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Gender Peer Effects Heterogeneity in Obesity *

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Résumé/abstract

This paper explores gender peer effects heterogeneity in adolescent Body Mass Index (BMI). We propose a utility-based non-cooperative social network model with effort technology. We allow the gender composition to influence peer effects. We analyze the possibility of recovering the fundamentals of our structural model from the best-response functions. We provide identification conditions of these functions generalizing those of the homogeneous version of the model. Extending Liu and Lee [2010], we consider 2SLS and GMM strategies to estimate our model using Add Health data. We provide tests of homophily in the formation of network and reject them after controlling for network (school) fixed effects. The joint (endogenous plus contextual) gender homogeneous model is rejected. However, we do not reject that the endogenous effects are the same. This suggests that the source of gender peer effects heterogeneity is the contextual effects. We find that peers' age, parents' education, health status, and race are relevant for the latter effects and are gender-dependent.

Mots clés/keywords: Obesity, Social Networks, Gender, Heterogeneity, Peer Effects, Identification, Add Health.

Codes JEL/JEL Codes: L12, C31, Z13, D85

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Abstract

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1 Introduction

Obesity has reached epidemic proportions in both children and adolescents in the United States, increasing from 5% in 1980 to 18% in 2010 (Ogden et al. [2012]). To explain such a phenomenon, a huge number of studies have focused on socio-economic factors such as growing unhealthy eating habits, the rise in time spent watching television, playing video games and using other types of media, the decline in time spent doing physical exercise such as walking or biking to school, and the reduction in physical education at school (see Papoutsi et al. [2013] for a survey).

Complementary to these views, health economics research has also attempted to investigate this obesity epidemic from the perspective of social interactions (*e.g.*, Christakis and Fowler [2007], Halliday and Kwak [2009], Trogdon et al. [2008], Cohen-Cole and Fletcher [2008], Yakusheva et al. [2014], Fortin and Yazbeck [2015]). Many of these studies document the presence of strong positive peer effects[1] which could create a synergic effect on the shocks (such as the technical progress in fast food preparation) which tend to increase obesity.

Many of these studies are based on linear social interactions functions, as proposed by Manski [1993], Bramoullé et al. [2009], and extended by Blume et al. [2015]. In the standard version of this approach, an individual's outcome linearly depends on his own characteristics, and on peer effects. The latter come from two different channels: the individual's reference group mean outcome - the so-called *endogenous peer effect* - and his/her reference group mean characteristics - known as the *contextual peer effects*.

However, one potentially important limitation of this framework is that it usually assumes that social interactions are *homogeneous*. This means that peer effects do not vary according to any particular type such as race or gender. However, this assumption is strong and may not be realistic. For example, Costa-Font and Jofre-Bonet [2013] provides evidence that the average Body Mass Index (BMI) of a young woman's female peers negatively influences her likelihood to suffer from anorexia.[2] Intuitively, one may expect the influence of her average male peers' BMI on this risk to be much less important. This example illustrates that imposing female and male peer effects to be the same on a young woman's chance to suffer from anorexia is likely to lead to severe estimating biases and therefore to inappropriate health policies.

[1] An exception is Cohen-Cole and Fletcher [2008] who find a non-statistically significant coefficient of the endogenous peer effect using standard econometric techniques. Fortin and Yazbeck [2015] find a significant but small peer effect.

[2] According to the American Psychiatric Association (2000), women account for 90% of the 8 million sufferers of anorexia.

This paper generalizes the homogeneous linear social interactions model on teenagers' BMI[3] by allowing for gender-dependent heterogeneity in peer effects.[4] Our model allows the influence of male peers on a student to be different from that of female peers, and depending on whether the student is a male or a female.[5] In our setting, we consider the population of interest as composed of a given number of networks (schools). Two types of individuals interact within the same network (*i.e*, male *vs.* female students). This defines two *within-* and two *between-* endogenous peer effects. Accordingly, we end up with four different endogenous (*mm, ff,mf,fm*) social interaction matrices.[6] The same reasoning applies for the contextual peer effects. Of course, the standard homogeneous model is nested into our heterogeneous model and therefore can be easily tested.

To estimate our model, we use data on students at the secondary level, arguably a period in which social interactions are potentially highly important to structure an individual's body. More precisely, we use the saturation sample (1996) of the National Longitudinal Study of Adolescent Health (Add Health) which focuses on 16 selected schools. In particular, students were asked for information on their height and weight. Using this information, we constructed students' BMI. Importantly, respondents from this sample were also asked to *name up to five male friends and up to five of their female friends within their school.* These data thus allow to recover the friendship social networks.

The contributions of the paper are both theoretical and empirical. At the theoretical level, we propose a non-cooperative (Nash) model of BMI outcome with gender-dependent heterogeneity in peer effects in a network context. Unless empirical evidence is provided that obesity is a virus, it is counterintuitive to think that one can gain weight by simply interacting with an obese person.[7] Therefore, our model takes into account the presence of a production function relating, among others, (unobservable) individual effort to BMI outcome. Our theoretical model is consistent with a mechanism of *strategic complementarity* or *synergy* in social interactions (*e.g.*, "I better like to eat in a fast food restaurant with a *buddy*"). Under the assumption that the best-response (reaction) functions are identified, and assuming a synergy mechanism, we show that we can recover all the primitives of our

[3] We also perform the analysis on zBMI (or z-score BMI), that is, the BMI standardized for gender and age.

[4] We leave the analysis of race or other characteristics heterogeneity in peer effects for further research.

[5] In a recent paper, Masten [2015] provides an analysis of endogenous peer effects with *unobservable* heterogeneity. In his framework, peer effects are introduced as random coefficients in a simultaneous equation model. Among other results, Masten shows that although the full joint distribution of structural unobservables is not point identified, some marginal distributions are point identified. The present paper can be seen as complementary to Masten [2015] approach.

[6] More details on these matrices will be given in the theoretical section.

[7] We acknowledge that some recent studies have pointed that obesity might be partially due to a virus ad-36 (see Rogers et al. [2007]).

structural model, *given* a proxy for the marginal productivity of effort on BMI. Therefore the so-called *identifiability problem* (see Chiappori and Ekeland [2009]) is (partly) solved, which may allow us to perform the analysis of shocks (*e.g.*, reforms) which affect social networks.

At the econometric level, one important contribution of our model is to show, using an approach similar to Bramoullé et al. [2009] but where we allow both for the presence of *isolated students*[8] and gender heterogeneity in peer effects, that we can derive sufficient identification conditions of the best-response functions. Our identification strategy is close to the one used by Liu and Lee [2010] but is generalized to the presence of gender heterogeneity. We show that these conditions are both based on restrictions on some of the parameters of our model, and on the structure of our four social interaction matrices. Our model is estimated using 2SLS and GMM methods inspired from Liu and Lee [2010]. Importantly, we rely on these estimators to account for the *Bonacich centrality* measure in a general case where interaction matrices are not row-normalized (due to the presence of isolated individuals in the networks).[9] In addition, our estimation strategy allows to account for network specific (or fixed) effects using a global transformation matrix.

Our approach is related to the one developed in Arduini et al. [2016]. In their paper, the authors focus on the identification and estimation of treatment response with heterogeneity using a network model. They propose a two-equation system based on the individual's type. However, their model specification and identification are different from ours. Our approach is based on a single structural heterogeneous peer effects model in a network context. To our knowledge, this is the first paper that uses our methodology. Also, while their estimation approach focuses on a 2SLS method, we provide both 2SLS and GMM estimators, the latter exploiting quadratic moments. Moreover, while Arduini et al. [2016] present Monte Carlo simulations of their approach, we provide an empirical application to peer effects heterogeneity in obesity.

One potentially important problem in the analysis of social interactions using non experimental data is the endogeneity in the formation of networks. A classical example is the presence of homophily where individuals with common characteristics tend to associate together. The introduction of network fixed effects (as it is done in our paper) may partly take this source of bias into account but not entirely as the formation of links within a network may still be endogenous. A recent literature relies on the presence of endogenous group or network formation and proposes models to simultaneously evaluate network formation and

[8]Isolated students are those who report having no friends. They represent about 23% of our sample.

[9]When an interaction matrix is row-normalized, the sum of its elements on each row is one. Of course, this is not possible when there is at least one individual i who has no friend. In this case, the latter sum for the row i is zero.

network effects [Conti et al., 2012, Goldsmith-Pinkham and Imbens, 2013, Badev, 2013, Hsieh and Lee, 2015, Hsieh and Lin, 2015, Boucher, 2016]. Goldsmith-Pinkham and Imbens [2013] argue that it is possible to test for the presence of endogeneity in network formation. Liu et al. [2013] propose a test based on the Goldsmith-Pinkham and Imbens [2013] approach as an attempt to test for the presence of possible network formation endogeneity using the Add Health dataset. They find no evidence of endogeneity in network formation. Patacchini and Rainone [2014] also find no evidence of the presence of endogenous network formation, while concentrating on peer effects in financial products. On the other hand, Hsieh and Lin [2015] finds the presence of homophily in the formation of network when the outcomes are students' achievement (GPA) or their smoking behavior. In our empirical application, we perform both a test of exogeneity inspired by Liu et al. [2013] and a more general one based on *visual* observation (see Boucher and Fortin [2016]). When we introduce network specific effects into the model, we do not reject the presence of exogeneity in the network formation associated with the BMI outcome.

Our main results based on a GMM approach reject the full (endogenous plus contextual) gender homogeneous model for the more general heterogeneous one. However, we do not reject that the endogenous peer effects are homogenous. This indicates that the source of heterogeneity is the contextual and not the endogenous peer effects. We find that peers' age, and parents' education, health status, and race are relevant for the latter effects and vary within and between gender.

Moreover, assuming gender peer effects homogeneity and strategic complementarity leads to a social multiplier equal to 1.20. This means that, under this assumption, the total impact of a common shock on the aggregate outcome in a network is 1.20 times the sum of its direct effects at the individual level, as it incorporates synergic effects stemming from social interactions. Interestingly, this figure is quite close to the one obtained by Fortin and Yazbeck [2015] (= 1.15) in an homogeneous model where peer effects are limited to fast food consumption.

The rest of the paper is organized as follows. Section 2 provides a short survey of the relevant literature on heterogeneity in peer effects. In section 3, we present a theoretical model of heterogeneous peer interactions. In section 4, we present our econometric model, our identification conditions and our estimation methods. Subsection 4.2 presents a particular case of the model. Data are presented in section 5. We also provide a test of network exogeneity. Section 6 presents our results based on gender decompositions. Section 7 concludes.

2 Previous Literature

Beside the study by Arduini et al. [2016] discussed in the introduction, a growing number of researches have focused on issues raised by heterogeneity in peer effects. Many papers assume that individuals interact in a network composed of groups[10] and analyze the impact of gender proportion on the outcomes of male and female individuals (*e.g.* Hoxby [2000], Whitmore [2005], and Lavy and Schlosser [2011]). One limitation of this approach is that it is generally not possible to separately identify endogenous and contextual effects. Manski [1993] named this failure the *reflection problem*.[11] To deal with this drawback, some studies assume no endogenous effect (which imposes untestable restrictions) and others focus only on the reduced form of the model. In the latter case, the model does not allow to recover the fundamentals of the structural model. Therefore, it may be difficult to estimate the impact of a reform which affects the structure of groups. However, Bramoullé et al. [2009] and Lee et al. [2010] have shown that the linear-in-means homogeneous model is *generically* identified when individuals interact through social networks that are not groups. Our approach extends this result to the heterogeneous peer effects model.

Kooreman and Soetevent [2007] investigate heterogeneity in peer effects using a binary choice model for different dichotomous outcomes. They estimate peer effects on girls, peer effects on boys and between peer effects where interaction occurs in group contexts. Their method consists in partitioning students into subgraphs using data from the Dutch National School Youth Survey. Using a simulation based method that accounts for multiple equilibria, they find that within-gender peer effects are larger than between-gender peer effects. They also find that boys tend to be more influenced by their peers than girls in terms of the different behavioral outcomes they consider (cigarettes smoking, for example).

Nonlinear tests of the nature of peer effects have been proposed by Sacerdote [2001]. His approach consists in grouping students and their peers into several categories and including in the regression all possible interactions of the student and the categories of his peers. Sacerdote [2001] finds that high ability students benefit each other more than high ability students benefit from average or low ability students in terms of school performance. In the same spirit, Lavy et al. [2009] find that high ability students positively affect girl performance and negatively affect boys performance. However, low ability students tend to negatively affect both girls and boys performance in their findings.

Renna et al. [2008] use the first wave of the Add Health dataset while controlling for

[10] In a group, all individuals are influenced by others in their group but by none outside of it.

[11] Note that Lee [2007] has shown that peer effects is theoretically identified when individuals do not belong to their own reference group and that there are a sufficient number of groups of different size. However, this approach often leads to weak identification, given the large average size of groups.

school fixed effects in a linear-in-means model. They find a positive peer effect coefficient where the outcome variable is weight of adolescents. They also find, using instrumental variables, that female adolescents are more responsive to the average body weight of their friends, and that the effects remain significant only for their same gender friends.

Yakusheva et al. [2014] use a natural experiment to identify peer effects in weight gain. Data are taken from roommates assignments of college students and they use the standard linear-in-means model of peer influences. They solve the correlated unobservables problem by taking the weight change of the *ego* from baseline to follow up. Their results show that there are little evidence of peer effects for males in weight gain, whereas effects are positive and significant for females. Their study thus provides evidence of heterogeneity in peer effects weight gain.

Hsieh and Lin [2015] estimate a high order spatial autoregressive model (SAR) to analyze gender and racial peer effects heterogeneity in the students' academic achievement (GPA) and smoking behavior. Their econometric model, using a Bayesian methodology, takes homophily in network formation into account. They find that within-gender are stronger than between-gender endogenous peer effects. Our approach uses a rather different methodology. First, since our tests reject homophily, we use a method of estimation based on 2SLS or GMM with quadratic moments. Also, our econometric model is derived from a structural Nash-theoretic approach. Finally we focus on a different outcome: students' BMI.

A recent paper by Beugnot et al. [2013] also provides evidence of heterogeneity of peer effects in work performance using a laboratory experiment where workers interact through networks. Their model is based on two different experiments: a *recursive experiment* in which participants play in isolation and take their decision without being influenced by their peers, and a *simultaneous experiment* where the same participants interact with other participants and where ties are undirected. They find positive and significant peer effects for male workers in the simultaneous game, while peer effects are not statistically significant in the case of female workers. They suggest that male workers appear to be more competitive than female workers.

3 Theoretical model

We consider a model extending Blume et al. [2015] in which n individuals interact through a social network. To make it more concrete, we assume a friendship network of n^m male and n^f female students ($n^m + n^f = n$), interacting within a school and whose weight (the outcome) can be influenced by their behaviour. To take observable heterogeneity into account, we define four network adjacency matrices: $\mathbf{A}_z (z = 1, \cdots, 4)$. We assume that links do not differ in strength. The network adjacency matrix $\mathbf{A}_1$ (resp., $\mathbf{A}_2$) is such that $a_{ij} = 1$ if the student i is a male student and is influenced by the male (resp., female) student j and $= 0$, otherwise. The matrices $\mathbf{A}_3$ and $\mathbf{A}_4$ are similarly defined for female students. The student i's reference group with size $n_{i,m}$ (resp. $n_{i,f}$) is the set of male (resp. female) students by which i is influenced.[12] The social interaction matrix $\mathbf{G}_z$ is a weighted adjacency matrix $\mathbf{A}_z$ such that, when $z = 1$ for instance, one has $g_{1ij} = 1/(n_{i,m} + n_{i,f})$ if i a male student and is influenced by the male student j, and 0, otherwise. For the moment, we suppose that all $\mathbf{G}_z$ are *non-stochastic* (or fixed) and known social interaction matrices. We also assume no *isolated* students.[13]

The BMI cannot be directly chosen by students but only indirectly through effort, that is, healthy life habits (*e.g.*, good dietary behaviour, physical exercise).[14] Moreover, it is counterintuitive to think that one can gain weight by simply interacting with an obese person. To account for such characteristics in our heterogeneous social network setting, we propose a non-cooperative (Nash) model in which every individual of each gender maximizes a quadratic utility function, separable in private and social sub-utilities, subject to a linear production function relating weight to effort and individual characteristics. Every individual of each type maximizes a utility function that is gender-dependent.[15] The maximization program of a type-m individual i is:

$$\max_{y_{i,m}, e_{i,m}} \quad U_{i,m}(e_{i,m}, \mathbf{y}) = -y_{i,m} - \frac{e_{i,m}^2}{2} + \psi_{mm} y_{i,m} \mathbf{g}'_{1i} \mathbf{y}_m + \psi_{mf} y_{i,m} \mathbf{g}'_{2i} \mathbf{y}_f,$$

$$s.t. \quad y_{i,m} = \alpha_0 - \alpha_1 e_{i,m} + \alpha_2 x_{i,m} + \eta_{i,m},$$

where $y_{i,j}$ is the outcome (BMI) of individual i in category j, $\mathbf{y}_m$ is the vector of outcomes in m category, $\mathbf{y}_f$ is the vector of outcomes in f category, $\mathbf{y}$ is the concatenated vector of outcomes in f and m categories, e_i stands for the (unobserved) effort of i, $\mathbf{g}'_{zi}$ is the i^{th} row of the social interaction matrix $\mathbf{G}_z$, x_i and $\eta_{i,m}$ are vectors of observable and unobservable characteristics, respectively. For notational simplicity, we assume only one observable characteristic.

The first two expressions in the utility function describe the private sub-utility. One

[12] For now on, we will assume that the student is excluded from his or her own reference group.

[13] Our econometric approach allows for isolated students since the social interactions matrices are not row-normalized.

[14] Fortin and Yazbeck [2015] assumes that visits in fast food restaurants is the main channel through which effort influences the students BMI.

[15] For notational simplicity, we ignore contextual peer effects, though our econometric model takes them into account.

assumes that an increase in BMI reduces the individual i's utility,[16] and $\frac{e_{i,m}^2}{2}$ represents the cost of effort (in term of utility) to reduce weight. One supposes that the marginal cost of effort is increasing with effort. The social sub-utility corresponds to the two last expressions. One assumes that social interactions influence preferences through a basic channel: *strategic complementarity* (or synergy) in BMI between a male student and his reference group of each type.[17] It means that an increase in the peers' average BMI of a given gender positively influences the marginal utility of his own BMI ($\psi_{mm} > 0; \psi_{mf} > 0$).[18] Heterogeneity in social interactions is reflected by the fact that ψ_{mm} and ψ_{mf} can be different.

The maximization program of type-f individuals can be written using a similar utility function, where social interaction parameters can differ from those of type-m. Hence, a type-f individual solves the following program :

$$\max_{y_{i,f}, e_{i,f}} \quad U_f(e_{i,f}, \mathbf{y}) = -y_{i,f} - \frac{e_{i,f}^2}{2} + \psi_{ff} y_{i,f} \mathbf{g}'_{3i} \mathbf{y}_f + \psi_{fm} y_{i,f} \mathbf{g}'_{4i} \mathbf{y}_m$$

$$s.t. \quad y_{i,f} = \alpha_0 - \alpha_1 e_{i,f} + \alpha_2 x_{i,f} + \eta_{i,f}$$

The first order conditions of the type-m maximization program lead to (in matrix notation):

$$\mathbf{y}_m = \alpha \boldsymbol{\iota}_m + \beta_{mm} \mathbf{G}_1 \mathbf{y}_m + \beta_{mf} \mathbf{G}_2 \mathbf{y}_f + \alpha_2 \mathbf{x}_m + \boldsymbol{\epsilon}_m \tag{1}$$

where $\alpha = \alpha_0 + \mu$, $\beta_{mm} = \mu \psi_{mm}$, $\beta_{mf} = \mu \psi_{mf}$, and $\boldsymbol{\epsilon}_m = \boldsymbol{\eta}_m$, with $\mu = \alpha_1^2$. Note that μ represents the squared marginal productivity of effort on weight level.

Similarly, the first order conditions for type-f individuals lead to:

$$\mathbf{y}_f = \alpha \boldsymbol{\iota}_f + \beta_{ff} \mathbf{G}_3 \mathbf{y}_f + \beta_{fm} \mathbf{G}_4 \mathbf{y}_m + \alpha_2 \mathbf{x}_f + \boldsymbol{\epsilon_f} \tag{2}$$

where $\beta_{ff} = \mu \psi_{ff}$, $\beta_{fm} = \mu \psi_{fm}$, $\gamma_f = \alpha_2$, and $\boldsymbol{\epsilon}_f = \boldsymbol{\eta}_f$. It is assumed that the absolute value of the β's is less than one.

Concatenating vectors and matrices from equations (1) and (2), we end up with the following best-response functions for the whole population of students, given the others'

[16] To simplify the model, we ignore a situation where very low weight negatively affects health (*e.g.*, anorexia).

[17] Our model is also consistent with a mechanism of *pure conformity* in social interactions. In that case, an individual's utility is positively affected by the degree to which he conforms with his peers' outcome or characteristics due for instance to the presence of social norms. Unfortunately, the present model cannot identify synergy from conformity (see Blume et al. [2015] and Boucher and Fortin [2016]) so that these two channels are *observationally equivalent*. Following Fortin and Yazbeck [2015], we assume in this paper that synergy is the relevant social interaction mechanism.

[18] An equivalent approach to introduce the social sub-utilities in the model would be to assume that the marginal utility of the male student's i effort increases with the average effort of his male or female peers ("I better like to go to a fast food restaurant with a friend").

weight level (Nash equilibrium):

$$\mathbf{y} = \alpha\boldsymbol{\iota} + \beta_{mm}\mathbb{G}_1\mathbf{y} + \beta_{mf}\mathbb{G}_2\mathbf{y} + \beta_{ff}\mathbb{G}_3\mathbf{y} + \beta_{fm}\mathbb{G}_4\mathbf{y} + \alpha_2\mathbf{x} + \boldsymbol{\epsilon}, \tag{3}$$

where the $\mathbb{G}_z$'s are $n \times n$ matrices. More precisely, $\mathbb{G}_1$ is the interaction matrix such that $\boldsymbol{g}_{1ij} = 1/(n_i^m + n_i^f)$ if i and j are male friends, and $= 0$ otherwise. Similarly, $\mathbb{G}_3$ is the interaction matrix such that $\boldsymbol{g}_{3ij} = 1/(n_i^m + n_i^f)$ if i and j are female friends, and $= 0$ otherwise. The same reasoning applies for $\mathbb{G}_2$ and $\mathbb{G}_4$, where $\boldsymbol{g}_{2ij} = 1/(n_i^m + n_i^f)$ if i and j are friends but where j is a female student while i is a male student, and $\boldsymbol{g}_{4ij} = 1/(n_i^m + n_i^f)$ if i and j are friends, where j is male student and i is a female student. It is clear that $\mathbb{G}_1 + \mathbb{G}_2 + \mathbb{G}_3 + \mathbb{G}_4 = \mathbb{G}$ where $\mathbb{G}$ is the row-normalized social interaction matrix for the whole population.

3.1 Identifiability

To evaluate the impact of an exogenous shock (*e.g.*, a new course providing information to improve health habits which influences the parameter α_0 of the BMI production function), on students' BMI, one must recover the fundamentals of our structural model from the knowledge of the coefficients of the best response functions (3). Unfortunately, a first result is negative: in the general case, if one does not impose more structure to the model, the fundamentals of the model are not all identified. The demonstration is simple: while the latter include seven coefficients (the four ψ's, and the three α's), equation (3) can identify only six coefficients (the four $\mu\psi$, $\alpha_0 + \mu$, and α_2). The basic problem is that $\mu = \alpha_1^2$ is not identifiable, as it is equal to the squared marginal productivity of effort on BMI, while effort is generally not observed. Of course, it is possible to recover the parameters of the preferences and the production function function, *for a given level of* μ. Indeed, each of the four social sub-utility parameters (the ψ's) are proportional to its corresponding β, the proportionality coefficient being μ^{-1}. Note also that if we have a good proxy for effort (*e.g.*, a measure of eating habits, physical exercise, *etc.*), it may help identify the fundamentals of the model.[19]

Our model provides a necessary condition for gender homogeneity in the peer effects. Homogeneity implies that all ψ's are equal ($= \psi$). In that case, one has: $\beta_{mm} = \beta_{mf} = \beta_{ff} = \beta_{fm} = \beta$. Therefore, in the absence of contextual effects, the model can be written as:

$$\mathbf{y} = \alpha\boldsymbol{\iota} + \beta\mathbb{G}\mathbf{y} + \gamma\mathbf{x} + \boldsymbol{\epsilon}. \tag{4}$$

[19] Fortin and Yazbeck [2015] used the number of weekly visits to a fast food restaurant by students to approximate eating habits.

When the model is homogeneous, the social multiplier corresponds to the impact of α_0 on the students' BMI, when the peer effects are taken into account. It is the same for all individuals and is equal to $1/(1-\beta)$ $[= 1/(1-\mu\psi)]$ in the absence of isolated individuals. Therefore, as long as the parameters of the best responses functions are identified, the social multiplier can be computed even if the social preference parameter ψ is not identifiable. Note however that the evaluation of the effect of a change in α_1 is not identifiable, at least as long as one does not have a good proxy for effort.

4 Econometric model

In this section, we provide an econometric version of the best-response functions. We now assume R networks, with $r = 1, \ldots, R$. We still suppose that individuals of each gender interact both with individuals of the same gender and with individuals of the other gender. $n_{i,r}^m$ and $n_{i,r}^f$ stand respectively for the number of male and female individuals influencing i in the network r. We now allow for isolated students, for which one has: $n_{i,r}^m = 0$ and $n_{i,r}^f = 0$. Also, there are n_r^m male individuals and n_r^f female individuals, where $n_r^m + n_r^f = n_r$. We introduce heterogeneous contextual effects that account for within- and between-gender peers characteristics in each network r. The best-response functions for the network r can be written as:

$$\begin{aligned} \mathbf{y_r} \;=\;& \boldsymbol{\iota}_{n_r}\alpha_r + \beta_{mm}\mathbb{G}_{1,r}\mathbf{y}_r + \beta_{mf}\mathbb{G}_{2,r}\mathbf{y}_r + \beta_{ff}\mathbb{G}_{3,r}\mathbf{y}_r + \beta_{fm}\mathbb{G}_{4,r}\mathbf{y}_r \\ &+ \gamma\,\mathbf{x}_r + \delta_{mm}\mathbb{G}_{1,r}\,\mathbf{x}_r + \delta_{mf}\mathbb{G}_{2,r}\,\mathbf{x}_r + \delta_{ff}\mathbb{G}_{3,r}\mathbf{x}_r + \delta_{fm}\mathbb{G}_{4,r}\,\mathbf{x}_r + \boldsymbol{\epsilon}_r \end{aligned} \quad (5)$$

with $r = 1, \ldots, R$, $\boldsymbol{\iota}_{n_r}$ is a $n_r \times 1$ vector of ones, and where α_r stands for a fixed effect specific to network r. Note that the $\mathbb{G}_{z,r}$'s, for $z = 1, \cdots, 4$, matrices are not row-normalized in the presence of isolated students. For sake of simplicity, we order vectors and matrices so that the first n_r^f rows correspond to type-f individuals of network r, and the remaining n_r^m rows are for type-m individuals in network r. Matrix ordering simplifies the identification conditions of our model.[20] In addition, for a sample with R networks, we stack up the data by defining $\mathbf{y} = (\mathbf{y}_1^{'}, ..., \mathbf{y}_R^{'})'$, $\mathbf{x} = (\mathbf{x}_1^{'}, ..., \mathbf{x}_R^{'})'$, $\boldsymbol{\epsilon} = (\boldsymbol{\epsilon}_1^{'}, ..., \boldsymbol{\epsilon}_R^{'})'$, $\bar{\mathbb{G}}_1 = D(\mathbb{G}_{1,1}, ..., \mathbb{G}_{1,R})$, $\bar{\mathbb{G}}_2 = D(\mathbb{G}_{2,1}, ..., \mathbb{G}_{2,R})$, $\bar{\mathbb{G}}_3 = D(\mathbb{G}_{3,1}, ..., \mathbb{G}_{3,R})$, $\bar{\mathbb{G}}_4 = D(\mathbb{G}_{4,1}, ..., \mathbb{G}_{4,R})$, $\boldsymbol{\iota} = D(\boldsymbol{\iota}_{n_1}, ..., \boldsymbol{\iota}_{n_R})$ and $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_R)'$ where $D(\mathbf{C}_1, ..., \mathbf{C}_R)$ is a block diagonal matrix. Finally, let $\bar{\mathbb{G}}(\boldsymbol{\beta}) = \beta_{mm}\bar{\mathbb{G}}_1 + \beta_{mf}\bar{\mathbb{G}}_2 + \beta_{ff}\bar{\mathbb{G}}_3 + \beta_{fm}\bar{\mathbb{G}}_4$ and $\bar{\mathbb{G}}(\boldsymbol{\delta}) = \delta_{mm}\bar{\mathbb{G}}_1 + \delta_{mf}\bar{\mathbb{G}}_2 + \delta_{ff}\bar{\mathbb{G}}_3 + \delta_{fm}\bar{\mathbb{G}}_4$ where $\boldsymbol{\beta} = (\beta_{mm}, \beta_{mf}, \beta_{ff}, \beta_{fm})'$ and $\boldsymbol{\delta} = (\delta_{mm}, \delta_{mf}, \delta_{ff}, \delta_{fm})'$. The best-response functions for

[20] Our vector and matrix ordering leads (by construction) to the following identities: $\mathbb{G}_{1,r}.\mathbb{G}_{4,r} = 0_{n_r}$, $\mathbb{G}_{3,r}.\mathbb{G}_{2,r} = 0_{n_r}$, $\mathbb{G}_{1,r}.\mathbb{G}_{3,r} = 0_{n_r}$, $\mathbb{G}_{3,r}.\mathbb{G}_{1,r} = 0_{n_r}$, $\mathbb{G}_{2,r}^{k\geq 2} = 0_{n_r}$, $\mathbb{G}_{4,r}^{k\geq 2} = 0_{n_r}$, $\mathbb{G}_{4,r}.\mathbb{G}_{3,r} = 0_{n_r}$ and $\mathbb{G}_{2,r}.\mathbb{G}_{1,r} = 0_{n_r}$.

the R networks are:

$$\mathbf{y} = \bar{\mathbb{G}}(\boldsymbol{\beta})\mathbf{y} + \gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota}\boldsymbol{\alpha} + \boldsymbol{\epsilon} \tag{6}$$

In this paper, we allow for network fixed effects. The latter take into account the unknown specificities which commonly influence the BMI of all students within a school. In this case, the fixed effects parameters α_r are specific to network r. Consequently, one has : $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_R)'$.

4.1 Identification

The aim of this section is to analyze conditions for the best-response functions of the model to be identified. The identification of these functions are necessary (but not sufficient)[21] to recover the fundamentals of our model. Here, identification means that a consistent estimator of these functions exist.

Let us first write the best-response functions (6) in their reduced form. This requires the matrix $\mathbf{S}(\boldsymbol{\beta}) = (\mathbf{I} - \bar{\mathbb{G}}(\boldsymbol{\beta}))$, where $\mathbf{I}$ is the identity matrix, to be invertible. Proposition (1) below provides sufficient conditions of invertibility of matrix $\mathbf{S}(\boldsymbol{\beta})$.

Proposition 1 *Suppose equation (6) holds. Suppose also that* $|\beta_{mm}| < 1$, $|\beta_{mf}| < 1$, $|\beta_{ff}| < 1$ *and* $|\beta_{fm}| < 1$. *Then matrix* $\mathbf{S}(\boldsymbol{\beta}) = (\mathbf{I} - \bar{\mathbb{G}}(\boldsymbol{\beta}))$ *is invertible.*[22]

The reduced form model, assuming conditions of proposition (1) are satisfied, is given by:

$$\mathbf{y} = \mathbf{S}(\boldsymbol{\beta})^{-1}\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota}\boldsymbol{\alpha}\right] + \mathbf{S}(\boldsymbol{\beta})^{-1}\boldsymbol{\epsilon}. \tag{7}$$

Using the reduced form model (7), $\bar{\mathbb{G}}_i\mathbf{y}$, $\forall\, \bar{\mathbb{G}}_i \in \{\bar{\mathbb{G}}_1, \bar{\mathbb{G}}_2, \bar{\mathbb{G}}_3, \bar{\mathbb{G}}_4\}$, can be expressed as :

$$\bar{\mathbb{G}}_i\mathbf{y} = \mathbf{W}_i(\boldsymbol{\beta})\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota}\boldsymbol{\alpha}\right] + \mathbf{W}_i(\boldsymbol{\beta})\boldsymbol{\epsilon}$$

where $\mathbf{W}_i(\boldsymbol{\beta}) = \bar{\mathbb{G}}_i\mathbf{S}(\boldsymbol{\beta})^{-1}$. It follows that $\forall\, i \in \{1, 2, 3, 4\}$, $\bar{\mathbb{G}}_i\mathbf{y}$ is correlated with $\boldsymbol{\epsilon}$ because $\mathbb{E}\left[(\mathbf{W}_i(\boldsymbol{\beta})\boldsymbol{\epsilon})'\boldsymbol{\epsilon}\right] \neq 0$. Thus, model (6) cannot be consistently estimated by OLS. On the other hand, 2SLS and GMM strategies can be used to estimate our model. We first consider a 2SLS approach and show that we can find instruments to obtain consistent estimates of our best response functions. Then we propose a GMM estimator of our heterogeneous model that generalizes our 2SLS estimator using additional quadratic moment equations. The latter provides asymptotically more efficient estimator than the 2SLS approach.

[21] The reason is that effort to reduce weight is generally unobservable.

[22] See proof in appendix A.

4.1.1 2SLS estimation

Following the same strategy as Liu et al. [2013], we re-write the best-response functions using our vector of parameters defined in $\boldsymbol{\theta} = (\boldsymbol{\beta}, \gamma, \boldsymbol{\delta})'$ and $\mathbf{Z} = \left[\bar{\mathbb{G}}_1\mathbf{y}, \bar{\mathbb{G}}_2\mathbf{y}, \bar{\mathbb{G}}_3\mathbf{y}, \bar{\mathbb{G}}_4\mathbf{y}, \mathbf{X}\right]$ where $\mathbf{X} = \left[\mathbf{x}, \bar{\mathbb{G}}_1\mathbf{x}, \bar{\mathbb{G}}_2\mathbf{x}, \bar{\mathbb{G}}_3\mathbf{x}, \bar{\mathbb{G}}_4\mathbf{x}\right]$. The resulting model is given by equation (8) below.

$$\mathbf{y} = \mathbf{Z}\boldsymbol{\theta} + \boldsymbol{\iota}\boldsymbol{\alpha} + \boldsymbol{\epsilon} \tag{8}$$

This simplified writing of our model allows us to derive our identification conditions in the case of 2SLS. A particularity of our model is however that it contains network fixed effects (included in $\boldsymbol{\alpha}$) that need to be accounted for in our estimation. Standard (homogeneous) linear-in-means social interaction models with network fixed effects usually perform a global or local transformation of the model in order to eliminate fixed effects and to avoid the *incidental parameters problem* to occur. The incidental parameters problem, as it was first defined by Neyman and Scott [1948], occurs whenever the data available for each group or network are finite. Consequently, it is sometimes not possible to consistently estimate the structural and incidental parameters of the model, although in some cases structural parameters can be consistently estimated. In such a situation however, *i.e.*, even when consistency is reached, efficiency is sometimes affected.

In order to avoid the incidental parameters problem in our case, we perform a global transformation on equation (8). For that purpose, let $\mathbf{J} = D(\mathbf{J}_1, ..., \mathbf{J}_R)$ where $\mathbf{J}_r = (\mathbf{I}_r - \frac{\boldsymbol{\iota}_r\boldsymbol{\iota}_r'}{n_r})\ \forall\ r \in \{1, ..., R\}$. $\mathbf{J}$ is a global transformation matrix such that $\mathbf{J}\boldsymbol{\iota}\boldsymbol{\alpha} = \mathbf{0}$. Our resulting (transformed) model is:

$$\mathbf{J}\mathbf{y} = \mathbf{J}\mathbf{Z}\boldsymbol{\theta} + \mathbf{J}\boldsymbol{\epsilon} \tag{9}$$

Following Liu and Lee [2010] strategy, the best IV matrix for $\mathbf{JZ}$ is given by :

$$\mathbf{J}\mathbb{E}(\mathbf{Z}) = \mathbf{J}\left[\{\mathbf{W_i}(\boldsymbol{\beta})\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota}\boldsymbol{\alpha}\right]\}_{\{i=1,2,3,4\}}, \mathbf{X}\right]$$

and $\mathbf{JZ} = \mathbf{J}\mathbb{E}(\mathbf{Z}) + \mathbf{J}\sum_{i=1}^{4}[\mathbf{W}_i\boldsymbol{\epsilon}]\mathbf{e}_i'$ where $\mathbf{e}_i$ is the i'th unit (column) vector of dimension $(k+4)$ with $k = dim(\mathbf{X})$. Letting $\mathbf{Q}_{i,\infty}^0 = \left[\mathbf{W}_i(\boldsymbol{\beta})\mathbf{x}, \mathbf{W}_i(\boldsymbol{\beta})\bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x}, \mathbf{W}_i(\boldsymbol{\beta})\boldsymbol{\iota}\right]$, $\forall\ i \in \{1, 2, 3, 4\}$, the associated set of instrumental variables is $\mathbf{Q}_\infty = \mathbf{J}\left[\{\mathbf{Q}_{i,\infty}\}_{i=1,2,3,4}, \mathbf{X}\right]$. It is important to note here the presence of variables characterized by the multiplication of our interaction matrices and the matrix $\boldsymbol{\iota}$, that account for the fact that all rows of our matrices do not sum to one (due to the presence of isolated people). This, as stated in Liu et al. [2013] refer to the Bonacich centrality measure that is shown, if included in the set of instruments, to increase the efficiency of our estimates.[23]

[23] Our empirical application accounts for the inclusion of the Bonacich centrality measure.

If conditions of proposition (1) are satisfied, one can use a series expansion of $\mathbf{S}(\boldsymbol{\beta})^{-1} = \sum_{k=0}^{\infty}[\bar{\mathbb{G}}(\boldsymbol{\beta})]^k$.[24] Using this expression, we can re-write, $\forall\ i \in \{1,2,3,4\}$:

$$\mathbf{Q}_{i,\infty} = [\mathbf{Q}^0_{i,\infty}\mathbf{x}, \mathbf{Q}^0_{i,\infty}\boldsymbol{\iota}]^{25}$$

Using a subset of $\mathbf{Q}_{\infty}$ including $\mathbf{X}$, we show $\mathbf{Q}_K = \mathbf{J}\left[\mathbf{Q}^1_K, \mathbf{Q}^2_K, \mathbf{Q}^3_K, \mathbf{Q}^4_K, \mathbf{X}\right]$ can be used as instruments, where $\mathbf{Q}^i_K$ is a subset of $\mathbf{Q}_{i,\infty}$, $\forall\ i \in \{1,2,3,4\}$ where K is the number of instruments.[26] In addition, let $\boldsymbol{\epsilon}(\boldsymbol{\theta}) = \mathbf{J}\left(\mathbf{y} - \mathbf{Z}\boldsymbol{\theta} - \boldsymbol{\iota}\boldsymbol{\alpha}\right)$. The moment conditions corresponding to the orthogonality between $\mathbf{Q}_K$ and $\mathbf{J}\boldsymbol{\epsilon}$ is $\mathbf{Q'}_K\boldsymbol{\epsilon}(\boldsymbol{\theta})$.

Proposition 2 *Suppose model (6) holds with correlated effects. Suppose also that* $(\delta_{mm} + \gamma\beta_{mm}) \neq 0$, $(\delta_{ff} + \gamma\beta_{ff}) \neq 0$, $(\delta_{mf} + \gamma\beta_{mf}) \neq 0$ *and* $(\delta_{fm} + \gamma\beta_{fm}) \neq 0$. *If vector columns of matrix* $\mathbf{Q}_K$ *are linearly independent, then social effects are identified.*[27]

Proposition 2 give conditions extending those proposed in Bramoullé et al. [2009] to the case of two-type (male-female) peer effects heterogeneity. In particular, we can note that there are some similarities in the restriction on our set of parameters, except that in our case, the restrictions are generalized to all categories of individuals and their associated parameters. In addition, the condition on linear dependencies of vector columns of matrix $\mathbf{Q}_K$ can be compared to the conditions on linear independency of the interaction matrices stated in Bramoullé et al. [2009]. In particular, the instruments that are used here are the characteristics of male friends of male friends of students, their female counterparts, the characteristics of female friends of friends of males who are females, *etc.* In summary, characteristics of friends at distance 2,3,4, *etc.* per categories may be used as instruments to properly estimate the model. The 2SLS estimator of model (6) is given by:

$$\hat{\boldsymbol{\theta}}_{2sls} = (\mathbf{Z}'\mathbf{P}_K\mathbf{Z})^{-1}\mathbf{Z}'\mathbf{P}_K\mathbf{y}$$

where $\mathbf{P}_K = \mathbf{Q}_K(\mathbf{Q'}_K\mathbf{Q}_K)^{-}\mathbf{Q'}_K$. The corresponding variance-covariance matrix of parameter estimates in this 2SLS setting is given by:

$$\hat{V}_{\hat{\boldsymbol{\theta}}_2sls} = (\mathbf{Z}'\mathbf{P}_K\mathbf{Z})^{-1}\mathbf{Z}^{'}\mathbf{D}\mathbf{Z}(\mathbf{Z}\mathbf{P}_K\mathbf{Z}')^{-1}$$

where $\mathbf{D}$ is an $n \times n$ diagonal matrix with entries given by the squared residuals from the estimation. Under plausible regularity conditions (see Liu and Lee [2010]), the 2SLS approach provides a consistent estimator of our model (6).

[24] Using the Newton's binomial formula and identities derived from our matrix ordering, one can re-write $\mathbf{S}(\boldsymbol{\beta})^{-1} = \mathbf{I} + \sum_{k=1}^{\infty}\sum_{i=0}^{k\geq 1}\binom{k}{i}\left[(\beta_{mm}\bar{\mathbb{G}}_1)^{k-i} + (k-i)\beta_{mf}(\beta_{mm}\bar{\mathbb{G}}_1)^{k-i-1}\bar{\mathbb{G}}_2\right].\left[(\beta_{ff}\bar{\mathbb{G}}_3)^i + i\beta_{fm}(\beta_{ff}\bar{\mathbb{G}}_3)^{i-1}\bar{\mathbb{G}}_4\right]$.

[25] Where $\mathbf{Q}^0_{i,\infty} = \left[\bar{\mathbb{G}}_i\bar{\mathbb{G}}_1, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_2, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_3, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_4, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_1^2, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4, \bar{\mathbb{G}}_i\bar{\mathbb{G}}_3^2, ...\right]$. See Appendix B for examples of $\mathbf{Q}_{i,\infty}$.

[26] A simple example for $\mathbf{Q}_K$ is $\mathbf{J}\left[\bar{\mathbb{G}}_1^2\mathbf{x}, \bar{\mathbb{G}}_1\boldsymbol{\iota}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\mathbf{x}, \bar{\mathbb{G}}_2\boldsymbol{\iota}, \bar{\mathbb{G}}_3^2\mathbf{x}, \bar{\mathbb{G}}_3\boldsymbol{\iota}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\mathbf{x}, \bar{\mathbb{G}}_4\boldsymbol{\iota}, \mathbf{X}\right]$.

[27] See proof in Appendix C.

4.1.2 GMM estimation

As shown in Liu and Lee [2010], the homogeneous version of the best-response functions (6) can be estimated using a GMM estimator. Following the same reasoning, we propose a GMM estimator of our heterogeneous model. For that purpose, we generalize the 2SLS estimator using additional quadratic moment equations. As argued in Liu and Lee [2010], the additional quadratic moments exploit the existing correlations between the error term of the reduced for model, thus provide more precision compared to the traditional 2SLS estimators. In addition, one of the advantages of using the GMM estimator instead of the 2SLS is that the objective function of the GMM estimator uses the optimal weighting matrix that allows the obtention of more efficient estimators.

In order to derive our GMM estimator in the context of heterogeneous peer effects, we first let the IV moments be given by $g_1(\boldsymbol{\theta}) = \mathbf{Q}'_K \boldsymbol{\epsilon}(\boldsymbol{\theta})$. The additional quadratic moments are given by $g_2(\boldsymbol{\theta}) = \left[\mathbf{U}'_1\boldsymbol{\epsilon}(\boldsymbol{\theta}), \mathbf{U}'_2\boldsymbol{\epsilon}(\boldsymbol{\theta}), ..., \mathbf{U}'_q\boldsymbol{\epsilon}(\boldsymbol{\theta})\right]' \boldsymbol{\epsilon}(\boldsymbol{\theta})$, where $\mathbf{U}_j$ is such that $tr(\mathbf{J}\mathbf{U}_j) = 0$.[28] For notational purpose, we also let $\mathbf{U}_j = \mathbf{J}\mathbf{U}_j\mathbf{J}$. In addition, let the combined vector of linear and quadratic empirical moments be given in $g(\boldsymbol{\theta}) = [g'_1(\boldsymbol{\theta}), g'_2(\boldsymbol{\theta})]$. Finally, let $\widetilde{\Omega} = \widetilde{\Omega}(\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$ where $\tilde{\sigma}^2$, $\tilde{\mu}_3$ and $\tilde{\mu}_4$ are initial estimators of the second, third and fourth moments of our the error term of our model. Following the strategy of Liu et al. [2013], extented to the case of heterogeneous peer effects, the optimal weighting matrix associated with our GMM estimation strategy is given by Ω taking the following form:

$$\Omega = Var\left[g(\boldsymbol{\theta})\right] = \begin{bmatrix} \tilde{\sigma}^2 \mathbf{Q}'_K \mathbf{Q}_K & \mu_3 \mathbf{Q}'_K \omega \\ \mu_3 \omega' \mathbf{Q}_K & (\mu_4 - 3\sigma^4)\omega'\omega + \sigma^4 \Upsilon \end{bmatrix}$$

where $\omega = [vec_D(\mathbf{U}_1), vec_D(\mathbf{U}_2), ..., vec_D(\mathbf{U}_q)]$, $\Upsilon = \frac{1}{2}\left[vec(\mathbf{U}^s_1), vec_D(\mathbf{U}^s_2), ..., vec_D(\mathbf{U}^s_q)\right]$ where $\forall$ square matrix $\mathbf{E}$ of size n , $\mathbf{E}^s = \mathbf{E} + \mathbf{E}'$ and $vec_D(\mathbf{A}) = (a_{11}, a_{22}, ..., a_{nn})$.

The feasible optimal GMM estimator is given by:

$$\hat{\boldsymbol{\theta}}_{gmm} = argmin\ _{\boldsymbol{\theta} \in \Theta} g'(\boldsymbol{\theta}) \widetilde{\Omega}^{-1} g(\boldsymbol{\theta})$$

Proposition 4 of Liu et al. [2013] state that under their assumptions $1-3$, $4'$, $5-9$, if $K/n -> 0$, and if $\widetilde{\Omega} = \widetilde{\Omega}(\tilde{\sigma}^2, \tilde{\mu}_3, \tilde{\mu}_4)$, our GMM estimator $\hat{\boldsymbol{\theta}}_{gmm}$ is consistent.

[28] Following Liu and Lee [2010], for any constant matrix $\mathbf{B}$, if we define $\mathbf{A} = \mathbf{B} - tr(\mathbf{J}\mathbf{B})\mathbf{I}/tr(\mathbf{J})$, then $tr(\mathbf{J}\mathbf{A}) = 0$. In our setting, we use $\mathbf{U}_1 = \bar{\mathbf{G}}_1 - tr(\mathbf{J}\mathbf{G}_1)\mathbf{I}/tr(\mathbf{J})$, $\mathbf{U}_2 = \bar{\mathbf{G}}_2 - tr(\mathbf{J}\mathbf{G}_1)\mathbf{I}/tr(\mathbf{J})$, $\mathbf{U}_3 = \bar{\mathbf{G}}_3 - tr(\mathbf{J}\mathbf{G}_1)\mathbf{I}/tr(\mathbf{J})$ and $\mathbf{U}_4 = \bar{\mathbf{G}}_4 - tr(\mathbf{J}\mathbf{G}_1)\mathbf{I}/tr(\mathbf{J})$.

4.2 Particular cases: homogeneous and gender-specific homogeneous effects

Two particular cases of our model may be recovered by restricting some parameter values: the baseline model of homogeneous peer effects and an intermediary case where effects are gender-specific homogeneous.

In the homogeneous case, we suppose that $\beta_{mm} = \beta_{mf} = \beta_{ff} = \beta_{fm} = \beta$. Similarly, $\delta_{mm} = \delta_{mf} = \delta_{ff} = \delta_{fm} = \delta$. The baseline model is $\mathbf{y} = \boldsymbol{\iota\alpha} + \beta\bar{\mathbb{G}}\mathbf{y} + \gamma\mathbf{x} + \delta\bar{\mathbb{G}}\mathbf{x} + \boldsymbol{\epsilon}$. This is similar to the standard model of Bramoullé et al. [2009] and our identification conditions are the same as in that paper.

In the gender-specific homogeneous case, we suppose that $\beta_{mm} = \beta_{mf} = \beta_m$ and $\beta_{ff} = \beta_{fm} = \beta_f$. Similarly, $\delta_{mm} = \delta_{mf} = \delta_m$ and $\delta_{ff} = \delta_{fm} = \delta_f$. The corresponding model is:

$$\mathbf{y} = \boldsymbol{\iota\alpha} \;+\; \beta_m\widetilde{\mathbb{G}}_1\mathbf{y} \;+\; \beta_f\widetilde{\mathbb{G}}_2\mathbf{y} \;+\; \gamma\,\mathbf{x} \;+\; \delta_m\widetilde{\mathbb{G}}_1\mathbf{x} \;+\; \delta_f\widetilde{\mathbb{G}}_2\mathbf{x} \;+\; \boldsymbol{\epsilon} \tag{10}$$

Using matrix $\widetilde{\mathbf{Z}} = [\widetilde{\mathbb{G}}_1\mathbf{y}, \widetilde{\mathbb{G}}_2\mathbf{y}, \widetilde{\mathbf{X}}]$ where $\widetilde{\mathbf{X}} = [\mathbf{x}, \widetilde{\mathbb{G}}_1\mathbf{x}, \widetilde{\mathbb{G}}_2\mathbf{x}]$ and $\tilde{\boldsymbol{\theta}} = (\tilde{\boldsymbol{\beta}}, \gamma, \tilde{\boldsymbol{\delta}})'$ with $\tilde{\boldsymbol{\beta}} = (\beta_m, \beta_f)'$ and $\tilde{\boldsymbol{\delta}} = (\delta_m, \delta_f)'$, the model can also be written as:

$$\mathbf{y} = \widetilde{\mathbf{Z}}\tilde{\boldsymbol{\theta}} + \boldsymbol{\iota\alpha} + \boldsymbol{\epsilon}$$

where subscripts $\widetilde{}$ on matrices and vectors have the same meaning as in the general model, except that there is no distinction between the type of friends for males (resp. for females).

Proposition 3 *If $|\beta_m| < 1$ and $|\beta_f| < 1$, matrix $\widetilde{\mathbf{S}} = (\mathbf{I} - \beta_m\widetilde{\mathbb{G}}_1 - \beta_f\widetilde{\mathbb{G}}_2)$ is invertible.*

If conditions of proposition 3 are satisfied, the reduced form model is:

$$\mathbf{y} = \widetilde{\mathbf{S}}^{-1}\left[\gamma\mathbf{x} + \widetilde{\mathbb{G}}(\tilde{\boldsymbol{\delta}}) + \boldsymbol{\iota\alpha}\right] + \widetilde{\mathbf{S}}^{-1}\boldsymbol{\epsilon} \tag{11}$$

where $\widetilde{\mathbb{G}}(\tilde{\boldsymbol{\delta}}) = \delta_m\widetilde{\mathbb{G}}_1\mathbf{x} + \delta_f\widetilde{\mathbb{G}}_2\mathbf{x}$.[29] We perform a global transformation of model and the transformed model is:

$$\mathbf{J}\mathbf{y} = \mathbf{J}\widetilde{\mathbf{Z}}\tilde{\boldsymbol{\theta}} + \mathbf{J}\boldsymbol{\epsilon}$$

The best IV matrix for $\mathbf{J}\widetilde{\mathbf{Z}}$ is given by $\mathbf{J}\mathbb{E}(\widetilde{\mathbf{Z}}) = \mathbf{J}\left[\{\widetilde{\mathbf{W}}_i(\boldsymbol{\beta})[\gamma\mathbf{x} + \widetilde{\mathbb{G}}(\tilde{\boldsymbol{\delta}})\mathbf{x} + \boldsymbol{\iota\alpha}]\}_{\{i=1,2\}}, \widetilde{\mathbf{X}}\right]$ and $\mathbf{J}\widetilde{\mathbf{Z}} = \mathbf{J}\mathbb{E}(\widetilde{\mathbf{Z}}) + \mathbf{J}\sum_{i=1}^{2}[\widetilde{\mathbf{W}}_i\boldsymbol{\epsilon}]\mathbf{e}_i'$ where $\mathbf{e}_i$ is the i'th unit (column) vector of dimension

[29] Matrix $\widetilde{\mathbf{S}}^{-1}$ can be re-written using a series expansion and the Newton binomial formula such that $\widetilde{\mathbf{S}}^{-1} = \mathbf{I} \;+\; \sum_{k=1}^{\infty}\sum_{i=0}^{k\geq 1}\binom{k}{i}\left(\beta_m\widetilde{\mathbb{G}}_1\right)^{(k-i)} \;\times\; \left(\beta_f\widetilde{\mathbb{G}}_2\right)^{i}$.

$(k+2)$ with $k = dim(\widetilde{\mathbf{X}})$. Letting $\widetilde{\mathbf{Q}}^0_{i,\infty} = \left[\widetilde{\mathbf{W}}_i(\boldsymbol{\beta})\mathbf{x}, \widetilde{\mathbf{W}}_i(\boldsymbol{\beta})\widetilde{\mathbb{G}}(\tilde{\boldsymbol{\delta}})\mathbf{x}, \widetilde{\mathbf{W}}_i(\tilde{\boldsymbol{\beta}})\boldsymbol{\iota}\right]$, $\forall\ i \in \{1, 2\}$. The set of instrumental variables of our model is $\widetilde{\mathbf{Q}}_\infty = \mathbf{J}\left[\{\widetilde{\mathbf{Q}}_{i,\infty}\}_{i=1,2}, \widetilde{\mathbf{X}}\right]$. Following the same method as in the general case, we end up with the following proposition.

Proposition 4 *Suppose model (10) holds with correlated effects. Suppose also that* $(\delta_m + \gamma\beta_m) \neq 0$ *and* $(\delta_f + \gamma\beta_f) \neq 0$. *If vector columns of matrix* $\widetilde{\mathbf{Q}}_K$ *are linearly independent, then social effects are identified.*

Using the same strategies, our model can be estimated using both GMM and 2SLS estimators.

5 Data

Our best-response model with heterogeneous peer effects is used to study the influences of peer outcomes and characteristics on the body weight of adolescents, using data from the National Longitudinal Study of Adolescent Health (Add Health). Add Health is a panel study of a nationally representative sample of adolescents in grades 7-12 in the United States. Mandated by the U.S. Congress to fund a study of adolescent health, the Carolina Population Center conducted the first wave during the 1994-1995 school year. The dataset comprises an In-School questionnaire that is administered to a nationally representative sample of students. People from the Add Health cohort are followed into young adulthood with four In-Home interviews: 1996, 2001-2002 and 2007-2008. The most recent In-Home interview was in 2008, when the sample was aged 24-32. Add Health combines data on respondents' social, economic, psychological and physical well-being with contextual data on family, neighbourhood, community, school, friendships, peer groups, and romantic relationships. The dataset thus provides tools to conduct studies designed to measure the effects of personal and contextual characteristics on behaviours that promote good health for instance, positioning the dataset at the top of the largest and most comprehensive longitudinal surveys of adolescents undertaken.

Wave I of Add Health consists of an In-school questionnaire that was filled out by 90,118 students in 145 schools and 80 communities. A subset of 20,745 students was then chosen for an in-depth In-Home survey. Wave II, which was held in 1996, includes an In-Home questionnaire that was completed by 14,738 students, a subset of the original 20,745 Wave I pupils. Students who were selected for the In-Home survey were asked for information on their height and weight. Using this information, we construct student body mass indices (BMI)[30] which is our dependent variable and an indicator of body fatness, according to the

[30]We do not use declared body mass indices although declared BMIs are shown to reflect real variables

formula: BMI = (weight in kilograms)/(height in meters)^2. Because Wave II of the Add Health dataset also comprises a nutrition section, we use variables from Wave II to further explore adolescents weight, as probably determined by their social ties. Covariates include age, racial background, grade variables, parents education and parents' health status. This leaves us with as many contextual peer effects coefficients as personal characteristics, for each type considered.

To account for social interactions, we also use information provided by Wave II of the Add Health dataset, in which respondents are asked to *name up to five male friends and up to five of their female friends within their school.* Provided information on their friendship links and on their type thus allows us to construct our friendship interaction matrices. The extensive questionnaire was also used to construct a saturation sample that focuses on 16 selected schools (about 3000 students). Every student attending these selected schools answered the detailed questionnaire. There are two large schools and 14 other small schools. We use the saturated sample in our estimations to deal with the problem of partial observability.

5.1 Descriptive statistics

Table (1) provides descriptive statistics of our sample. The sample comprises 2220 students in all 16 schools of the In-Home survey. Average BMI is 23.14 with a standard deviation of 4.72. This reveals that on average, the population considered is *normal* in terms of weight. In terms of individual characteristics, we can see that the male-female population is equally distributed, and that mean age is about 16. White students are more represented (61%) than the other racial communities. The percentage of Black and Asian students is respectively 15% and 14%. In addition, 18% of students in the sample are of Hispanic origin. 61% of students in our sample attend grade 11 or 12 and 27% are in grade 9 or 10. Most of the parents hold at least a high school degree and 18% of mothers hold a college degree compared to 15% of fathers of the students in our sample. Almost all parents work for pay, 92% of mothers report being in good health compared to 76% of fathers. Reported (directed) network statistics indicate that the average number of friends is 2 and is equally distributed between male friends and female friends. However, considering undirected networks increases the average number of friends to 5. This indicates that the constraint put in the number of friends by the Add Health study is not binding, and individuals actually report having less friends than the number of allowed nominations during the survey. Consequently, the partial observability of networks (see for example

in the case of Add-Health.

Chandrasekhar and Lewis [2011]) is not problematic in our study, even when undirected networks are considered. 509 students of our sample (about 23%) are isolated. We address the issue of network endogeneity in the following section.

5.2 Network endogeneity

In the presence of self selection into networks, identification may be hindered because endogenous effects cannot be separated from correlated effects, even when performing our global transformation that captures only part of this selection bias - i.e. the one that is due to the fact that individuals in the same network face a common environment. Network endogeneity may be the source of potentially important biases whenever there are unobservables at the individual level that determine network formation and that influence the outcome of interest at the same time. The presence of homophily where individuals with common characteristics tend to associate together is an example of such a situation.

The network endogeneity issue has been addressed by a number of recent papers. The main strategy consists in including a network formation model and using bayesian techniques to estimate the parameters of interest (see for example Patacchini and Rainone [2014] and Hsieh and Lee [2015]). A recent paper of Goldsmith-Pinkham and Imbens [2013] argues that the presence of endogeneity in network formation is testable. Following this argument, Liu et al. [2013] proposes a test for the presence of endogeneity in networks using the Add Health dataset and applies their approach to the allocation of time in sleep. They find no evidence of endogeneity of networks. Patacchini and Rainone [2014] also find no evidence of the presence of endogenous network formation, while focusing on peer effects on financial products.

We perform a series of tests based on the Goldsmith-Pinkham and Imbens [2013] idea and the best response functions in the homogeneous model. We argue that lack of evidence of network endogeneity in the homogeneous model suggests that network endogeneity is not an important concern in our heterogeneous model of peer interactions. We first follow the strategy of Liu et al. [2013] and, in a second approach, we consider a Goldsmith-Pinkham and Imbens [2013] "inspired" test in a more general fashion.

Liu et al. [2013] adopt the following strategy based on the Goldsmith-Pinkham and Imbens [2013] approach. The underlying idea is simple. Suppose that the best response functions in network r are given by :

$$\mathbf{y}_r = \boldsymbol{\iota}_{n_r}\boldsymbol{\alpha}_r + \beta\bar{\mathbb{G}}_r\mathbf{y}_r + \gamma\mathbf{x}_r + \delta\bar{\mathbb{G}}_r\mathbf{x}_r + \boldsymbol{\epsilon}_r$$

Suppose the error term is the sum of unobserved characteristics at the individual level $\mathbf{v}_r$ and random disturbances $\mathbf{e}_r$ such that $\boldsymbol{\epsilon}_r = \pi\mathbf{v}_r + \mathbf{e}_r$, where π is the effect of the unobserved individual characteristic on the outcome of interest, $\mathbf{y}_r$. Let consider a network formation model explaining the probability of observing a link between two individuals i and j. It is assumed that the link formation process depends on distances between observed and unobserved characteristics between any two individuals. For simplicity, we assume that there is only one unobserved variable that drives both network formation and the outcome variable. The network formation model is thus given by equation (12) below:

$$g_{ij,r} = \kappa + \sum_{k=1}^{K} \zeta_m |x_{i,r}^k - x_{j,r}^k| + \phi|v_{i,r} - v_{j,r}| + \kappa_r + u_{ij,r}. \tag{12}$$

Following this model, if there is homophily in the unobserved characteristics, then $\phi < 0$ i.e. the closer two individuals are in terms of unobservables, the higher the probability that they become friends. If, in addition, $\pi \neq 0$, these unobservables have a direct effect on $\mathbf{y}_r$ as well. Liu et al. [2013] argue that if the data reveal a positive and statistically significant correlation between the predicted probability (using probit or logit estimation) to observe a link between the two individuals ($\hat{g}_{ij,r}$) and the difference between the residuals of the two individuals in the outcome equation ($|\hat{\epsilon}_{i,r} - \hat{\epsilon}_{j,r}|$), when a link is really observed ($g_{ij,r} = 1$), then we should not reject the presence of endogeneity in network formation. In the same spirit, if a positive and statistically significant correlation is found between the predicted probability to observe a link and the difference of residuals in the outcome equation, if no link is observed in the reality ($g_{ij,r} = 0$), then the same conclusion holds. Following this idea, we first perform a naive regression of the predicted probability to observe a link ($\hat{g}_{ij,r}$) and differences in residuals for the entire sample ($|\hat{\epsilon}_i - \hat{\epsilon}_j|$). We also include the variable indicating whether there is a link or not, g_{ij} and we differentiate between cases where fixed effects are included and cases where there are no fixed effects. Our results are reported on table (2) and suggest that, in the absence of network fixed effects, there is a negative and significant effect of differences in residuals and the predicted probability to observe a link. However, whenever network fixed effects are accounted for, this significant effect vanishes.

As an alternative test, we propose to concentrate on the whole distribution of predicted probabilities. Our test is based on a visual observation strategy aiming at detecting the presence of endogeneity in network formation. The idea is that if the estimated kernel densities are visually similar for both $g_{ij,r} = 1$ and $g_{ij,r} = 0$, then there is no evidence of network endogeneity. Figure (1) summarizes the results of our non parametric estimation without fixed effects. We can see that the two kernel density estimates are not similar without the inclusion of school fixed effects. However, once we control for school fixed effects using a semi-parametric model (see figure 2 above), one can see that densities are visually similar. Accordingly, there is no significant effect of differences in residuals of the outcome and the predicted link values. This analysis suggests that there is no evidence of the presence of network endogeneity as related to students' BMI in our data. It also provide evidence that the fixed effect strategy is quite efficient in reducing the selection bias associated with the confounding variables influencing both the network formation and the students' weight.

6 Results

In this section, we discuss estimates of the best response functions on the weight of adolescents. We first present results from the homogeneous model of peer interactions. In the second subsection, we explore the heterogenous model.

6.1 Homogenous peer effects and BMI

Table (4) summarizes our results using a 2SLS estimator. The first two columns report the estimates and standard errors of individual characteristics, and columns 3 and 4 report the associated contextual peer effects. For robustness purposes, we also distinguish results while excluding a dummy for the gender variable, which corresponds to a full homogeneous model (see specification (1)), or including it and excluding race variables (see specification (2)). Focusing on (1), results indicate that the endogenous peer effect is not significant at 5%. On the other hand, some contextual peer variables influence an individual's BMI. In particular, having friends whose mother has a college or and advanced level of education strongly reduces a student's BMI.[31] This reveals the importance of the mothers' education and may indicate a transmission of information on good health habit from friends' mother to a student (learning effect). Regarding individual effects, being enrolled in grades 11 or 12 reveal positive and significant effects on a student BMI compared to students who are in grades 7 or 8 (the reference). As expected, the impact of being a female student is negative on BMI in specification (2) which introduces a dummy for gender. Other estimates obtained specifications (1) and (2) are very similar.

Table (5) provides results based on GMM rather than on 2SLS. We can see that estimates are now much more precise (as argued by Liu et al. [2013]), as the GMM approach exploits additional (quadratic) moments conditions and the optimal weighting matrix.[32] The endogenous peer effect is now statistically significant at the 5% level and is equal to 0.205. This may reveal the presence of strategic complementarity between one's BMI and the BMI of their other friends. This means that peers' BMI positively affect own BMI,

[31] The mothers' reference group is *don't know*.

[32] See section 4.1.2 for more details.

through channels documented in section such as a higher incentive to go to a fast food restaurant with a friend (3). Under a strategic complementarity mechanism, this leads to a social multiplier equal to 1.20 ($= \frac{1}{1-0.205} \times 0.77 + 1 \times 0.23$).[33] Our results are in accordance with the recent literature that reports evidence of a positive but small endogenous peer effects on weight. For instance, Fortin and Yazbeck [2015], using a different econometric approach based on Add Heath data but limiting the effort to reduce BMI on limiting visits in fast food restaurants, estimates the social multiplier to 1.15. The second specification (see (2)) also reveals similar effects, with an endogenous effect equal to 0.203 and a similar social multiplier as in specification (1). In addition, as in 2SLS specification (2), female students tend to have a lower BMI compared to male students.

As regards contextual effects, some additional estimates are now statistically significant compared to estimates obtained using the 2SLS estimation strategy. In particular, in specification (1), peers' average age has a negative and significant impact on a student's BMI. Also having friends whose father has some college education now strongly and negatively affects a student's BMI. The effect of mother education is also amplified, with the effect of having a friend's mother holding a college degree becoming negative and statistically significant at 1% on BMI. Individual characteristics effects also become much more precise than estimates obtained previously, and grade 9 or 10 students also have a higher BMI than grades 7 or 8 students. Race also seems to have an important role, as white and black students have a lower BMI relative to their Hispanic, Asian or American Indian friends, though their estimated coefficients are significant at only 10%.

6.2 Gender heterogeneity and BMI

In this subsection, we generalize our econometric model to allow for within- and between-gender heterogeneity. We also provide Wald statistics to test the standard homogenous model as compared with our more general heterogeneity model.

Table (6) provides the results from 2SLS estimation. Column 1 provides estimates of the individual effects. Columns 3, 5, 7, and 9 report coefficients associated with the effects of male peers characteristics on the BMI of male students (M-M), the effects of female peers characteristics on the BMI of male students (M-F), the effects of female peers characteristics on the BMI of female students (F-F) and the effects of male peers characteristics on the BMI of female students (F-M). The lower panel provides the four corresponding endogenous peer effects. Standard errors of the estimates are reported in the adjacent columns.

[33] Recall that 23% of our sample are isolated students. For them, the social multiplier is 1.

As in the case of the homogenous model, the 2SLS endogenous peer estimates are not significant at the 5% level. As regards contextual effects, having male peers whose mother holds a college or an advanced degree has a negative impact on a male student's BMI. Again, this confirms the importance of the mothers' education and may reflect the transmission of information on the benefits of good health habits. However, this effect is significant only in the case of male-male ties. Furthermore, having male peers whose father holds some college degree negatively affect male students' BMI. Besides, an increase the percentage of their white or black female friends positively influences females' BMI. Estimates of individual characteristics are very similar to those of the homogeneous model. In particular, grade 11-12 students are the ones who report a higher BMI, even when controlling for age. The other individual effects coefficients are not significant.

Table (7) reports the estimated coefficients based on the GMM approach. As it is the case with the homogenous model, the use of this method makes our estimates more precise. Importantly, our results reported on the last panel of table (7) suggest that both within- and between-gender endogenous interactions influence own BMI. Thus within male-male and female-female endogenous effects exhibit positive and significant coefficients. The female-female endogenous effect (= 0.216) is a little higher than the male-male one (= 0.202). A female student who interacts with female students with high (low) BMI has more chance to have a high (low) BMI, as it is the case for a male student interacting with male students. Moreover, between female-male interactions (= 0.216) and male-female (= 0.287) interactions appear to influence a student's BMI. However, performing a Wald test (see Table 8) leads us not to reject that all the endogenous peer effects are the same (statistics =0.55 as compared with a critical χ^2 value of 7.89, at the 5% level). One thus concludes that, at least as far as the endogenous peer effects are concerned, gender homogeneity is a plausible hypothesis.

The next important issue is to check whether gender homogeneity also characterizes the contextual peer effects. Our results reveal an important number of differences in the contextual effects depending on the nature (within and between) of social interactions. First, although the age of female peers has a negative impact on both the BMI of male and female students, the age of male peers has no impact on students' BMI, whatever their gender. In addition, contextual effects are heterogeneous in the percentage of white students with whom a student interacts. Our results suggest that this effect appears positively for female students who interact with white female students but negatively for male students who interact with white male students. Importantly, having male peers whose mother hold less than a high school degree, a high school, a college or an advanced degree negatively affects male BMI. Also, having male peers whose mother hold an advanced education negatively influence female BMI. Besides, peers' mother education has no effect on students' BMI at the 5% level, irrespective of their gender. In addition, a female student's BMI is positively affected by male peers whose father holds less than a high school. Also, a male student's BMI is negatively influenced by male peers whose father holds a high school or some college level. Moreover, having male peers whose mother holds an advanced degree increases a female student's BMI. Finally, female peers whose father is in good health negatively influence a female student's BMI. Based on a Wald test, (see Table 8), we reject joint homogeneity in contextual and endogenous effects (statistics =114.89 as compared with a critical χ^2 value of 72.15 at the 5% level). In other words, while we do not reject homogeneity of endogenous peer effects, homogeneity of contextual effects is rejected.

As regards the individual effects, our results report estimates quite close to those of the GMM homogenous model (see Table 5). They indicate that being enrolled in grade 11-12 or grade 9-10 has a positive and significant effect on a student's BMI. Also, students whose father has no high school education or does not have a good health have a higher BMI.

We also perform a robustness analysis of our results when using the z-BMI instead of absolute BMI. The 2SLS and GMM estimation strategies reveals similar patterns. The Wald tests statistics for the z-BMI results are provided in Table (8 and indicate that, as it is the case for BMI, while full (endogenous plus contextual) gender homogeneity is rejected, but not endogenous gender homogeneity.

7 Conclusion

This paper proposes a non-cooperative model of the Body Mass Index (BMI) outcome with effort technology in a network context. We allow for intra- and inter-gender heterogeneity in endogenous and contextual peer effects. We analyze the possibility of recovering the fondamentals of our structural model (individual preferences and production function of effort on BMI). We show that as long that effort is not observable, the latter is only partly identified. However, having good proxies for effort (*e.g.*, good eating habits, physical exercise) helps completely identify the fondamentals of the model. Also, we show that identification conditions of the best response (reaction) functions depend on the value of some coefficients and on the properties of the social interactions matrices defined within the model. Interestingly, particular cases of our model, including the traditional network interaction model, can be shown as particular cases of our model. We first test the exogeneity of network formation as regards its effect on BMI. We do not reject exogeneity, at least when network-specific effects are introduced in the model. Then we estimate a standard homogenous version of our model, using adolescents' BMI in the Add Health dataset. Using GMM estimators based on Liu and Lee [2010], we find a significant but small endogenous effect, meaning that students tend to influence each other in terms of weight. The peer effect is equal to 0.205, which corresponds to a social multiplier of 1.20, assuming that the basic channel of social interactions is synergy (strategic complementarity). We then estimate our more general model with gender-dependent heterogeneity. Surprisingly, we do not reject that the within- and between-gender endogenous peer effects are the same. However, contextual effects differ significantly within and between gender as related with age, parents' education, parents health status and race. One important conclusion is therefore that gender-dependent heterogeneity is present not in the endogenous but in the contextual peer effects. We thus reject the full homogeneity model in BMI peer effects.

At the policy level, one interest of our approach is to introduce observable heterogeneity in the model (here, gender-dependent heterogeneity). At the theoretical level, this may help policy makers to use our method to better analyze the impact of reforms on adolescent obesity and to find the most appropriate tracking of students to reach the optimal outcome level.

Many extensions of our approach are possible. Firstly, following Hsieh and Lin [2015], one could introduce (and test) additional observable categories such as race or age in the model. Secondly, unobservable peer effects heterogeneity could also be taken into account (see Masten [2015]). Thirdly, more attention could be put on the mechanisms by which peers' BMI may influence individuals' BMI (through eating habits, physical exercise, social norms, etc.). Finally, developing and estimating a complete model of peer effects with heterogeneity and endogeneity in the network formation would be a most relevant research topic.

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Table 1: Descriptive statistics

	Mean	Standard deviation	Min	Max
Weight status				
BMI	23.14	4.72	13.25	46
zBMI	0.55	1.17	-3.74	4.44
Individual characteristics				
Age	16.36	1.43	13	20
Female	0.49	0.50	0	1
White	0.61	0.48	0	1
Black	0.15	0.36	0	1
American Indian	0.03	0.17	0	1
Asian Pacific	0.14	0.34	0	1
Hispanic origin	0.18	0.38	0	1
Grades 7-8	0.12	0.32	0	1
Grades 9-10	0.26	0.44	0	1
Grades 11-12	0.62	0.48	0	1
Eating habits				
Own decision to eat	0.85	0.36	0	1
Parents present when eat	4.47	2.38	0	7
Mother education and health status				
No high school	0.13	0.34	0	1
High school	0.35	0.48	0	1
Some college	0.19	0.39	0	1
College	0.18	0.38	0	1
Advanced	0.06	0.24	0	1
Don't know	0.04	0.19	0	1
Good health status	0.92	0.27	0	1
Father education and health status				
No high school	0.11	0.31	0	1
High school	0.25	0.43	0	1
Some college	0.14	0.35	0	1
College	0.15	0.36	0	1
Advanced	0.07	0.25	0	1
Don't know	0.05	0.23	0	1
Good health status	0.76	0.42	0	1
Network statistics				
Average number of friends	2.28	1.94	0	10
Number of female friends	1.16	1.27	0	5
Number of male friends	1.12	1.30	0	5
No friend	509			
N=2220				

Table 2: Endogenous network - Dep. var.: predicted link probability

	(1)		(2)	
	Coefficient	**(Std. Err.)**	**Coefficient**	**(Std. Err.)**
$\lvert\hat{\epsilon}_i - \hat{\epsilon}_j\rvert$	-0.00041 ***	(0.000005)	-0.000002	(0.0000019)
g_{ij}	0.03703 ***	(0.00017)	0.02047 ***	(0.00013)
Intercept	0.00449 ***	(0.000017)	0.07755 ***	(0.00018)
Network fixed effects	No	No	Yes	Yes
Observations	1,120,936		1,120,936	

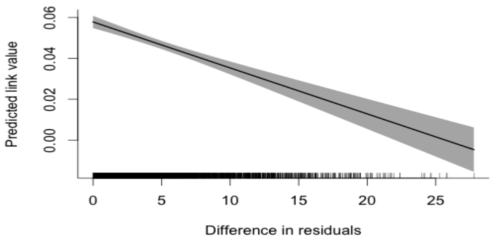
*** p<0.01, ** p<0.05, * p<0.1.

Table 3: Endogenous network formation: Liu et al. [2013] test

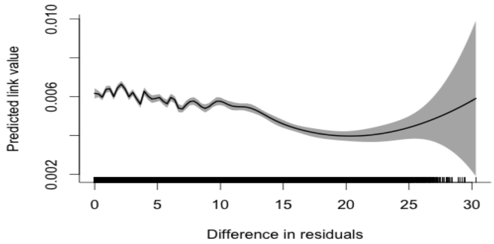
Panel A: $g_{ij,r} = 1$				
	T = 25%	T = 35%	T=45%	T=60%
$\lvert\hat{\epsilon}_i - \hat{\epsilon}_j\rvert$	0.0000009	0.0000127	0.0000087	-0.0000403**
	(0.000006)	(0.00000783)	(0.000011)	(0.0000161)
Intercept	0.002142***	0.004237**	0.01019***	0.01124***
	(0.000046)	(0.001578)	(0.001365)	(0.00200)
Network fixed effects	Yes	Yes	Yes	Yes
Observations	1,249	1,750	2,249	3000
Panel B: $g_{ij,r} = 0$				
	T = 95%	T = 85%	T=75%	T=60%
$\lvert\hat{\epsilon}_i - \hat{\epsilon}_j\rvert$	0.000003	0.000007	0.000002	0.000001
	(0.000031)	(0.000011)	(0.000007)	(0.000004)
Intercept	0.007777***	0.07437***	0.07440***	0.07440***
	(0.000671)	(0.00043)	(0.000342)	(0.000276)
Network fixed effects	Yes	Yes	Yes	Yes
Observations	55,792	167,390	278,984	446,367

*** p<0.01, ** p<0.05, * p<0.1. Standard errors are in parentheses.

Figure 1: Estimation without school fixed effects

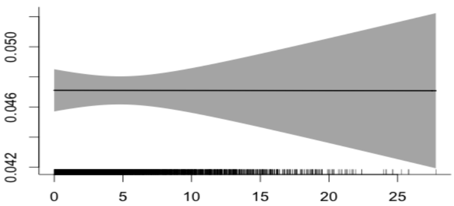


$g_{ij,r} = 1$

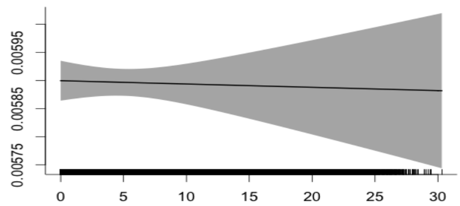


$g_{ij,r} = 0$

Figure 2: Estimation with school fixed effects



$g_{ij,r} = 1$



$g_{ij,r} = 0$

Table 4: 2SLS estimation of homogeneous peer effects - BMI

	(1)				(2)			
	Individual effects		Contextual effects		Individual effects		Contextual effects	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Endogenous effect	0.125	0.104	—	—	0.123	0.097	—	—
Personal characteristics								
Age	0.099	0.133	-0.130	0.153	0.047	0.134	-0.108	0.150
Female = 1	—	—	—	—	-0.723 ***	0.227	0.014	0.368
White = 1	-0.425	0.353	0.355	0.450	—	—	—	—
Black = 1	-0.519	0.445	0.557	0.686	—	—	—	—
Grade 9-10	0.910	0.586	0.309	0.734	0.982 *	0.580	0.313	0.732
Grade 11-12	1.562 **	0.716	0.028	0.841	1.712 **	0.708	-0.023	0.829
Mother education								
Mother no High School	-0.439	0.493	-1.845 **	0.954	-0.108	0.454	-1.629 *	0.939
Mother High School	-0.024	0.401	-1.100	0.883	0.103	0.394	-1.006	0.871
Mother Some College	0.163	0.431	-0.364	0.922	0.319	0.427	-0.230	0.910
Mother College	0.265	0.449	-1.965 **	0.872	0.340	0.446	-1.976 **	0.851
Mother Advanced	-0.326	0.536	-3.288 ***	1.036	-0.223	0.529	-3.120 ***	1.020
Father education								
Father no High School	0.747	0.478	0.310	0.719	—	—	—	—
Father High School	0.041	0.323	-0.382	0.587	-0.206	0.304	-0.482	0.519
Father Some College	0.143	0.359	-0.985	0.601	-0.120	0.345	-1.158 **	0.531
Father College	-0.112	0.368	0.181	0.642	-0.397	0.357	-0.061	0.581
Father Advanced	0.172	0.455	-0.427	0.788	-0.086	0.442	-0.493	0.743
Parents health status								
Mother in Good Health	-0.185	0.386	0.777	0.704	-0.209	0.379	0.749	0.690
Father in Good Health	-0.397	0.303	-0.460	0.534	-0.192	0.286	-0.385	0.497
N=2220								

*** p<0.01, ** p<0.05, * p<0.1.

Table 5: GMM estimation of homogeneous peer effects - BMI

	(1)				(2)			
	Individual effects		**Contextual effects**		**Individual effects**		**Contextual effects**	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Endogenous effect	0.205 ***	0.021	—	—	0.203 ***	0.021	—	—
Personal characteristics								
Age	0.118	0.088	-0.230 ***	0.054	0.069	0.088	-0.212 ***	0.052
Female = 1	—	—	—	—	-0.728 ***	0.158	0.033	0.247
White = 1	-0.439 *	0.241	0.308	0.300	—	—	—	—
Black = 1	-0.505 *	0.300	0.407	0.397	—	—	—	—
Grade 9-10	0.991 **	0.448	0.227	0.540	1.061 **	0.447	0.236	0.540
Grade 11-12	1.637 ***	0.509	0.022	0.582	1.779 ***	0.507	-0.013	0.574
Mother education								
Mother no High School	-0.420	0.333	-1.952 ***	0.562	-0.081	0.320	-1.755 ***	0.547
Mother High School	-0.008	0.286	-1.162 **	0.498	0.116	0.284	-1.096 **	0.495
Mother Some College	0.162	0.306	-0.486	0.535	0.310	0.304	-0.397	0.531
Mother College	0.291	0.314	-2.049 ***	0.524	0.368	0.312	-2.082 ***	0.520
Mother Advanced	-0.287	0.395	-3.316 ***	0.687	-0.182	0.393	-3.151 ***	0.686
Father education								
Father no High School	0.771 ***	0.286	0.239	0.456	—	—	—	—
Father High School	0.062	0.228	-0.396	0.368	-0.192	0.204	-0.469	0.325
Father Some College	0.151	0.256	-0.961 **	0.403	-0.119	0.236	-1.101 ***	0.366
Father College	-0.107	0.262	0.248	0.429	-0.398	0.244	0.052	0.393
Father Advanced	0.193	0.336	-0.439	0.562	-0.068	0.323	-0.474	0.538
Parents health status								
Mother in Good Health	-0.213	0.276	0.761	0.475	-0.233	0.274	0.738	0.473
Father in Good Health	-0.410 **	0.205	-0.458	0.329	-0.194	0.190	-0.385	0.307
N=2220								

*** $p<0.01$, ** $p<0.05$, * $p<0.1$.

Table 6: 2SLS estimation of gender heterogeneous peer effects model - BMI

	Individual effects		**Contextual effects**							
			M - M		M - F		F - F		F - M	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Personal characteristics										
Age	0.058	0.134	-0.015	0.187	-0.573	0.388	-0.286	0.227	-0.004	0.322
White = 1	-0.390	0.356	-0.691	0.628	-0.783	1.222	1.691 **	0.694	-0.373	1.181
Black = 1	-0.489	0.444	-1.602	1.055	1.036	1.723	1.770 **	0.938	0.207	1.795
Grade 9-10	0.973	0.591	1.014	0.966	-1.938	1.740	1.319	1.149	0.623	1.708
Grade 11-12	1.642 **	0.718	0.472	1.055	-0.228	1.899	1.259	1.281	-1.125	1.733
Mother education										
Mother no High School	-0.466	0.494	-3.51 **	1.449	-0.376	2.355	0.229	1.408	-2.665	3.333
Mother High School	-0.025	0.405	-1.854	1.328	0.462	2.338	-0.108	1.289	-1.142	3.024
Mother Some College	0.197	0.437	-0.366	1.506	0.888	2.360	0.311	1.378	-0.185	3.221
Mother College	0.298	0.449	-2.961 **	1.294	-0.535	2.631	-1.272	1.417	-1.684	3.000
Mother Advanced	-0.246	0.545	-3.687 **	1.576	-4.617	3.070	-1.409	1.631	-4.278	3.381
Father education										
Father no High School	0.664	0.488	-0.886	1.184	-0.007	1.667	0.599	1.233	2.718	1.959
Father High School	0.014	0.329	-1.224	0.884	1.801	1.945	-0.141	1.032	-0.266	1.517
Father Some College	0.155	0.360	-1.924 **	0.880	0.367	2.062	-0.336	1.061	-1.782	1.701
Father College	-0.173	0.376	-0.880	0.952	0.600	2.453	0.255	0.965	2.488	2.016
Father Advanced	0.181	0.455	-0.877	1.052	1.715	2.590	-0.030	1.418	-0.601	1.952
Parents health status										
Mother in Good Health	-0.149	0.390	0.953	1.225	2.266	1.731	0.408	0.882	0.760	2.555
Father in Good Health	-0.336	0.303	-0.735	0.801	0.583	1.619	-1.245	0.932	-0.382	1.315
			Endogenous effects							
			M - M		M - F		F - F		F - M	
			0.134	(0.112)	0.343	(0.274)	0.115	(0.164)	0.068	(0.210)
N=2220										

*** p<0.01, ** p<0.05, * p<0.1. Standard errors are in parentheses.

Table 7: GMM estimation of gender heterogeneous peer effects model - BMI

	Individual effects		Contextual effects							
			M - M		M - F		F - F		F - M	
	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.	Coef.	S.E.
Personal characteristics										
Age	0.078	0.089	-0.103	0.085	-0.502 **	0.208	-0.415 ***	0.094	-0.213	0.157
White = 1	-0.405 *	0.242	-0.757 *	0.454	-0.732	0.844	1.677 ***	0.468	-0.383	0.887
Black = 1	-0.483	0.302	-1.764 ***	0.678	1.151	1.064	1.624 ***	0.571	-0.012	1.103
Grade 9-10	1.009 **	0.451	0.973	0.743	-1.990	1.253	1.324	0.889	0.668	1.125
Grade 11-12	1.684 ***	0.512	0.468	0.804	-0.398	1.345	1.344	0.976	-0.900	1.184
Mother education										
Mother no High School	-0.465	0.333	-3.553 ***	0.882	-0.305	1.731	0.056	0.974	-3.068 *	1.740
Mother High School	-0.019	0.286	-1.885 **	0.734	0.492	1.617	-0.212	0.922	-1.367	1.365
Mother Some College	0.179	0.308	-0.390	0.840	0.905	1.669	0.016	0.948	-0.271	1.524
Mother College	0.307	0.314	-2.970 ***	0.784	-0.518	1.741	-1.529	0.968	-1.838	1.412
Mother Advanced	-0.206	0.396	-3.663 ***	1.101	-4.562 *	2.345	-1.508	1.171	-4.431 **	1.821
Father education										
Father no High School	0.708 **	0.288	-0.976	0.780	-0.086	1.349	0.443	0.718	2.738 **	1.327
Father High School	0.044	0.228	-1.224 **	0.598	1.745	1.154	-0.175	0.605	-0.393	1.029
Father Some College	0.184	0.256	-1.918 ***	0.644	0.275	1.222	-0.250	0.668	-2.103 *	1.195
Father College	-0.149	0.263	-0.818	0.685	0.545	1.473	0.324	0.713	2.303 *	1.201
Father Advanced	0.212	0.336	-0.931	0.890	1.557	1.989	-0.049	0.917	-0.616	1.464
Parents health status										
Mother in Good Health	-0.178	0.277	0.951	0.749	2.434 *	1.419	0.357	0.765	0.671	1.467
Father in Good Health	-0.357 *	0.207	-0.745	0.540	0.614	0.983	-1.195 **	0.537	-0.347	0.936
			Endogenous effects							
			M - M		M - F		F - F		F - M	
			0.202 ***	(0.039)	0.287 **	(0.119)	0.216 ***	(0.030)	0.228 ***	(0.082)
N=2220										

*** p<0.01, ** p<0.05, * p<0.1. Standard errors are in parentheses.

Table 8: Wald Statistics for Gender Homogeneity in Peer Effects

		2SLS		GMM	
		BMI	z-BMI	BMI	z-BMI
Full Homogeneity: $\beta_{mm} = \beta_{mf} = \beta_{ff} = \beta_{fm} / \delta_{mm} = \delta_{mf} = \delta_{ff} = \delta_{fm}$	Gender	54.679	57.979	114.89***	117.22***
Only-Endogenous Homogeneity: $\beta_{mm} = \beta_{mf} = \beta_{ff} = \beta_{fm}$	Gender	0.700	0.322	0.550	0.329

Full-gender model: 54 d.o.f., threshold values respectively 81.06, 72.15 and 67.67 for 0.01%, 0.05% and 10% significance.

6cOnly-endogenous model: 3 d.o.f., threshold values respectively 11.34, 7.81 and 6.25 for 0.01%, 0.05% and 0.10% significance.

Appendices

A Proof of proposition 1: invertibility condition

Recall that matrices $\bar{\mathbb{G}}_1$, $\bar{\mathbb{G}}_2$, $\bar{\mathbb{G}}_3$ and $\bar{\mathbb{G}}_4$ are ordered so that the n^f first rows are for type-f individuals and the remaining n^mrows correspond to type-m individuals. In this setting, the degree of $\bar{\mathbb{G}}(\boldsymbol{\beta})$ are, for type-f individuals, equal to $(n_i^f \beta_{ff} + n_i^m \beta_{fm})/(n_i^m + n_i^f)$. For type-$m$ individuals, it is equal to $(n_i^m \beta_{mm} + n_i^f \beta_{mf})/(n_i^m + n_i^f)$. Thus, the degree vector of $\bar{\mathbb{G}}(\boldsymbol{\beta})$ is a vector containing each of these unique values that depend on the number of friends of each type for each individual. Let $\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_n$ be the spectrum of $\bar{\mathbb{G}}(\boldsymbol{\beta})$. The determinant of $\mathbf{S}$ can be written as the product of eigenvalues of the matrix. Given that eigenvalues of $\mathbf{I} + \bar{\mathbb{G}}(\boldsymbol{\beta})$ are equal to $(1 + \lambda_i)$, one has $det(\mathbf{S}) = \prod_{i=1}^n (1 + \lambda_i)$. The maximum degree is:

$$\varDelta(\bar{\mathbb{G}}) = \max \left(\max_{i \in N^f} \left[\frac{n_i^f \beta_{ff} + n_i^m \beta_{fm}}{n_i^m + n_i^f} \right], \max_{j \in N^m} \left[\frac{n_j^m \beta_{mm} + n_j^f \beta_{mf}}{n_j^m + n_j^f} \right] \right)$$

We have the two following inequalities: $\lambda_1(\bar{\mathbb{G}}) \leq \varDelta(\bar{\mathbb{G}})$ and $\lambda_n(\bar{\mathbb{G}}) \geq -\lambda_1(\bar{\mathbb{G}})$. A sufficient condition for matrix $\mathbf{S}$ to be invertible is that its determinant is positive. Taking this condition into account, developping by using the upper inequalities, we end up with the following inequality, $\forall i / in \{1, ..., n\}$:

$$1 - \varDelta(\bar{\mathbb{G}}) \leq \lambda_i \leq 1 + \varDelta(\bar{\mathbb{G}})$$

Two of the following cases may apply:

- If $\varDelta(\bar{\mathbb{G}}) = \max_{i \in N^f} \left[\frac{n_i^f \beta_{ff} + n_i^m \beta_{fm}}{n_i^m + n_i^f} \right]$, then two upper cases are β_{ff} or β_{fm}. Thus, sufficient invertibility condition are $|\beta_{ff}| < 1$ or $|\beta_{fm}| < 1$;
- If $\varDelta(\bar{\mathbb{G}}) = \max_{j \in N^m} \left[\frac{n_j^m \beta_{mm} + n_j^f \beta_{mf}}{n_j^m + n_j^f} \right]$, then two upper cases are β_{mm} or β_{mf}. Thus, sufficient invertibility condition are $|\beta_{mm}| < 1$ or $|\beta_{mf}| < 1$.

□

B Example of IV matrix $\mathbf{Q}_{i,\infty}$

Examples of $\mathbf{Q}_{i,\infty}$ are:

$$\begin{cases} \mathbf{Q}_{1,\infty} &= \bar{\mathbb{G}}_1^2\mathbf{x}, \bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\mathbf{x}, \bar{\mathbb{G}}_1^3\mathbf{x}, \bar{\mathbb{G}}_1^2\bar{\mathbb{G}}_2\mathbf{x}, \bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\mathbf{x}, \bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\mathbf{x}, ..., \bar{\mathbb{G}}_1\boldsymbol{\iota}, \bar{\mathbb{G}}_1^2\boldsymbol{\iota}, \bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\boldsymbol{\iota}, ..., \\ \mathbf{Q}_{2,\infty} &= \bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\mathbf{x}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\mathbf{x}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2\mathbf{x}, ..., \bar{\mathbb{G}}_2\boldsymbol{\iota}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\boldsymbol{\iota}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\boldsymbol{\iota}, \bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2\boldsymbol{\iota}, ..., \\ \mathbf{Q}_{3,\infty} &= \bar{\mathbb{G}}_3^2\mathbf{x}, \bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\mathbf{x}, \bar{\mathbb{G}}_3^3\mathbf{x}, ..., \bar{\mathbb{G}}_3\boldsymbol{\iota}, \bar{\mathbb{G}}_3^2\boldsymbol{\iota}, \bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\boldsymbol{\iota}, \bar{\mathbb{G}}_3^3\boldsymbol{\iota}, ..., \\ \mathbf{Q}_{4,\infty} &= \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\mathbf{x}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\mathbf{x}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^2\mathbf{x}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\mathbf{x}, ..., \bar{\mathbb{G}}_4\boldsymbol{\iota}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\boldsymbol{\iota}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\boldsymbol{\iota}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^2\boldsymbol{\iota}, \bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\boldsymbol{\iota}, ... \end{cases}$$

C Proof of proposition 2: identification of the best response model

Proof C.1 *To prove our proposition, we use the formula of the inverse of matrix $\mathbf{S}(\boldsymbol{\beta})$ established using the Newton Binomial formula, the identities given in footnote (20), and the conditions of invertibility (see Proposition 1). We also use the expression of $\mathbf{JZ}$ given by equation $\mathbf{JZ} = \mathbb{E}(\mathbf{Z}) + \mathbf{J}\sum_{i=1}^{4}[\mathbf{W}_i\boldsymbol{\epsilon}]\mathbf{e}_i'$ where $\mathbf{J}\mathbb{E}(\mathbf{Z}) = \mathbf{J}\left[\{\mathbf{W_i}(\boldsymbol{\beta})\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota\alpha}\right]\}_{\{i=1,2,3,4\}}, \mathbf{X}\right]$. The following steps are necessary to prove our proposition:*

1. *Let $k = 1, 2, 3, 4, ...$ and derive the expression of $\mathbf{S}_k(\boldsymbol{\beta})^{-1}$ using:*

$$\mathbf{S}_k(\boldsymbol{\beta}) = \sum_{i=0}^{k\geq 1}\binom{k}{i}\left[(\beta_{mm}\bar{\mathbb{G}}_1)^{k-i} + (k-i)\beta_{mf}(\beta_{mm}\bar{\mathbb{G}}_1)^{k-i-1}\bar{\mathbb{G}}_2\right].\left[(\beta_{ff}\bar{\mathbb{G}}_3)^{i} + i\beta_{fm}(\beta_{ff}\bar{\mathbb{G}}_3)^{i-1}\bar{\mathbb{G}}_4.\right]$$

2. *Sum over all k's and re-write $\mathbf{S}(\boldsymbol{\beta})^{-1}$ such that $\mathbf{S}(\boldsymbol{\beta})^{-1} = \mathbf{I} + \sum_{k=1}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})$.*

3. *Using the latter expression, derive an expression of $\mathbf{W}_i(\boldsymbol{\beta}) = \bar{\mathbb{G}}_i\mathbf{S}(\boldsymbol{\beta})^{-1}$ and $\mathbf{W}_i(\boldsymbol{\beta})\bar{\mathbb{G}}(\boldsymbol{\delta})$ $\forall i \in \{1, 2, 3, 4\}$.*

4. *Write $\{\mathbf{W_i}(\boldsymbol{\beta})\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota\alpha}\right]\}_{\{i=1,2,3,4\}}$ as a function of instruments and extract intruments and the associated restrictions on the parameters of the model, premultiplied by matrix $\mathbf{J}$.*

For sake of simplicity, let susbscripts mm, mf, ff and fm in $\boldsymbol{\beta}$ be replaced by $1, 2, 3, 4$ respectively. Using the steps enumerated above and developing for $k \in 1, 2, 3, 4$, one can write $\mathbf{S}_k(\boldsymbol{\beta})$ using the expression below:

— —

$$\mathbf{S}_1(\boldsymbol{\beta}) = [\beta_1\bar{\mathbb{G}}_1 + \beta_2\bar{\mathbb{G}}_2] \times [\beta_3\bar{\mathbb{G}}_3 + \beta_4\bar{\mathbb{G}}_4]$$

— —

$$\mathbf{S}_2(\boldsymbol{\beta}) = \left[\beta_1^2\bar{\mathbb{G}}_1^2 + 2\beta_1\beta_2\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\right] + 2\left[\beta_1\bar{\mathbb{G}}_1 + \beta_2\bar{\mathbb{G}}_2\right] \times \left[\beta_3\bar{\mathbb{G}}_3 + \beta_4\bar{\mathbb{G}}_4\right] + \left[\beta_3^2\bar{\mathbb{G}}_3^2 + 2\beta_3\beta_4\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\right]$$

— —

$$\begin{aligned}\mathbf{S}_3(\boldsymbol{\beta}) = &\quad \left[\beta_1^3\bar{\mathbb{G}}_1^3 + 3\beta_1^2\beta_2\bar{\mathbb{G}}_1^2\bar{\mathbb{G}}_2\right] + 3\left[\beta_1^2\bar{\mathbb{G}}_1^2 + 2\beta_1\beta_2\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\right] \times \left[\beta_3\bar{\mathbb{G}}_3 + \beta_4\bar{\mathbb{G}}_4\right] \\ &+ 3\left[\beta_1\bar{\mathbb{G}}_1 + \beta_2\bar{\mathbb{G}}_2\right] \times \left[\beta_3^2\bar{\mathbb{G}}_3^2 + 2\beta_3\beta_4\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\right] + \left[\beta_3^3\bar{\mathbb{G}}_3^3 + 3\beta_3^2\beta_4\bar{\mathbb{G}}_3^2\bar{\mathbb{G}}_4\right]\end{aligned}$$

— —

$$\begin{aligned}\mathbf{S}_4(\boldsymbol{\beta}) = &\quad \left[\beta_1^4\bar{\mathbb{G}}_1^4 + 4\beta_1^3\beta_2\bar{\mathbb{G}}_1^3\bar{\mathbb{G}}_2\right] + 4\left[\beta_1^3\bar{\mathbb{G}}_1^3 + 3\beta_1^2\beta_2\bar{\mathbb{G}}_1^2\bar{\mathbb{G}}_2\right] \times \left[\beta_3\bar{\mathbb{G}}_3 + \beta_4\bar{\mathbb{G}}_4\right] \\ &+ 6\left[\beta_1^2\bar{\mathbb{G}}_1^2 + 2\beta_1\beta_2\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\right] \times \left[\beta_3^2\bar{\mathbb{G}}_3^2 + 2\beta_3\beta_4\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\right] + 4\left[\beta_1\bar{\mathbb{G}}_1 + \beta_2\bar{\mathbb{G}}_2\right] \\ &\times \left[\beta_3^3\bar{\mathbb{G}}_3^3 + 3\beta_3^2\beta_4\bar{\mathbb{G}}_3^2\bar{\mathbb{G}}_4\right] + \left[\beta_3^4\bar{\mathbb{G}}_3^4 + 4\beta_3^3\beta_4\bar{\mathbb{G}}_3^3\bar{\mathbb{G}}_4\right]\end{aligned}$$

— —

We then write $\mathbf{S}^{-1}(\boldsymbol{\beta}) = \mathbf{I} + \mathbf{S}_1(\boldsymbol{\beta}) + \mathbf{S}_2(\boldsymbol{\beta}) + \mathbf{S}_3(\boldsymbol{\beta}) + \mathbf{S}_4(\boldsymbol{\beta}) + \sum_{k=5}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})$ using the expressions of $\mathbf{S}_k(\boldsymbol{\beta})$ given above. We are then able to write, $\forall i \in \{1, 2, 3, 4\}$, $\mathbf{W_i}(\boldsymbol{\beta})\left[\gamma\mathbf{x} + \bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x} + \boldsymbol{\iota\alpha}\right]$ as:

$$
\begin{aligned}
\mathbf{W_1}(\boldsymbol{\beta})\left[\gamma\mathbf{x}+\bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x}+\boldsymbol{\iota\alpha}\right]=\gamma\bar{\mathbb{G}}_1\mathbf{x} \quad &+ \quad (\gamma\beta_1+\delta_1)\left[\bar{\mathbb{G}}_1^2+\beta_1\bar{\mathbb{G}}_1^3+\beta_1^2\bar{\mathbb{G}}_1^4+\beta_1^5\bar{\mathbb{G}}_1^2\right]\mathbf{x}\\
&+ \quad (\gamma\beta_2+\delta_2)\left[\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\right]\mathbf{x}+\beta_1(2\gamma\beta_2+\delta_2)\left[\bar{\mathbb{G}}_1^2\bar{\mathbb{G}}_2\right]\mathbf{x}\\
&+ \quad \beta_2(2\gamma\beta_3+\delta_3)\left[\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\right]\mathbf{x}+\beta_2(2\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \left[\bar{\mathbb{G}}_1+\beta_1\bar{\mathbb{G}}_1^2+\beta_2\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2+\beta_1^2\bar{\mathbb{G}}_1^3+2\beta_1\beta_2\bar{\mathbb{G}}_1^2\bar{\mathbb{G}}_2+...\right]\boldsymbol{\iota\alpha}\\
&+ \quad \bar{\mathbb{G}}_1\sum_{k=5}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})\left[(\gamma+\bar{\mathbb{G}}(\boldsymbol{\delta}))\mathbf{x}+\boldsymbol{\iota\alpha}\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{W_2}(\boldsymbol{\beta})\left[\gamma\mathbf{x}+\bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x}+\boldsymbol{\iota\alpha}\right]=\gamma\bar{\mathbb{G}}_2\mathbf{x} \quad &+ \quad (\gamma\beta_3+\delta_3)\left[\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3+\beta_3\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2+\beta_3^2\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^3+\beta_3^3\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^3\right]\mathbf{x}\\
&+ \quad (\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\right]\mathbf{x}+\beta_3(2\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \beta_3^2(3\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2\bar{\mathbb{G}}_4\right]\mathbf{x}+\beta_3^3(4\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^3\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \left[\bar{\mathbb{G}}_2+\beta_3\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3+\beta_3^2\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2+2\beta_3\beta_4\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4+...\right]\boldsymbol{\iota\alpha}\\
&+ \quad \bar{\mathbb{G}}_2\sum_{k=5}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})\left[(\gamma+\bar{\mathbb{G}}(\boldsymbol{\delta}))\mathbf{x}+\boldsymbol{\iota\alpha}\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{W_3}(\boldsymbol{\beta})\left[\gamma\mathbf{x}+\bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x}+\boldsymbol{\iota\alpha}\right]=\gamma\bar{\mathbb{G}}_3\mathbf{x} \quad &+ \quad (\gamma\beta_3+\delta_3)\left[\bar{\mathbb{G}}_3^2+\beta_3\bar{\mathbb{G}}_3^3+\beta_3^2\bar{\mathbb{G}}_3^4+\beta_3^3\bar{\mathbb{G}}_3^5\right]\mathbf{x}\\
&+ \quad (\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4\right]\mathbf{x}+\beta_3(2\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_3^2\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \beta_3^2(3\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_3^3\bar{\mathbb{G}}_4\right]\mathbf{x}+\beta_3^3(4\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_3^4\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \left[\bar{\mathbb{G}}_3+\beta_3\bar{\mathbb{G}}_3^2+\beta_4\bar{\mathbb{G}}_3\bar{\mathbb{G}}_4+2\beta_3\beta_4\bar{\mathbb{G}}_3^2\bar{\mathbb{G}}_4+...\right]\boldsymbol{\iota\alpha}\\
&+ \quad \bar{\mathbb{G}}_3\sum_{k=5}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})\left[(\gamma+\bar{\mathbb{G}}(\boldsymbol{\delta}))\mathbf{x}+\boldsymbol{\iota\alpha}\right]
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{W_4}(\boldsymbol{\beta})\left[\gamma\mathbf{x}+\bar{\mathbb{G}}(\boldsymbol{\delta})\mathbf{x}+\boldsymbol{\iota\alpha}\right]=\gamma\bar{\mathbb{G}}_4\mathbf{x} \quad &+ \quad (\gamma\beta_1+\delta_1)\left[\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1+\beta_1\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^2+\beta_1^2\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^3+\beta_1^3\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^4\right]\mathbf{x}\\
&+ \quad (\gamma\beta_2+\delta_2)\left[\bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\right]\mathbf{x}+\beta_1(2\gamma\beta_2+\delta_2)\left[\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\right]\mathbf{x}\\
&+ \quad \beta_2(2\gamma\beta_3+\delta_3)\left[\bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3\right]\mathbf{x}+\beta_2(2\gamma\beta_4+\delta_4)\left[\bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\right]\mathbf{x}\\
&+ \quad \left[\bar{\mathbb{G}}_4+\beta_1\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1+\beta_2\bar{\mathbb{G}}_4\bar{\mathbb{G}}_2+\beta_1^2\bar{\mathbb{G}}_4\bar{\mathbb{G}}_1^2+...\right]\boldsymbol{\iota\alpha}\\
&+ \quad \bar{\mathbb{G}}_4\sum_{k=5}^{\infty}\mathbf{S}_k(\boldsymbol{\beta})\left[(\gamma+\bar{\mathbb{G}}(\boldsymbol{\delta}))\mathbf{x}+\boldsymbol{\iota\alpha}\right]
\end{aligned}
$$

Using the above expressions, we can derive sufficient conditions of identification of our parameters using the IV method. These conditions are similar to the ones obtained in Bramoullé et al. [2009] regarding the independence of the interaction matrices of our model and restrictions on our parameters.

Specifically, considering the expressions given above, we can see that naturally occuring intruments of our endogenous variables include different order of our interaction matrices and interactions of different orders of these matrices. For example, intruments of our first endogenous variable include $\mathbf{JG_1x}$*,* $\mathbf{JG_1}^2\mathbf{x}$*,* $\mathbf{JG_1}^3\mathbf{x}$ *and higher degrees of the matrix* $\mathbf{JG}_1$ *multiplied by vector* $\mathbf{x}$ *of characteristics if both* $(\gamma\beta_1 + \delta_1) \neq 0$ *and matrices* $\mathbf{G_1}, \mathbf{G_1}^2, \mathbf{G_1}^3, \mathbf{G_1}^4,$ *etc. are linearly independent. Following the same method and using the other expressions above, we can see that minimal conditions for IV variables to work for each of the four endogenous variables are* $(\gamma\beta_2+\delta_2) \neq 0$, $(\gamma\beta_3+\delta_3) \neq 0$ *and* $(\gamma\beta_4+\delta_4) \neq 0$*. In addition,* γ *needs to be different from zero and matrices* $\bar{\mathbb{G}}_1$*,* $\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_4$*,* $\bar{\mathbb{G}}_1^2$*,* $\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_3^2$*,* $\bar{\mathbb{G}}_1^3$*, ...,* $\mathbf{I}$ *need to be independent, which corresponds to the condition that vector columns of matrix* $\mathbf{Q}_K$ *of instruments should be linearly independent.*

Additional conditions appear whenever one is concerned about adding instruments of higher order of interaction matrices multiplication. In this case, the additional conditions on parameters of the model take the form of $\beta_i \neq 0\ \forall i \in \{2, 3, 4\}$ *and* $((j-1)\gamma\beta_l + \delta_l) \neq 0$ *and linear independence of* j^{th} *order interaction of social interaction matrices such that* $\mathbf{CG}_i\bar{\mathbb{G}}_l$ *adds up to the independence conditions stated above, where* $\mathbf{C}$ *is either a single interaction matrix or a non-zero product of interaction matrices. For example,* $\mathbf{JG}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4\mathbf{x}$ *may be used as an instrument if* $\beta_2 \neq 0$*,* $(2\gamma\beta_4 + \delta_4) \neq 0$ *and matrices* $\bar{\mathbb{G}}_1$*,* $\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_4$*,* $\bar{\mathbb{G}}_1^2$*,* $\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_3^2$*,* $\bar{\mathbb{G}}_1^3$*, ...,* $\mathbf{I}$ *and* $\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2\bar{\mathbb{G}}_4$ *are linearly independent. Also,* $\mathbf{JG}_4\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2\mathbf{x}$ *may be used as an additional instrument if* $\beta_2 \neq 0$*,* $\beta_3 \neq 0$*,* $(3\gamma\beta_3 + \delta_3) \neq 0$ *and matrices* $\bar{\mathbb{G}}_1$*,* $\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_4$*,* $\bar{\mathbb{G}}_1^2$*,* $\bar{\mathbb{G}}_1\bar{\mathbb{G}}_2$*,* $\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3$*,* $\bar{\mathbb{G}}_3^2$*,* $\bar{\mathbb{G}}_1^3$*, ...,* $\mathbf{I}$ *and* $\bar{\mathbb{G}}_4\bar{\mathbb{G}}_2\bar{\mathbb{G}}_3^2$ *are linearly independent.*

QED