θ^H are asked to behave efficiently. Since $S' \in [\theta^-, \theta^+]$ (Proposition 4), $\theta^L < S'$ and $\theta^H > S'$, so that this rule requires low θ types to buy capital and high θ types to sell. In contrast, types whose θ is closer to S' are being asked to deviate from that rule by an amount proportional to l or h. Since l is non increasing and non positive, and h is non decreasing and non negative, the wedge is wider, the closer θ is to the relevant switching value (θ^-or θ^+). This is illustrated in Figure 2 for a uniform distribution of θ over $[1, 2]$

with $b = 1$. Applying rule (24) beyond type θ^+ (θ^-) would imply requesting a positive (negative) investment from a type that would be asked to sell (buy) capital under full information at identical marginal value of capital. The optimal contract reflects the fact that it is less costly in terms of efficiency loss to choose pooling instead in the vicinity of S'. Thus there is a third instance of (apparent) efficient behavior: when θ is equal to S' the agent invests zero as would be the case under full information.

As hinted above one would be mistaken to believe that types θ^L θ^H and S' are asked to invest as under full information. Although the rule is the same as under full information for these types there is inefficiency arising from the fact that at any given k S' differs from its full information counterpart $\Gamma^{s\prime}$. This difference appears because whatever its current type there is a strictly positive probability (unless T is known to be within two periods) that in the future an agent will be asked to invest a different amount than would be warranted under symmetric information.

The distinction between S' and $\Gamma^{s\prime}$ also has implications for interpreting Tobin's q. Under symmetric information according to the q theory of investment the stock exchange valuation V of a firm provides a measure of the total value of its assets. Under appropriate assumptions (Hayashi 1982; Abel and Eberly 1994) this also applies at the margin so that $\frac{V}{k}$ provides a measure of the contribution to V of the marginal (and average) unit of capital as it is perceived by the market[7]. In our notation $\frac{V}{k}$ would then provide a measure of $\Gamma^{s\prime}$. According to the q theory investment should be chosen in such a way as to equate the marginal cost of acquiring one unit of capital with $\Gamma^{s\prime}$ and the unobserved $\Gamma^{s\prime}$ should be replaced with $\frac{V}{k}$ in the investment equation: $bi_t^s + \theta = \frac{V}{k}$ where θ is interpreted to include the market purchase price p_K. Under asymmetric information the shadow price of capital S' cannot be measured in the same way: the reason is that S is a surplus *to be shared* between the principal and the agent ($S \equiv \Gamma^a + \psi^a$) while

[7]The identification of marginal q with average q is a restriction that can be circumvented; see Abel and Blanchard (1986). This does not affect our argument.

the stock exchange measures only the portion Γ^a that accrues to the principal under asymmetric information. Furthermore it is clear in our simple specification of the asymmetric information model that the relative size of Γ^a and ψ^a is sensitive to the parameters of the problem. Consequently the proportional error made by measuring S using stock exchange valuation is likely to be highly variable both across firms across sectors and over time[8]. This might be yet another explanation for the lack of reliability of q models of investment. In fact there is evidence that Tobin's q as measured by the ratio of Γ^a over the replacement cost of tangible assets might be sensitive to information asymmetries. The literature on diversification and performance provides evidence that Tobin's q is negatively correlated with the degree of diversification when diversification is into unrelated businesses. This evidence is discussed and complemented by Lang and Stulz (1994). It may imply that agency costs are higher in firms that diversify into unrelated business than in more focused firms (i.e. $S < \Gamma^s$) but also that the share of S appropriable by shareholders Γ^a which is what markets measure is lower in unfocused firms than in focused ones.

As was mentioned earlier the qualitative nature of our results is unaffected if θ is defined to include besides an idiosyncratic component the observable asset price of equipment p_K. The pooling that occurs under asymmetric information provides a new explanation for investment to be insensitive to variations in the cost of capital and in the shadow value of capital over certain ranges. Abel and Eberly (1994) summarize conditions under which the standard cost of adjustment model of investment involves $i = 0$ over an asset price region. In the absence of any non negativity constraint one possibility is the presence of a fixed cost for any non zero level of investment; in that case however there are discontinuities in the optimal investment function which do not arise in our model. Another possibility is the presence of a kink in the cost

[8]There is evidence of such variability in the recent empirical literature. See Demers et *al.* (1994).

of investment function at $i = 0$Γ either due to the different nature of negativeΓ versus positiveΓ investmentΓ or to a difference between purchase and resale prices. HereΓ there is no discontinuity in the optimal investment function so that its qualitative propertiesΓ as a relationship between investment and its costΓ are similar to the properties of i^a. Although not distinguishable in that respectΓ the two models differ in the investment uncertainty relationship. This is discussed further below.

Besides the standard neoclassical theory of investmentΓ the optimality of not reacting to a changing environmentΓ over a certain rangeΓ has been identified and discussed in a growing literature on hysteresis. As far as investment theory is concernedΓ this literature is best presented in Dixit and Pindyck (1994). Faced with an irreversible decision to take under uncertaintyΓ firms value positively the option of waiting for more information. This introduces a wedgeΓ the option valueΓ between the cost of investment and its expected marginal product value. This option value explains whyΓ over a certain rangeΓ firms optimally choose not to react to variations in the cost of investment or in the expected marginal product value. As when fixed costs are associated with non null investment levelsΓ which indeed makes the investment decision irreversibleΓ there are discontinuities between inaction regimes and regimes of active investmentΓ as opposed to the asymmetric information model.

Thus our model implies an observationally different investment behavior than the most well-known alternatives. One apparent exception is the cost-of-adjustment model when there is a kink in the cost of adjustment at $i = 0$. Both models imply inaction over some range in the cost of capital or its shadow valueΓ and a progressive departureΓ without discontinuityΓ from that situation at values outside the inaction range. HoweverΓ the two models differ in the way investment is affected by uncertainty. Comparisons are not straightforward thoughΓ because the types of uncertainties envisaged in both models are somewhat different: uncertainty about future capital productivity in the standard cost of adjustment model; uncertainty about the current cost of capital in the asymmetric information model. In the asymmetric information modelΓ an increase in uncertaintyΓ taking the form of a wider spread in the distribution of θΓ will normally reduce the absolute value of investmentΓ as illustrated in Figure 3 for the uniform distribution. In contrastΓ in the kinked cost-of-adjustment modelΓ the result of Abel (1983) and Caballero (1991) apply: they find that increased uncertainty increases the investment of competitive firms with constant returns to scaleΓ at least when the random shocks are idiosyncratic to individual firmsΓ as they are in our model. Such reversals in the positive correlation between uncertainty and investment as implied by our asymmetric information model have been encountered in other contextsΓ and discussed extensively by Caballero. He observes that adjustment-cost asymmetryΓ combined with imperfect competitionΓ produce this reversal in symmetric information modelsΓ underlining that imperfect competition "is also the paramount factor". Our model exhibits this property without adjustment-cost asymmetry or imperfect competition.

5 Conclusion

Investment theoryΓ especially the body of literature underlying the study of business fixed investment spendingΓ largely neglects issues of agency and information. In this paperΓ we have introduced asymmetry of information between shareholders and managers into an otherwise standard cost of adjustment model of investment.

This produces an investment equation with clearly distinguishable features. The most remarkable one is a new form of hysteresis which results from conflicting incentives to misrepresent costs for certain types. Hysteresis arises when the conflict is resolved by the use of an inflexible rule as in Lewis and Sappington (1989). Departures from the inaction regime are smooth as in the model of Abel and Eberley (1994) involving adjustment costs with a kink at $i = 0$. HoweverΓ their model can be distinguished from ours by the nature and role of uncertainty.

Our model also casts a new light on the q theory of investment. It shows that, if information asymmetries are present, the shadow value of capital should be defined to include rents accruing to managers. Failing that, q is poorly and inconsistently measured.

From the point of view of agency theory, the investment model turns out to have interesting peculiarities. First, our model introduces a form of dynamics which has been neglected sofar in principal-agent models, although it is standard in other fields of economics. The intertemporal link is provided by capital and investment, but might as well involve learning by doing or R&D. At first sight, this type of dynamics does not appear to affect the result, pervasive in static agency theory, according to which the behavior of 'good' types is the same as under full information. Thus the lowest-cost manager is asked to chose i so as to equate marginal cost to the shadow price of capital as under full information. However, since there is a positive probability of not being lowest-cost in some future period, distortions will occur in the future almost certainly, so that the shadow price of capital is different than under full information: the same investment rule yields a different investment level.

Second, depending on the cost of capital, positive or negative investment may be desirable to shareholders and managers. As a result 'good' types, to whom a full information investment rule applies, may coexist at both ends of the type range.

LEMMAS AND PROOFS[9]

A Lemma 1

Lemma 1 *There exists one and only one interval* $[\theta^-, \theta^+]$*, possibly reduced to a single point but necessarily different from* $\left[\theta^L, \theta^H\right]$*.*

Proof. In order to show that there exists at least one intervalΓ we show that $\Pi > 0\ \forall\theta$ is impossible. Suppose that $\Pi > 0\ \forall\theta$; then μ is continuous over $\left[\theta^L, \theta^H\right]$ andΓ by (14)Γ is strictly monotonic. However the transversality conditions corresponding to $\Pi\left(\theta^L\right) > 0$ and $\Pi\left(\theta^H\right) > 0$Γ $\mu\left(\theta^L\right) = 0$ and $\mu\left(\theta^H\right) = 0$ respectivelyΓ cannot be both satisfied by a monotonicΓ continuous trajectory. Now suppose that there is more than one interval satisfying the definition of $[\theta^-, \theta^+]$; by definition the intervals must be separated by intervals over which $\Pi > 0$; thus there exists $\theta_1 < \theta_2 < \theta_3$ such that $\Pi(\theta_1) = 0$Γ $\Pi(\theta_2) > 0$Γ and $\Pi(\theta_3) = 0$. ConsequentlyΓ as θ increases from θ_1 to θ_3Γ the continuous function $\Pi(\theta)$ must first riseΓ which requires $i < 0$ by (8)Γ then diminishΓ which requires $i > 0$. This violates (7). It remains to show that $[\theta^-, \theta^+]$ is different from $\left[\theta^L, \theta^H\right]$. Suppose otherwise; thenΓ by (8)Γ in order to maintain $\Pi = 0$ over the whole intervalΓ $i = 0\ \forall\theta$. Since the objective to maximize is $\int_{\theta^L}^{\theta^H} \left\{pg(k) - \theta i(\theta) - \frac{1}{2}bi(\theta)^2 - [1-\alpha]\Pi(\theta) + S(k + i(\theta))\right\} f(\theta)\, d\theta$Γ and since either $S'(k + i(\theta)) > \theta_L$Γ or $S'(k + i(\theta)) < \theta_H$Γ or bothΓ the program $i = \Pi = 0\ \forall\theta$ may be strictly improvedΓ either (a) by setting $i > 0$ over a neighborhood of θ^LΓ or (b) by setting $i < 0$ over a neighborhood of θ^HΓ or both. Let us show that this is feasible under the constraints imposed by asymmetric information. Thus suppose (a) applies and choose

[9] Propositions or Lemmas are stated in the Appendix only if they are not stated in the main text.

some $i_1 > 0$Γ $\Delta > 0$Γ and Θ_1 such thatΓ for $\theta \in \left[\theta^L, \Theta_1\right]$Γ the difference between the contribution of the perturbed program and the original candidate satisfies[10]

$$pg(k) - \theta i_1 - \frac{1}{2} b i_1^2 - [1-\alpha]\,\Pi^1(\theta) + S(k+i_1) - pg(k) - S(k) > \Delta > 0$$

where $\Pi^1(\theta)$ is such that (8) is satisfied and $\Pi^1(\Theta_1) = 0$: $\Pi^1(\theta) = \Pi_L - \theta i_1$. The principal may ask agents of type $\theta < \Theta_1$ to set $i = i_1$Γ offering them $\Pi^1(\theta)$Γ rather than asking them to set $i = 0$ and offering them $\Pi = 0$. They will find it in their interest to acceptΓ each yielding an increment of at least Δ to the objective of the principal. Thus $i = 0 = \Pi\ \forall\theta$ cannot be optimalΓ implying that $[\theta^-, \theta^+] \subset \left[\theta^L, \theta^H\right]$. ■

B Proposition 2

We start from (14) and use the transversality conditions corresponding to each case in order to eliminate the constant of integration A. The assumed continuity of μ implies that there is only one such constant of integration in each case. In cases 1-3Γ $\Pi\left(\theta^H\right)$ is freeΓ so that $\mu\left(\theta^H\right) = 0$; (14) implies

$$A = \Lambda\left(\theta^H\right) - [1-\alpha]$$

Substituting into (14)Γ recognizing that $\Lambda\left(\theta^H\right) = \Lambda(\theta^+)$Γ gives (16). In cases 4-5Γ $\Pi\left(\theta^L\right)$ is freeΓ so that $\mu\left(\theta^L\right) = 0$; it follows from (14) that $A = 0$ which in turn implies (17).

$\Lambda(\theta) = 0$ by definition in cases 1 and 5. In cases 2 and 3Γ $\theta^+ < \theta^H$Γ so thatΓ by definitionΓ $\Lambda(\theta) = \Lambda(\theta^+)\ \forall\ \theta \geq \theta^+$. For $\theta \leq \theta^-$Γ by definitionΓ $\Lambda(\theta) = 0$. For

[10] if no such triplet $\left(i^1, \Theta_1, \Delta\right)$ may be foundΓ then it is certain that a similar triplet corresponding to (b)Γ $i^2 < 0$Γ Θ_2Γ $\Delta > 0$ can be found such thatΓ for $\theta \in \left[\Theta_2, \theta^H\right]$

$$pg(k) - \theta i_2 - \frac{1}{2} b i_2^2 - [1-\alpha]\,\Pi^2(\theta) + S(k+i_2) - pg(k) - S(k) > \Delta > 0$$

$\theta^- < \theta < \theta^+$Γ by Proposition 1Γ $i = 0$Γ which impliesΓ substituting (16) into (12)

$$0 = \frac{1}{b}\left\{[S' - \theta] - \frac{[1-\alpha][F(\theta) - 1] + \Lambda(\theta^+) - \Lambda(\theta)}{f(\theta)}\right\}$$

from which (18) follows. $\Lambda(\theta^+)$ is obtained as follows. In case 2Γ writing (18) at $\theta^- = \theta^L$Γ with $\Lambda(\theta^-) = 0$ by definitionΓ gives $\Lambda(\theta^+) = \left[S' - \theta^L\right] f\left(\theta^L\right) + [1-\alpha]$. In case 3Γ since $\Pi\left(\theta^L\right)$ is freeΓ we further haveΓ as a transversality conditionΓ that $\mu\left(\theta^L\right) = 0$. Writing (16) at θ^LΓ with $\Lambda\left(\theta^L\right) = \Lambda(\theta^-) = 0$ by definitionΓ yields $\Lambda(\theta^+) = [1-\alpha]$.

We turn to establishing (19): this is done by substituting (17) into (12)Γ and setting $i^a = 0$. ■

C Proposition 3

Since θ^- is the lowest level of θ at which constraint (9) is bindingΓ $\Lambda(\theta^-) = 0$. By Proposition 1Γ at θ^-Γ $i = 0$. Writing (12) at θ^-Γ while substituting the formulas for μ and Λ given in Proposition 2 for cases 3 and 4Γ impliesΓ in both casesΓ that θ^- must satisfy

$$0 = \frac{1}{b}\left\{\left[S' - \theta^-\right] - \frac{[1-\alpha]F(\theta^-)}{f(\theta^-)}\right\}$$

which reduces to (20). SimilarlyΓ by Proposition 1Γ at θ^+Γ $i = 0$. Writing (12) at θ^+Γ while substituting the formulas for μ and Λ given in Proposition 2 for cases 2 and 3Γ impliesΓ in both casesΓ that θ^+ must satisfy

$$0 = \frac{1}{b}\left\{\left[S' - \theta^+\right] - \frac{[1-\alpha]F(\theta^+) - [1-\alpha]}{f(\theta^+)}\right\}$$

which reduces to (21). ■

D Proposition 4

It is useful to refer to Figure 1 in order to see how the various cases are related to each other. The switch from Case 1 to Case 2 occurs when the value S_1' of S' is such that θ^L solves the definition of $\theta^+\Gamma$ i.e. when

$$0 = \left[S_1' - \theta^L\right] f\left(\theta^L\right) + 1 - \alpha$$

This implies $S_1' < \theta^L$. The switch between Case 2 and Case 3 occurs when the value S_2' of S' is such that θ^L solves the definition of θ^-

$$0 = \left[S_2' - \theta^L\right] f\left(\theta^L\right) \tag{25}$$

which implies $S_2' = \theta^L$. The switch between Case 3 and Case 4 occurs when the value S_3' of S' is such that θ^H solves the definition of θ^+

$$0 = \left[S_3' - \theta^H\right] f\left(\theta^H\right) \tag{26}$$

which implies $S_3' = \theta^H$. The switch between Case 4 and Case 5 occurs when value S_4' of S' is such that θ^H solves the definition of θ^-

$$0 = \left[S_4' - \theta^H\right] f\left(\theta^H\right) - [1 - \alpha]$$

which implies $S_4' > \theta^H$.

In cases 2 and 3Γ (21) applies; it follows that $S' \leq \theta^+$ and that the inequality is strict unless $\theta^+ = \theta^H$; similarlyΓ in cases 3 and 4Γ (20) implies $S' \geq \theta^-$ and the inequality is strict unless $\theta^- = \theta^L$. ■

E Lemma 2

Lemma 2 *A sufficient condition for the candidate solution described in propositions 1-4 to solve the principal's problem is for f to satisfy Assumption 1.*

Proof. The candidate solution described in propositions 1-4 exists since it was obtained by construction. It satisfies all conditions for an optimum except that (7) was not imposedΓ that the monotonicity of Λ was not verifiedΓ and that the condition $\theta^- < \theta^+$ was not verified. We have to show that these last three properties are verified. When i is constant at zeroΓ (7) is satisfied. Let us consider other situations. We start with Case 1Γ as well as cases 2 and 3 for $\theta \geq \theta^+$. Substituting the appropriate values of μ and Λ from Proposition 2 into (12)

$$i(\theta) = \frac{1}{b}\left[S' - \theta + [1-\alpha]\frac{1-F(\theta)}{f(\theta)}\right]$$

Since $\frac{1-F(\theta)}{f(\theta)} \equiv l(\theta)$, a sufficient condition for $\frac{di}{d\theta} \leq 0$ is l to be non increasing. The other cases where i is non constant are cases 3 and 4Γ for $\theta \leq \theta^-$Γ and Case 5. After appropriate substitutionsΓ (12) gives

$$i(\theta) = \frac{1}{b}\left[S' - \theta - [1-\alpha]\frac{F(\theta)}{f(\theta)}\right]$$

Since $\frac{F(\theta)}{f(\theta)} \equiv h(\theta)$Γ a sufficient condition for $\frac{di}{d\theta} \leq 0$ is f to be non decreasing.

It is immediate to verifyΓ using (18)Γ that the monotonicity of Λ is implied by the monotonicity of l in cases 2 and 3Γ whileΓ in case 4Γ it is implied by (19) and the monotonicity of h. SimilarlyΓ it can be verified using (20) or (21) that the monotonicity of h implies $\theta^- < \theta^+$. ■

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Case 1: $S' < S'_1$

Case 2: $S'_1 < S' < \theta^L$

Case 3: $\theta^L < S' < \theta^H$

Case 4: $\theta^H < S' < S'_4$

Case 5: $S'_4 < S'$

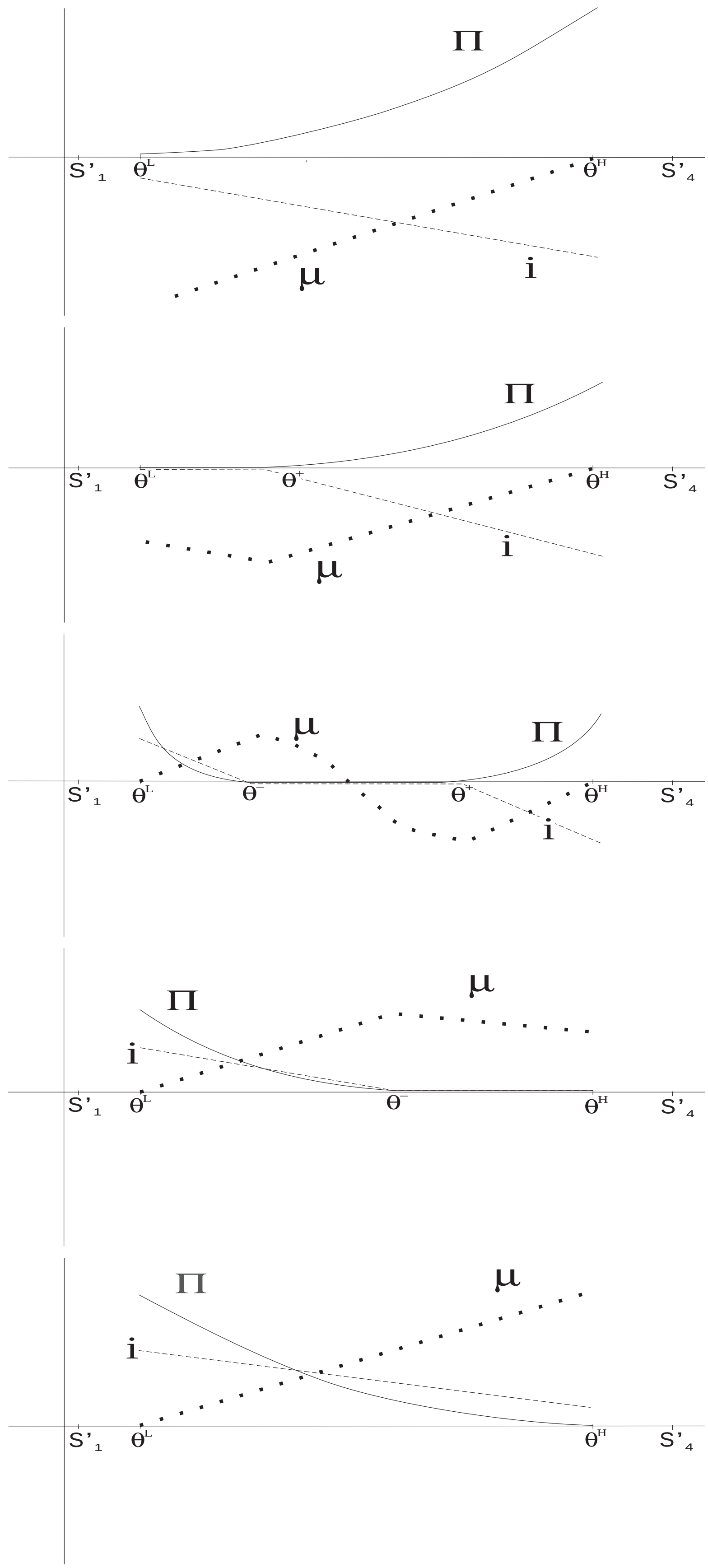

Figure 1: Optimal investment, profit, and shadow price of profit

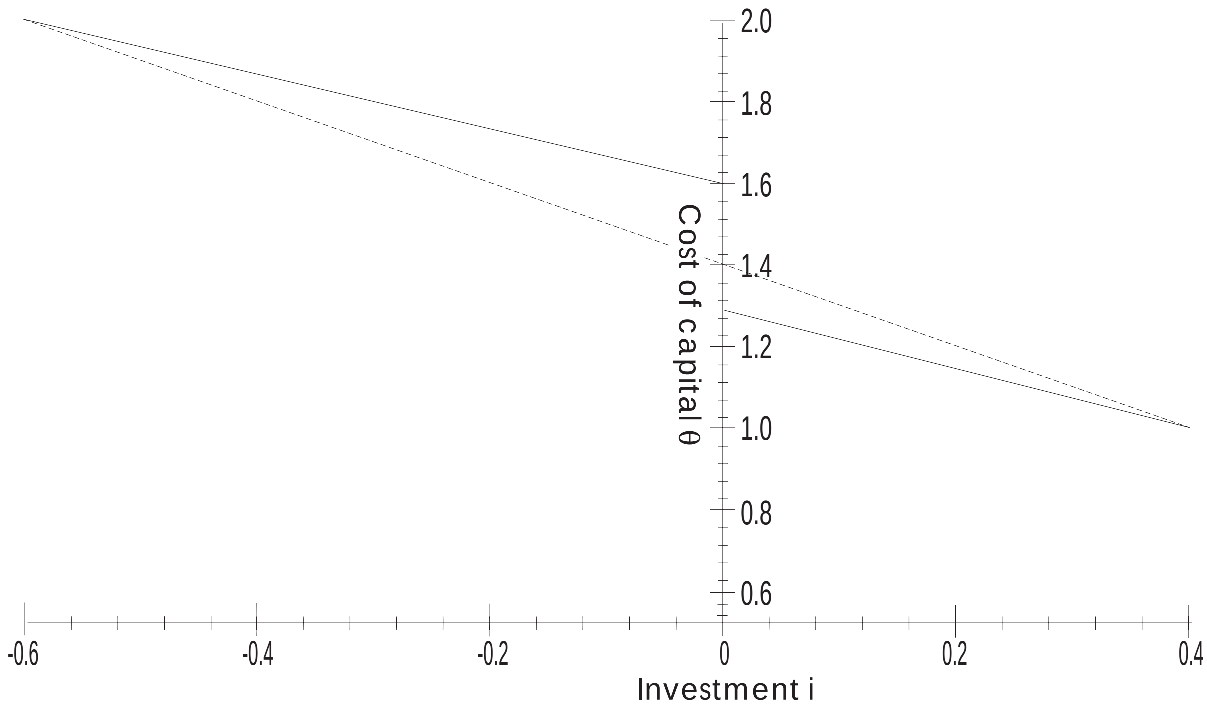

Figure 2: Inverse investment demand curve: asymmetric information (continuous line) and symmetric information (dotted line), S’ = 1.4

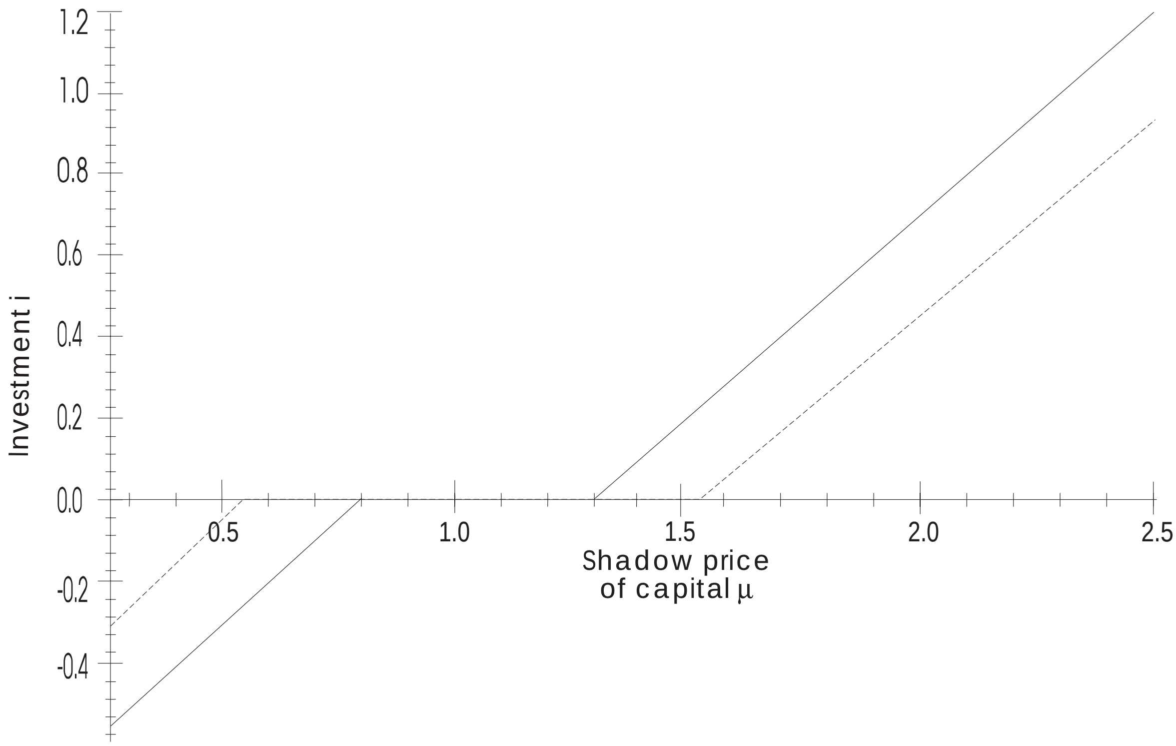

Figure 3: Investment as a function of S' under asymmetric information: high uncertainty (dotted line) and low uncertainty (continuous line), q = 1.2