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Development Aid in the Presence of Corruption: Differential Games among Donors

Murray C. Kemp[*], *Ngo Van Long*[†]

Résumé / *Abstract*

On présente deux modèles d'aide internationale dans lesquels deux pays avancés s'engagent dans un jeu dynamique. Dans le premier modèle, les aides apportent aux donateurs des gains moraux. On montre qu'une hausse de la corruption du pays sous-développé peut augmenter les aides. Il y a une multiplicité d'équilibres de Nash, qui peuvent être ordonnés sous le critère de Pareto. Dans le deuxième modèle, les pays donateurs cessent de donner aussitôt que le niveau du développement atteint un but fixé. On montre que l'équilibre de ce modèle implique que le flux d'aide devient de plus en plus faible au fur et à mesure que le niveau de développement s'approche du but fixé. Les pays avancés donnent plus si le taux de corruption augmente.

Mots clés : aide internationale, corruption, jeux dynamiques, jeux différentiels

In this paper, we complement the work of Kemp and Shimomura (2002) by considering the case of many donors playing a dynamic non-cooperative game of foreign aid. We consider two models. Model 1 deals with the case where donor countries continually feel the warm glow of from the act of giving. Model 2 postulates that donors will stop giving aid when a target level of development is reached. One of the main results of Model 1 is that there are multiple equilibria that can be Pareto ranked. Another interesting result is that an increase in the level of corruption in the recipient country will reduce the aid level of the low aid equilibrium, but increase that of the high aid equilibrium. In Model 2, the equilibrium strategies are non-linear functions of the level of development. The flow of aid falls at a faster and faster rate as the target is approached. An increase in corruption will increase the flow of aid in this model.

Keywords: *development aid, corruption, dynamic games, differential games*

Codes JEL : O11

[*] Department of Economics, Macquarie University, Sydney, Australia, Email: murraykemp@bigpond.com, or mkemp@efs.mq.edu.au.

[†] CIRANO and CIREQ, Department of Economics, McGill University, 855 Sherbrooke St West, Montreal, Quebec, H3A 2T7, Canada. Email:ngo.long@mcgill.ca.

1 Introduction

In a recent contribution to the theory of foreign aid (i.e., voluntary and unrequited international transfer), Kemp and Shimomura (2002) remarked that the theory rests on two incompatible assumptions: (i) each country is indifferent to the wellbeing of other countries, and (ii) voluntary unrequited international transfers do take place. They therefore proposed a more satisfactory model that would allow for the possibility that the wellbeing of each country is influenced by the wellbeing of other countries, and addressed the issue of the extent of foreign aid, optimally chosen by the donor[1]. They focused on the static case with two countries: a donor and a recipient[2].

In this paper, we complement the work of Kemp and Shimomura (2002) by considering the case of many donors playing a dynamic and non-cooperative game of foreign aid to a given recipient. In working with a dynamic model, we are paying tribute to the late Koji Shimomura, who made substantial contributions to the literature on differential games, both at the theoretical level and at the level of applications[3]. Following Kemp and Shimomura (2002), we assume that the donors care about the wellbeing of the recipient.

We consider two models. Model 1 deals with the case where donor countries continually feel the warm glow of from the act of giving. Model 2 postulates that donors will stop giving aid when a target level of development is reached. One of the main results of Model 1 is that there are multiple equilibria that can be Pareto ranked. Another interesting result is that an

[1] In a complementary piece, Kemp and Shimomura (2003) considered the case of involuntary unrequited transfers, e.g., war reparations, under the assumption that the wellbeing of each country is negatively influenced by the wellbeing of the other country.

[2] For a dynamic formulation with a donor and a recipient, see Kemp, Long and Shimomura (1992). That paper did not deal with a differential game between donors, which is the subject matter of the present paper.

[3] See, for example, Shimomura (1991), Kemp, Long and Shimomura (1993, 2001), Long and Shimomura (1998), Long, Shimomura and Takahashi (1999). For expositions of differential games, see Clemhout and Wan (1994), and Dockner et al. (2000).

increase in the level of corruption in the recipient country will reduce the aid level of the low aid equilibrium, but increase that of the high aid equilibrium. In Model 2, the equilibrium strategies are non-linear functions of the level of development. The flow of aid falls at a faster and faster rate as the target is approached. An increase in corruption will increase the flow of aid in this model.

We do not wish to comment on the related empirical literature, except to mention that Alesina and Weder (2002) found that "Scandinavian countries (plus Australia) seem to give more to less corrupt governments" while "at the opposite extreme, more US foreign aid goes to more countries that are corrupt" (p. 1133-4). They explained this by appealing to historical factors, which are beyond the scope of our paper. Both in their papers and in ours, corruption is supposed to happen only in the recipient country. In practice, corruption can also occur in the donor countries and might even involve collaboration between officials of donor and recipient countries. This is a topic for future research.

2 Model 1: Aid giving with warm glow

2.1 The game among donors

There are n donor countries, and one recipient country. To keep things simple, the only state variable in our model is the stock of capital of the recipient country, which we denote by $X(t)$. Here, capital should be interpreted in a broad sense. For example, it may be a composite indicator of the country's physical and human capital, including health, infrastructure, education system and other aspects of human development. Assume that the country's gross output is

$$Y(t) = F(X(t))$$

where $F(0) = 0$ and $F'(X) > 0$. A constant fraction s of gross output is saved. Let $A_i(t)$ be the flow of aid from donor country i, which is supposed to be used for investment. Assume that corrupt officials in the recipient country siphon off a fraction $(1 - \varepsilon_i)$ of this aid, and only the remaining part, $\varepsilon_i A_i(t)$, is used for capital accumulation. (The parameter ε_i may be different for different donors, because they may have different auditing practices and therefore impose different degrees of deterrence on the potential corrupt officials of the recipient country.) The rate of growth of X is assumed to be

$$\dot{X}(t) = sF(X(t)) + \sum_{i=1}^{n} \varepsilon_i A_i(t) - \delta X(t) \tag{1}$$

The parameter $\delta > 0$ represents the rate of depreciation. (One could modify the transition equation (1) by multiply s to all terms on the right-hand side; the interpretation would then be slightly different, but the main results would be essntially unchanged.)

Let $B_i(t)$ denote the donor country i's net satisfaction level at time t derived from giving the amount $A_i(t)$. We assume $B_i(t)$ consists of two terms. The first term, denoted by $G_i(A_i(t), X(t))$, is the satisfaction that the donor country derives from (a) seeing that the recipient country has accumulated a stock $X(t)$, and (b) the "warm glow" of giving the amount $A_i(t)$. The second term, denoted by $\kappa_i A_i(t)$, represents the opportunity cost of giving, namely the amount of consumption foregone by the donor. Here, $\kappa_i > 0$ measures the foregone domestic consumption for each dollar of aid sent abroad. It is often argued that $\kappa_i > 1$, because the marginal cost of public funds includes distortion costs that arise from raising revenues using distorting taxes. In our model, we only require that $\kappa_i > 0$. The net satisfaction is

$$B_i(t) = G_i(A_i(t), X(t)) - \kappa_i A_i(t)$$

We assume that $G_i(A_i(t), X(t))$ is an increasing function of both arguments.

We consider a non-cooperative differential game among the donor countries. Suppose donor country i knows, in equilibrium, that other donor countries j (where $j \neq i$) use a decision rule $A_j(t) = \phi_j(X(t))$, i.e., they use a stationary feedback strategy that assigns, for each value of the state variable X, a non-negative amount of aid $A_i = \phi_j(X)$. Country i then seeks to solve the following optimization problem. Find a non-negative time path $A_i(t)$ that maximizes

$$\int_0^\infty e^{-\rho_i t} \left[G_i(A_i(t), X(t)) - \kappa_i A_i(t)\right] dt \tag{2}$$

subject to

$$\dot{X}(t) = sF(X(t)) + \varepsilon_i A_i(t) + \sum_{j \neq i}^{n} \varepsilon_j \phi_j(X(t)) - \delta X(t)$$

where $X(0) = X_0$.

Suppose this optimization problem yields an optimal time path $A_i^*(.)$ for the control variable, and an associated time path $X^*(.)$ for the state variable. Then one can express $A_i^*(t)$ as a function of $X^*(t)$:

$$A_i^*(t) = \phi_i(X^*(t))$$

The function $\phi_i(.)$ is called donor country i's "best reply" to the $n-1$ decision rules $\phi_j(.), j \neq i$.

A *Markov-perfect Nash equilibrium*[4] of this differential game is an n-tuple of decision rules, $(\phi_1, \phi_2, ..., \phi_n)$ such that each decision rule is a best reply to the other $n-1$ decision rules, for all possible initial dates, and any initial level of the stock. Our task is to investigate whether, under certain assumptions, there exist one or several Markov-perfect Nash equilibria, and to study their properties.

[4]See Docker et al. (2000) for a precise definition with explanation of Markov-perfect equilibrium, Long and Sorger (2006) for a brief definition, and Maskin and Tirole (2001) for some discussion.

2.2 Analysis

We make the following assumptions on the functions $F(X)$ and $G_i(A_i, X)$.

Assumption A0: *The function $F(X)$ is linear: $F(X) = rX$ where r is small relative to the rate of discount of the donor countries, so that $\rho > sr - \delta$.*

This linearity assumption on the production function of the "poor" country is borrowed from Tornell and Velasco (1992), Lane and Tornell (1996), and Tornell and Lane (1999)[5]. It simplifies the analysis a great deal. The additional assumption that the inequality $\rho > sr - \delta$ holds will be used ensure that the integral in (2) converges.

Assumption A1: *The function G_i is increasing, concave, and homogeneous of degree one in (A_i, X).*

Assumption A1 allows us to write the donor's satisfaction as

$$G_i(A_i, X) = XG_i\left(\frac{A_i}{X}, 1\right) \equiv Xg_i(\alpha_i)$$

where $\alpha_i(t) \equiv A_i(t)/X(t)$. Again, this serves to simplify the analysis. The role of linear homogeneity in differential games was explored in detail in Long and Shimomura (1998), and Long, Shimomura, and Takahashi (1999). For a recent application using this assumption, see Long and Sorger (2006).

Assumption A2: *$g_i(\alpha_i)$ is strictly concave and increasing, with the Inada properties:*

$$\lim_{a_i \to 0} g_i'(\alpha_i) = \infty \text{ and } \lim_{\alpha_i \to \infty} g_i'(\alpha_i) = 0$$

2.2.1 Existence of Markov-perfect Nash equilibria

Suppose player i knows that all other players use a linear stationary Markovian strategy

$$A_j(t) = \alpha_j X(t)$$

[5]These authors focused on the decisions on rent-extraction by corrupt officials of developing countries, and did not model foreign aid decisions.

where α_j is a positive constant. Let ψ_i be the co-state variable for player i's optimal control problem. The Hamiltonian of player i is

$$H_i = G_i(A_i, X) - \kappa A_i + \psi_i \left[(sr - \delta + (n-1)\varepsilon\alpha_j)X + \varepsilon A_i\right]$$

The Hamiltonian is jointly concave in (A_i, X). This ensures that the necessary conditions are also sufficient. The necessary conditions are

$$\frac{\partial H_i}{\partial A_i} = G_{A_i}(A_i, X) - \kappa_i + \varepsilon\psi_i = 0$$

$$\dot{\psi}_i = \rho_i\psi_i - \frac{\partial H_i}{\partial X} = \psi_i(\delta + \rho_i - sr - (n-1)\varepsilon\alpha_j) - G_X(A_i, X)$$

The transversality conditions are

$$\lim_{t\to\infty} \psi_i(t)e^{-\rho_i t} \geq 0, \ \lim_{t\to\infty} X(t)\psi_i(t)e^{-\rho_i t} = 0 \tag{3}$$

Let $\alpha_i = A_i/X$. Then $G_{A_i}(A_i, X) = g_i'(\alpha_i)$ and $G_X(A_i, X) = g_i(\alpha_i) - \alpha_i g_i'(\alpha_i) > 0$. The necessary conditions become

$$g_i'(\alpha_i) - \kappa_i + \varepsilon\psi_i = 0 \tag{4}$$

$$\dot{\psi}_i = \psi_i(\delta + \rho_i - sr - (n-1)\varepsilon\alpha_j) - [g_i(\alpha_i) - \alpha_i g_i'(\alpha_i)]$$

Since X is a "good" stock (i.e., it contributes to the welfare of the donor country i), we expect the shadow price ψ_i to be positive. The positivity of ψ_i in turn implies that $\kappa - g_i'(\alpha_i) > 0$ along an optimal path. This means that the optimal $\alpha_i(t)$ exceeds the level that maximizes static satisfaction, $G_i(A_i, X) - \kappa A_i$. The marginal net current benefit of aid, $g_i'(\alpha_i) - \kappa_i$, is thus negative at the optimum. This is because the donor rationally takes into account the effect of current aid on the recipient's future level of capital which contributes to the donor's future satisfaction.

Let us try a solution where α_i =constant. The constancy of α_i implies, via (4), that $\dot{\psi}_i = 0$. This in turn implies that

$$\psi_i = \frac{g_i(\alpha_i) - \alpha_i g_i'(\alpha_i)}{\delta + \rho_i - sr - (n-1)\varepsilon\alpha_j}$$

Combining this equation with (4), we get the condition

$$\kappa_i - g_i'(\alpha_i) = \frac{\varepsilon_i \left(g_i(\alpha_i) - \alpha_i g_i'(\alpha_i)\right)}{\rho_i - [sr + (n-1)\varepsilon\alpha_j - \delta]} \tag{5}$$

The interpretation of condition (5) is as follows. Suppose α_i is optimally set at a constant level. Then a dollar of additional aid would equate the current net marginal cost, $\kappa_i - g_i'(\alpha_i)$, with the present value of the future stream of marginal benefits, which is the right-hand side of equation (5). This stream arises from the fact that a dollar of aid will lead to an investment of ε, which yields a stream of future marginal enjoyment, $G_X = g_i(\alpha_i) - \alpha_i g_i'(\alpha_i)$, to the donor. Note that the numerator of the right-hand side of equation (5) is equal to $\varepsilon_i G_X$, and the denominator is the rate of discount ρ_i minus the net rate of return of the capital stock (the expression inside the square brackets). The equilibrium α_i is chosen to equate the current-period net marginal cost with the present-value of future marginal benefits.

In this sub-section, we focus on the case where all players (donor countries) have the same functional form for $g_i(.)$ and the same values of κ, ρ and ε. Let us restrict attention to symmetric equilibria, i.e., $\alpha_i = \alpha_j = \alpha^*$ for all i, j. Then we must look for a fixed point of the following equation

$$\kappa - g'(\alpha) = \frac{\varepsilon \left(g(\alpha) - \alpha g'(\alpha)\right)}{\rho - [sr + (n-1)\varepsilon\alpha - \delta]} \tag{6}$$

Since $\kappa - g'(\alpha) = \psi$ which is positive , and $g(\alpha) - \alpha g'(\alpha)$ is positive for all $\alpha \geq 0$, if equation (6) has a fixed point $\alpha^* > 0$, it must be the case that $\rho > (sr - \delta) + (n-1)\varepsilon\alpha^*$.

With $\dot{X}/X = (sr-\delta) + n\varepsilon\alpha^*$, the transversality conditions (3) are satisfied if $\rho > (sr - \delta) + n\varepsilon\alpha^*$. Thus we restrict our search to α that satisfies the condition

$$\alpha < \frac{\rho - (sr - \delta)}{n\varepsilon} \equiv \frac{\mu}{n\varepsilon} \equiv \frac{z}{n} \equiv \widetilde{\alpha} < \widehat{\alpha} \equiv \frac{z}{(n-1)} \tag{7}$$

Define α_κ as the solution of the equation

$$g'(\alpha) = \kappa \tag{8}$$

We are interested only in $\alpha > \alpha_\kappa$, because otherwise the shadow price ψ would be negative.

Assumption A3: *The marginal cost of public finance, κ, is sufficiently high, such that the following relationship between $\widetilde{\alpha}$ and α_κ(as defined by equations (7) and (8) respectively) is satisfied*

$$\alpha_\kappa < \widetilde{\alpha}$$

Now re-write equation (6) as

$$(\kappa - g'(\alpha))\,[z - (n-1)\alpha] = g(\alpha) - \alpha g'(\alpha) \tag{9}$$

where z is defined by equation (7).

Proposition 1: *Under assumptions A0 to A3, if $(\rho + \delta - sr)/\varepsilon$ is sufficiently great, there exists at least one symmetric Markov-perfect Nash equilibrium in which all donor countries use a linear strategy $A_i = \alpha^* X$, where $\alpha^* \in (\alpha_\kappa, \widetilde{\alpha})$.*

Proof: Consider equation (9). The left-hand side is the product of two terms. The first term is positive for all $\alpha > \alpha_\kappa$ and is zero at $\alpha = \alpha_\kappa$. The second term is positive for all $\alpha < \widehat{\alpha}$ and is zero at $\alpha = \widehat{\alpha}$. The left-hand side is equal to zero at $\alpha = \alpha_\kappa$ and also at $\widehat{\alpha}$. Over the interval $(\alpha_k, \widehat{\alpha})$, the left hand side is positive and is shaped like an inverted U, and the height of its graph is increasing in z. On the other hand, the right-hand side of (9) is always positive and is a decreasing function of α. It follows that if z is a sufficiently large positive number, the curve that represents the right-hand side will intersect the curve that represents the left-hand side at least twice over the interval $(\alpha_k, \widehat{\alpha})$, and at least one of these intersections is at some value $\alpha^* \in (\alpha_\kappa, \widetilde{\alpha})$.

Remark: If Assumption A3 is not satisfied, equilibria in linear strategies may not exist.

2.2.2 Multiplicity of Markov-perfect Nash equilibria in linear strategies

In this sub-section, we investigate further the possible multiplicity of Markov-perfect Nash equilibria in linear strategies. Note that we do not force countries to use only linear strategies. (Under certain assumptions, Long and Shimomura (1998) show that best replies to linear strategies are linear strategies). Let us specialize to the case where $n = 2$ and $g(\alpha) = (1/\beta)\alpha^\beta$ where $0 < \beta < 1$. Then

$$\alpha_\kappa = \left(\frac{1}{\kappa}\right)^{1/(1-\beta)}$$
$$\widetilde{\alpha} = \frac{z}{2} < \widehat{\alpha} = z \tag{10}$$

Assumption A3 implies

$$\left(\frac{1}{\kappa}\right)^{1/(1-\beta)} < \frac{z}{2} \tag{11}$$

Equation (5) becomes, for player 1,

$$\alpha_2 = z - \frac{1-\beta}{\beta}\left[\frac{\alpha_1}{\kappa\alpha_1^{1-\beta} - 1}\right] \equiv J(\alpha_1) \tag{12}$$

Equation (12) gives player 1's "best reply correspondence" $R_1(\alpha_2)$ to player 2's α_2, where $\alpha_2 \in [0, \widehat{\alpha}]$. Let us consider the graph of $J(\alpha_1)$, which is the inverse map of the correspondence $R_1(\alpha_2)$. In the space (α_1, α_2), as α_1 falls toward $(1/\kappa)^{1/(1-\beta)} \equiv \alpha_\kappa$, $J(\alpha_1)$ approaches minus infinity. For $\alpha_1 > \alpha_\kappa$, the function $J(\alpha_1)$ is single-peaked, and is strictly concave. Its derivative is

$$J'(\alpha_1) = -\left(\frac{1-\beta}{\beta}\right)\left[\frac{-1 + \beta\kappa\alpha_1^{1-\beta}}{\left(\kappa\alpha_1^{1-\beta} - 1\right)^2}\right]$$

Define $\overline{\alpha}$ as the value of α_1 at which $J(\alpha_1)$ attains its maximum:

$$\overline{\alpha} \equiv \left(\frac{1}{\beta\kappa}\right)^{1/(1-\beta)} > \alpha_\kappa \tag{13}$$

Then $J'(\alpha_1) > 0$ for $\alpha_1 \in (\alpha_\kappa, \overline{\alpha})$ and $J'(\alpha_1) < 0$ for $\alpha_1 > \overline{\alpha}$. At $\alpha_1 = \overline{\alpha}$, $J(\alpha_1)$ attains its maximum:

$$J(\overline{\alpha}) = z - \left(\frac{1-\beta}{\beta}\right)\left[\frac{\overline{\alpha}}{\frac{1}{\beta} - 1}\right] = z - \overline{\alpha} \tag{14}$$

If $z - \overline{\alpha}$ is sufficiently large, the curve $\alpha_2 = J(\alpha_1)$ intersects the line 45 degree line $\alpha_2 = \alpha_1$ at exactly two points, denoted by L and H, where at L, $(\alpha_1, \alpha_2) = (\alpha_L, \alpha_L)$, and at H, $(\alpha_1, \alpha_2) = (\alpha_H, \alpha_H)$. The values α_H and α_L are positive solutions of the equation

$$(\alpha - z)(\kappa\alpha^{1-\beta} - 1) + \left(\frac{1-\beta}{\beta}\right)\alpha = 0 \ , \ (\alpha > \alpha_\kappa). \tag{15}$$

The points L and H are potential symmetric equilibria[6]. Let us consider two cases. In case A, $J(\overline{\alpha}) < \overline{\alpha}$, so that the peak of the curve $\alpha_2 = J(\alpha_1)$ is below the 45 degree line $\alpha_2 = \alpha_1$. In case B, $J(\overline{\alpha}) > \overline{\alpha}$, so that the peak of the curve $\alpha_2 = J(\alpha_1)$ is above the 45 degree line $\alpha_2 = \alpha_1$.

Case A: $J(\overline{\alpha}) < \overline{\alpha}$

It follows from equation (14) that Case A applies if and only if $z < 2\overline{\alpha}$. There are two subcases. In subcase A1, $J(\overline{\alpha})$ is sufficiently close to $\overline{\alpha}$ to ensure that the curve $J(\alpha_1)$ has two intersections with the 45 degree line, at α_H and α_L where $\overline{\alpha} > \alpha_H > \alpha_L > \alpha_\kappa$. See Figure 1. For both α_H and α_L to be equilibrium values, we require $\alpha_H < \widetilde{\alpha} = z/2$. We will refer to the point $(\alpha_1, \alpha_2) = (\alpha_L, \alpha_L)$ as the "*low-aid equilibrium*" and to the point $(\alpha_1, \alpha_2) = (\alpha_H, \alpha_H)$ as the "*high aid equilibrium*".

[6] Strictly speaking, an equilibrium is a pair of strategies (feedback rules) that are best replies to each other. Here, since we are dealing with linear strategies, we can afford a slight abuse of words and refer to a vector (α_1, α_2) as a potential "equilibrium".

In subcase A2, $J(\overline{\alpha})$ is sufficiently close to zero, for example because z is sufficiently small, so that the whole curve $J(\alpha_1)$ lies below the 45 degree line for all $\alpha_1 > \alpha_\kappa$, and hence no Nash equilibrium (in linear strategies) exists. (See Figure 2).

PLEASE PLACE FIGURE 1 HERE

PLEASE PLACE FIGURE 2 HERE

Remark: One might be tempted to say that in Figure 1, the low-aid equilibrium $(\alpha_1, \alpha_2) = (\alpha_L, \alpha_L)$ is "stable" and the high-aid equilibrium $(\alpha_{1,}\alpha_2) = (\alpha_H, \alpha_H)$ as "unstable", by appealing to the Cournot-type analysis which is common in economic textbooks. However, such stability consideration is based on some sort of myopic adjustment process which has no place in game theory.

Case B: $J(\overline{\alpha}) > \overline{\alpha}$. This case applies if and only if $z > 2\overline{\alpha}$. Then the curve $\alpha_2 = J(\alpha_1)$ cuts the 45 degree line $\alpha_2 = \alpha_1$ at two values of α_1, say α_L and α_H where

$$\alpha_H > \overline{\alpha} > \alpha_L > \alpha_\kappa$$

Figure 3 illustrates Case B.

PLEASE PLACE FIGURE 3 HERE

Proposition 2: *In the case of two donor countries, under certain restrictions on parameter values, there exist two symmetric equilibria, one with low aid, and one with high aid. At both equilibria, each country uses a linear Markov-perfect strategy,* $A = \alpha X$.

2.2.3 Numerical examples

Example 1: We set $\beta = 1/2$, $\kappa = 1$, $sr - \delta = 0$, $\rho = 0.07$, and $\varepsilon = 0.01$.Then $\mu = 0.07$, $z = 7$, $\overline{\alpha} = 4/\kappa^2 = 4$, $\alpha_\kappa = 1 < \widetilde{\alpha} = 3.5$, and $J(\overline{\alpha}) = 7 - 4 < \overline{\alpha}$. (Thus we are in Case A). Equation (15) has two positive roots, $\alpha_L = 1.8412$ and $\alpha_H = 2.8629$. (This is subcase A1).

The growth rate of the capital stock of the recipient country at the low-aid equilibrium is

$$\pi_L = (sr - \delta) + 2\varepsilon\alpha_L = 0.02\alpha_L = 0.036824$$

and that at the high-aid equilibrium is $\pi_H = 0.02\alpha_H = 0.057258$. The welfare of the donor at the low-aid equilibrium is

$$W_L = \int_0^\infty \exp(-\rho t) \left[Xg(\alpha_L) - \kappa X \alpha_L \right] dt = X(0) \frac{g(\alpha_L) - \kappa\alpha_L}{\rho - \pi} =$$

$$\frac{2\sqrt{1.8412} - 1.8412}{0.07 - 0.036824} X(0) = 26.303 X(0)$$

The donor's welfare at the high-aid equilibrium is

$$W_H = \frac{2\sqrt{2.8629} - 2.8629}{0.07 - 0.057258} X(0) = 40.898 X(0)$$

Example 2: We set $\beta = 1/2$. Set $\kappa = 1$ and $z = 6$. Then $\alpha_\kappa = 1 < \widetilde{\alpha} = 3$,and $J(\overline{\alpha}) = 6 - 4 < \overline{\alpha}$. (Thus we are in Case A).Equation (15) has no positive roots. (This is subcase A2). Thus there is no Markov-perfect Nash equilibrium in linear strategies.

2.2.4 Properties of the low aid equilibrium and the high aid equilibrium

We will refrain from using the words "comparative statics." But it is still meaningful to ask: does a lower degree of corruption in the recipient country (a higher ε) result in a higher position of the low aid equilibrium (along the 45 degree line) and a lower position of the high aid equilibrium? The answer is in the affirmative. To prove this, observe that a higher ε implies a lower z, which means that the curve $J(\alpha_1)$ is shifted downwards, therefore α_L takes on a higher value than before, and α_H takes on a lower value than before.

Proposition 3: *A lower degree of corruption in the recipient country is associated with a higher low-aid equilibrium, and with a lower high-aid equilibrium.*

Example 3: Modify Example 1, by reducing z from 7 to 6.76. This may result from $sr - \delta = 0$, $\rho = 0.07$, $\mu = 0.07$ and $\varepsilon = \mu/6.76 = 0.01355$, which means the corruption coefficient $(1 - \varepsilon)$, is lower than that of Example 1. Then $\alpha_\kappa = 1 < \widetilde{\alpha} = 3.38$, and $J(\overline{\alpha}) = 6.76 - 4 < \overline{\alpha}$. (Thus we are in Case A). Equation (15) has two positive roots, $\alpha_L = 2.1538 > 1.8412$ and $\alpha_H = 2.3541 < 2.8629$.

The growth rate at the low-aid equilibrium is now $\pi_L = 2\varepsilon\alpha_L = 0.058368$ and that at the high-aid equilibrium is $\pi_H = 0.063796$, which is higher than that of example 1. While the recipient country receives a lower α_H than that of Example 1, its growth rate is higher, because less aid is siphoned off by corrupt officials.

The welfare of each donor country at the low-aid equilibrium is

$$W_L = \frac{2\sqrt{2.1538} - 2.1538}{0.07 - 0.058368} X(0) = 67.174 X(0)$$

This is greater than in example 1. The donor disburses more, and the numerator of the expression for W_L is smaller than in Example 1, but the denominator is also smaller, resulting in higher welfare.

The welfare of each donor country at the high-aid equilibrium is

$$W_H = \frac{2\sqrt{2.3541} - 2.3541}{0.07 - 0.063796} X(0) = 115.17 X(0)$$

which is again higher than the corresponding one in Example 1.

2.3 Heterogeneous Donor Countries

Let us relax the assumption that donor countries are identical. Suppose there are two donor countries, 1 and 2, with distinct parameter values. The reaction correspondence of country 1 is given implicitly by the equation

$$\kappa_1 - g'(\alpha_1) = \frac{\varepsilon_1 \left[g(\alpha_1) - \alpha_1 g'(\alpha_1)\right]}{\rho_1 - (sr - \delta) - \varepsilon_2 \alpha_2} \equiv \frac{\varepsilon_1 \left[g(\alpha_1) - \alpha_1 g'(\alpha_1)\right]}{\mu_1 - \varepsilon_2 \alpha_2}$$

and that of country 2 by

$$\kappa_2 - g'(\alpha_2) = \frac{\varepsilon_2 \left[g(\alpha_2) - \alpha_2 g'(\alpha_2)\right]}{\rho_2 - (sr - \delta) - \varepsilon_1 \alpha_1} \equiv \frac{\varepsilon_2 \left[g(\alpha_2) - \alpha_2 g'(\alpha_2)\right]}{\mu_2 - \varepsilon_1 \alpha_1}$$

Take the special case where $g(\alpha) = (1/\beta)\alpha^\beta$. Then country 1's reaction correspondence $R_1(\alpha_2)$ is

$$\alpha_2 = \frac{\mu_1}{\varepsilon_2} - \frac{\varepsilon_1(1-\beta)}{\varepsilon_2 \beta}\left[\frac{\alpha_1}{\kappa_1 \alpha_1^{1-\beta} - 1}\right] \equiv J_1(\alpha_1)$$

A similar equation applies to country 2. If the differences between ρ_1 and ρ_2, ε_1 and ε_2, and κ_1 and κ_2, are not too great, we will have equilibria that are very close to the symmetric equilibria reported in the preceding section. For example, if $\kappa_1 > \kappa_2$ while other parameters are the same, the curve $\alpha_2 = J_1(\alpha_1)$ will shift up, which means that at both the low-aid equilibrium and the high-aid equilibrium, country 1 gives less than country 2, i.e., $\alpha_{L1} < \alpha_{L2}$, and $\alpha_{H2} < \alpha_{H1}$.

Example 4: This is a slight departure from Example 1. We increase κ_1 from 1 to 1.01, while keepng κ_2 at the same level, 1. We set $\beta = 1/2$, $\kappa_1 = 1.01, \kappa_2 = 1, sr - \delta = 0$, $\rho_1 = \rho_2 = 0.07$, and $\varepsilon_1 = \varepsilon_2 = 0.01$. We require $\alpha_i \geq \alpha_{\kappa_i}$ where

$$\alpha_{\kappa_1} = \left(\frac{1}{\kappa_1}\right)^2 = 0.980\,30$$

$$\alpha_{\kappa_2} = \left(\frac{1}{\kappa_2}\right)^2 = 1$$

Here we have a system of two equations

$$(\alpha_2 - 7)(1.01\sqrt{\alpha_1} - 1) + \alpha_1 = 0$$

$$(\alpha_1 - 7)(\sqrt{\alpha_2} - 1) + \alpha_2 = 0$$

There are two admissible equilibria, $(\alpha_{1L}, \alpha_{2L}) = (1.7554, 1.8085)$ and $(\alpha_{1H}, \alpha_{2H}) = (2.8944, 2.9705)$. As expected, the country with higher cost of public finance gives less aid than the other country, at both the low-aid equilibrium and the high-aid equilibrium. At the high-aid equilibrium, both countries give more than in Example 1.

Example 5 (mean-preserving spread): This is another slight departure from Example 1. We increase κ_1 to 1.01 and decrease κ_2 to 0.99, so that $\kappa_1 + \kappa_2 = 2$ as in Example 1.

Here we have a system of two equations

$$(\alpha_2 - 7)(1.01\sqrt{\alpha_1} - 1) + \alpha_1 = 0$$

$$(\alpha_1 - 7)(0.99\sqrt{\alpha_2} - 1) + \alpha_2 = 0$$

There are two admissible equilibria, $(\alpha_{1L}, \alpha_{2L}) = (1.7881, 1.8997)$ and $(\alpha_{1H}, \alpha_{2H}) = (2.7839, 2.9371)$. Here, the sum of aids at the low-aid equilibrium is $\alpha_{1L} + \alpha_{2L} = 3.6878 > 3.6824$. At the high-aid equilibrium, the sum of aids is $5.721 < 5.7258$. Thus a mean-preserving spread of the κ_i increases the sum of aids at the low-aid equilibrium, but decreases the sum of aids at the high-aid equilibrium.

2.4 An extension: status-conscious donors

So far, we have assumed that a donor country's benefits from aid giving depend only on its aid amount, and on the stock of capital of the recipient. It may be argued that donor countries may compare aid levels, and derive satisfaction from being a more generous giver than the world average. In the two-donor countries case, this consideration might be captured by specifying that the gross benefit function of donor i is no longer the function $G(A_i, X)$ but is rather a new function $\widehat{G}(A_i, A_j, X)$ where

$$\widehat{G}(A_i, A_j, X) = \left(\frac{A_i}{A_j}\right)^\gamma G(A_i, X) \text{ with } 0 < \gamma < 1$$

where, as before, the function $G(A_i, X)$ is concave, increasing, and homogeneous of degree one in (A_i, X). Furthermore, we assume that $\widehat{G}(A_i, A_j, X)$ is strictly concave in A_i.

In this case, assuming that country j uses a stationary linear feedback strategy $A_j = \alpha_j X$, the Hamiltonian of country i is

$$H_i = \left(\frac{A_i}{\alpha_j X}\right)^\gamma G(A_i, X) - \kappa_i A_i + \psi_i \left[(sr - \delta + \varepsilon_j \alpha_j)X + \varepsilon_i A_i\right]$$

The necessary conditions are

$$\frac{\partial H_i}{\partial A_i} = \left(\frac{A_i}{\alpha_j X}\right)^\gamma G_{A_i}(A_i, X) + \gamma A_i^{-1} \left(\frac{A_i}{\alpha_j X}\right)^\gamma G(A_i, X) - \kappa_i + \varepsilon_i \psi_i = 0$$

$$\dot{\psi}_i = \psi_i(\delta + \rho - sr - \varepsilon_j \alpha_j) - \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma G_X(A_i, X) + \gamma \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma X^{-1} G(A_i, X)$$

Let $\alpha_i = A_i/X$. Then $G_{A_i}(A_i, X) = g'(\alpha_i)$ and $G_X(A_i, X) = g(\alpha_i) - \alpha_i g'(\alpha_i) > 0$. The necessary conditions become

$$\left(\frac{\alpha_i}{\alpha_j}\right)^\gamma g'(\alpha_i) + \gamma \alpha_i^{-1} \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma g(\alpha_i) - \kappa_i + \varepsilon \psi_i = 0$$

$$\dot{\psi}_i = \psi_i(\delta + \rho - sr - \varepsilon_j \alpha_j) - \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma [g(\alpha_i) - \alpha_i g'(\alpha_i)] + \gamma \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma g(\alpha_i)$$

Again, let us try a linear strategy $A_i = \alpha_i X$, where α_i is a positive constant. Then

$$\psi_i = \frac{1}{\varepsilon_i} \left[\kappa_i - \left(\frac{\alpha_i}{\alpha_j}\right)^\gamma \left(g'(\alpha_i) - \gamma \alpha_i^{-1} g(\alpha_i)\right)\right]$$

which implies $\dot{\psi}_i = 0$ and thus

$$\psi_i = \frac{\left(\frac{\alpha_i}{\alpha_j}\right)^{\gamma} [g(\alpha_i)(1-\gamma) - \alpha_i g'(\alpha_i)]}{\delta + \rho - sr - \varepsilon_j \alpha_j}$$

For simplicity, consider the case of identical donor countries, and restrict attention to symmetric equilibria. Then we must find fixed points of the following equation:

$$\left[\kappa - g'(\alpha) + \gamma\alpha^{-1} g(\alpha)\right](z - \alpha) = g(\alpha)(1-\gamma) - \alpha g'(\alpha) \tag{16}$$

where $z \equiv (\rho - sr + \delta)/\varepsilon > 0$.

Take the case $g(\alpha) = (1/\beta)\alpha^{\beta}$. Strict concavity in α_i requires $\beta + \gamma < 1$. Then equation (16) becomes

$$\left[\kappa\alpha^{1-\beta} - \left(1 + \frac{\gamma}{\beta}\right)\right](z - \alpha) = \left(\frac{1-\beta-\gamma}{\beta}\right)\alpha \tag{17}$$

For an admissible equilibrium, we require a positive shadow price, which implies that

$$\alpha > \alpha_{\min} \equiv \left(\frac{1 + (\gamma/\beta)}{\kappa}\right)^{1/(1-\beta)}$$

and, since $1 - \beta - \gamma > 0$, we must have $\alpha < z$. If γ is sufficiently small, and z sufficiently large, the equation (17) has two fixed-points. To consider a numerical example, let $\beta = 1/2$ and $\gamma/\beta \equiv \omega$. Here ω is an index of the extent of "status-consciousness" of the donors. We then have the equation

$$\alpha^{1.5} + \frac{(2\omega)}{\kappa}\alpha - \frac{z}{\kappa}\alpha^{0.5} + (1+\omega)z = 0$$

Let us define $y = \sqrt{\alpha}$. Consider the polynomial

$$p(y) = y^3 + \frac{(2\omega)}{\kappa}y^2 - \frac{z}{\kappa}y + z$$

Since $p(0) > 0$ and $p(-\infty) < 0$, there is a negative root. Since $p(\infty) = \infty$, the remaining two roots are real if and only if there is some point $\overline{y} > 0$ such that $p(\overline{y}) \leq 0$. This happens if z is sufficiently large. Then we have two positive real roots y_L and y_H, and we can compute $\alpha_L = y_L^2$ and $\alpha_H = y_H^2$. Provided that $\alpha_{\min} < \alpha_L < \alpha_H < z/2$, these roots are the equilibria we are looking for.

It is clear from the properties of the polynomial $p(y)$ that, for $0 \leq \omega$, a small increase in ω will lead to a small increase in α_L and a small decrease in α_H. It follows that, for this example, *the higher is the extent of status-consciousness of the donors, the greater is the sum of aids at the low-aid equilibrium, and the smaller is the sum of aids at the high-aid equilibrium.*

3 Model 2: Foreign Aid with a Development Target

There are 2 donor countries, and one recipient country. To keep things simple, the only state variable in our model is the "level of development" of the recipient country, which we denote by $X(t)$. Assume that when $X = \widehat{X}$, the recipient country can take off and achieve sustained growth without help from abroad. The donor countries want the recipient to achieve the target $\widehat{X}$, and the game ends when this target is reached.

Let $A_i(t)$ be the flow of aid from donor country i. We assume that there is an upper bound on aid, so that $0 \leq A_i(t) \leq \overline{A}$. Without loss of generality, we normalize $\overline{A} = 1$.

Starting from any $X < \widehat{X}$, the level of development $X(t)$ evolves according to the following dynamic law

$$\dot{X} = \beta_1(X)A_1 + \beta_2(X)A_2 + \omega(X)A_1A_2 - \delta(X) \text{ for } 0 \leq X < \widehat{X}$$

where $\beta_i(X) > 0$ is the effectiveness of country i's aid. The term $\omega(X) \geq 0$ represents the interactive effect of the two flows of aids. The function $\delta(X)$

represents the depreciation of X. All the functions $\beta_i(.), \omega(.)$ and $\delta(.)$ are differentiable, and bounded, for all $X \in \left[0, \widehat{X}\right]$.

If country i gives the maximum level of aid, i.e., $A_i = 1$, while country j gives nothing, the above dynamic equation becomes

$$\dot{X} = \beta_i(X) - \delta(X)$$

We assume that, $\beta_i(X) - \delta(X) > 0$ for all $X \geq 0$. This means that even if one donor gives no aid, the recipient's level of development will grow, provided that the other donor gives the maximum aid. This assumption implies that

$$0 < \frac{\delta(X)}{\beta_i(X)} < 1 \tag{18}$$

Let us turn to the objective of the donors. We assume that, according to the rules of the game, both donors terminate the aid program whenever the level of development reaches the take-off level $\widehat{X}$. Let T denote the time at which X reaches $\widehat{X}$. The payoff of donor i is assumed to be

$$J_i = K_i(X(T)) - \int_0^T c_i A_i(t) dt$$

where $c_i > 0$ is the cost per unit of aid, and $K_i(.)$ is the psychological reward at the end of the program. We normalize this function, so that $K_i(0) = 0$ and $K_i(\widehat{X}) = \widehat{X}$. Each donor country maximizes its payoff J_i, subject to the dynamic law and the target $X(T) = \widehat{X}$, and given that the other donor uses a feedback strategy $A_j(t) = \phi_j(X(t))$. We look for a Nash equilibrium in feedback strategies.

It is useful to define

$$f_1(X, A_1) \equiv \beta_1(X)A_1 + \beta_2(X)\phi_2(X) + \omega(X)A_1\phi_2(X) - \delta(X)$$

$$f_2(X, A_2) \equiv \beta_1(X)\phi_1(X) + \beta_2(X)A_2 + \omega(X)A_2\phi_1(X) - \delta(X)$$

Let $V_i(X)$ denote the value function of donor i. The solution of donor 1's problem must satisfy the following Hamilton-Jacobi-Bellman equation:

$$\begin{cases} V_1(X) = K_1(X) & \text{if } X = \widehat{X} \\ 0 = \max_{A_1}\left[-c_1 A_1 + V_1'(X) f_1(X, A_1)\right] & \text{if } 0 \leq X < \widehat{X} \end{cases}. \tag{19a}$$

where $0 \leq A_1(t) \leq 1$. (Note that the second part of the equation is equivalent to the condition that the Hamiltonian is equal to zero, a condition that follows from the fact that each donor is solving a free-time problem, and the discount rate is zero.)

Similarly, for donor 2,

$$\begin{cases} V_2(X) = K_2(X) & \text{if } X = \widehat{X} \\ 0 = \max_{A_2}\left[-c_2 A_2 + V_2'(X) f_2(X, A_2)\right] & \text{if } 0 \leq X < \widehat{X} \end{cases}. \tag{20}$$

Proposition 4: *The following pair of strategies constitutes a Markov perfect Nash equilibrium:*

$$\phi_i(X) = \frac{\delta(X)}{\beta_i(X)},\ i = 1, 2 \tag{21}$$

and the value function of donor country i is

$$V_i(X) = K_i\left(\widehat{X}\right) - \int_X^{\widehat{X}} \frac{\beta_j(x) c_i}{\beta_1(x)\beta_2(x) + \omega(x)\delta(x)} dx,\ i = 1, 2 \tag{22}$$

Proof:

It is straightforward to verify that the value functions given by equation (22) has the derivative

$$V_i'(X) = \frac{\beta_j(X) c_i}{\beta_i(X)\beta_j(X) + \omega(X)\delta(X)} \tag{23}$$

Substituting $\phi_j(.)$, as given by (21), and $V_i'(.)$, as given by (23), into the Hamilton-Jacobi-Bellman equation for donor i, it is easy to see that $A_i = \phi_i(X)$ is indeed an optimal control.

Remark:

(i) The strategy (21) is, in general, non-linear in X. For example, consider the following specification of $\widehat{X}$, $\delta(.)$ and $\beta_i(.)$:

$$\widehat{X} = 1$$

$$\delta(X) = 1 - \exp\left[X - \widehat{X}\right] \text{ for all } X \in \left[0, \widehat{X}\right]$$

$$\beta_i(X) = \alpha_i + \delta(X)$$

where $\alpha_i > 0$. Then it is easy to see that $\phi_i'(X) < 0$ and $\phi_i''(X) < 0$, that is, as the recipient country's level of development grows, aid from each donor falls at a faster and faster rate.

(ii) The equilibrium growth rate of the stock X is

$$\begin{aligned} \dot{X} &= \beta_1(X)A_1^* + \beta_2(X)A_2^* + \omega(X)A_1^*A_2^* - \delta(X) \\ &= \delta(X) + \omega(X)\frac{[\delta(X)]^2}{\beta_1(X)\beta_2(X)} \end{aligned} \tag{24}$$

until $\widehat{X}$ is attained.

(iii) An increase in corruption can be represented as a fall in β_i. It follows from (21) and (24) that the higher is the level of corruption, the greater is the flow of aid, and the greater is the growth rate of the stock X (unless $\omega(X) = 0$, in which the growth rate of the stock is independent of the level of corruption.)

4 Concluding Remarks

We have shown that non-cooperative games of foreign aid between donor countries may have multiple Markov-perfect Nash equilibria. These equilibria are Pareto rankable, implying the possibility of co-ordination failures, where the inferior equilibrium may be picked. Even if coordination failures can be avoided, i.e., the high aid equilibrium is picked, this is still inferior to full cooperation (maximization of joint welfare by determining the aid amounts collectively). A higher degree of corruption can lead to more aid in both models.

We have restricted consideration to international transfers. Other policies may be preferable. For example, if the choice is between more aid or more trade (i.e., lifting barriers to trade), it may turn out that the world would be better off with more trade. This point has been raised in Kemp and Shimomura (1991). Another alternative to foreign aid is the relaxation of immigration law. For example, as Lance Pritchett (2006) pointed out, "the industrial world currently transfers something on the order of $70 billion a year in overseas development assistance...A recent World Bank study has estimated the benefits of the rich countries allowing just a 3 percent rise in their labour force through relaxing restrictions. The gains from even this modest increase to poor-country citizens are $300 billion...The current rich-country residents (also) benefit from this relaxation."

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References

[1] Alesina, A and B. Weder, 2002, Do corrupt governments receive less foreign aid? *American Economic Review*, Vol. 92(4), 1120-1137.

[2] Clemhout, S. and H. Wan Jr., 1994, Differential games: economic applications, in R. J. Aumann and S. Hart (eds.), *Handbook of Game Theory with Economic Applications,* Volume 2, Elsevier, Amsterdam,

[3] Dockner, E., S. Jorgensen, N.V. Long, and G. Sorger, 2000, *Differential Games in Economics and Management Science*, Cambridge University Press, Cambridge, UK.

[4] Kemp, M. C., N.V. Long, and K. Shimomura, 1993, Cyclical and non-cyclical redistributive taxation, *International Economic Review* 34, 415-429.

[5] Kemp, M. C., N.V. Long, and K. Shimomura, 1992, A dynamic formulation of the foreign aid process, in G. Feichtinger, ed., *Dynamic Economic Models and Optimal Control*, North- Holland, Amsterdam, 191-203.

[6] Kemp, M. C., N.V. Long, and K. Shimomura, 2001, A Differential Game Model of Tariff War, *Japan and the World Economy,* 13 (3), 2001, 279-298

[7] Kemp, M. C. and K. Shimomura, 1991, Trade or Aid?, in A.Takayama, M.Ohyama and H.Ohta eds., *Trade, Policy, and International Adjustments*, 19-35, Academic Press, NY.

[8] Kemp, M. C. and K. Shimomura, 2002, A theory of voluntary unrequited international transfers, *Japanese Economic Review* 53, 290-300.

[9] Kemp, M. C. and K. Shimomura, 2003, A theory of involuntary unrequited transfers, *Journal of Political Economy* 111, 686-692.

[10] Lane, P., and A.Tornell, 1996, Power, growth, and the voracity effect, *Journal of Economic Growth* 1, 213-241.

[11] Long N. V. and K. Shimomura, 1998, Some results on the Markov equilibria of a class of homogeneous differential games, *Journal of Economic Behavior and Organization* 33, 557-566.

[12] Long, N. V., K. Shimomura, and H. Takahashi, 1999, Comparing open-loop with Markov equilibria in a class of differential games, *Japanese Economic Review* 50, 457-469.

[13] Long, N. V. and G. Sorger, 2006, Insecure property rights and growth: the role of appropriation costs, wealth effects, and heterogeneity, *Economic Theory* 28(3), 513-529.

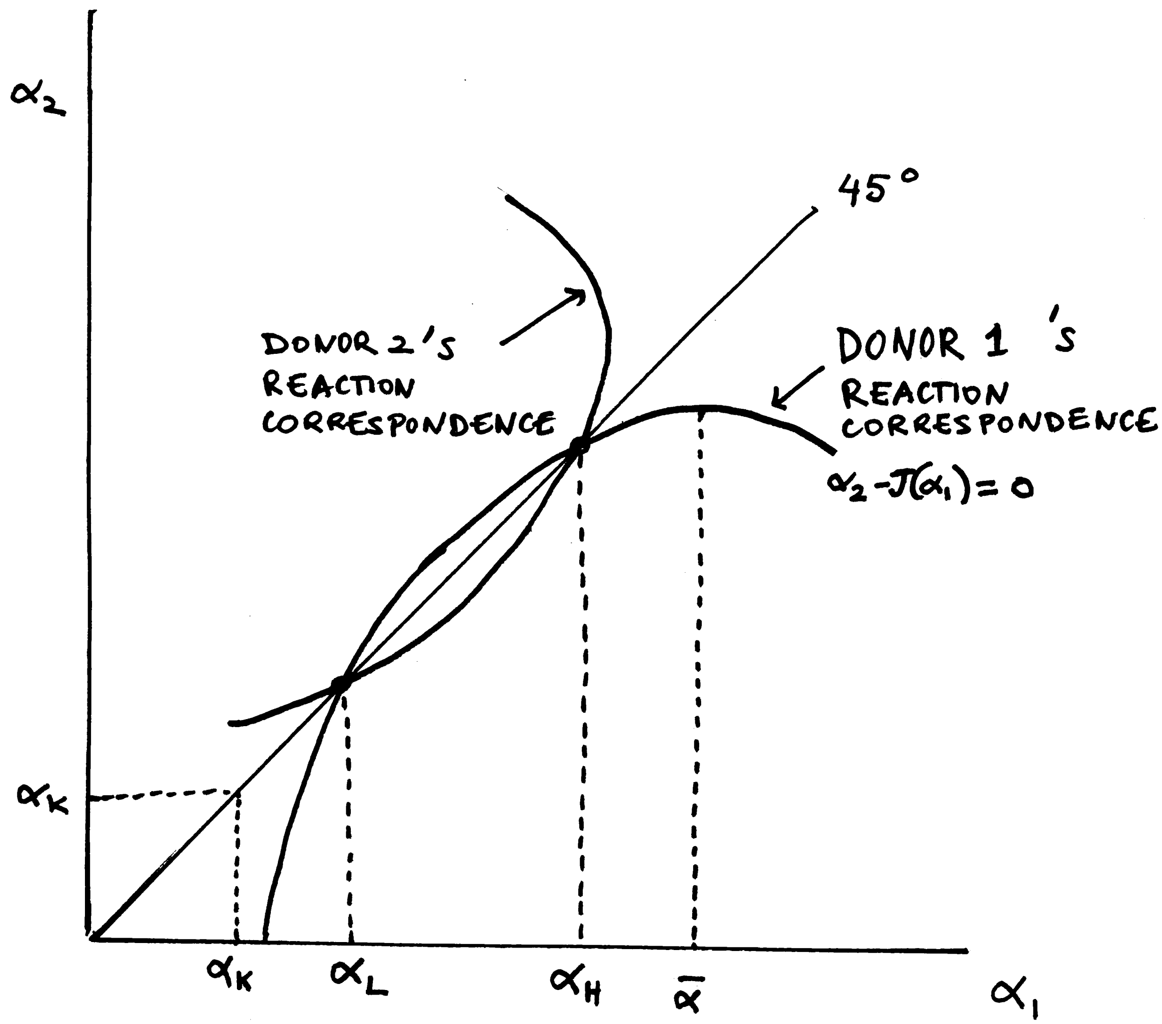
[14] Maskin, E. and J. Tirole, 2001, Markov-Perfect Equilibrium I: Observable Actions, *Journal of Economic Theory*, 100, 191-219.

[15] Pritchett, L., 2006, *Let Their People Come: Breaking the Gridlock on Global Labor Mobility*, Center for Global Development, 151pp. (www.cgdev.org/content/publications/detail/10174)

[16] Shimomura, K., 1991, Feedback equilibria in a differential game of capitalism, *Journal of Economic Dynamics and Control* 15, 317-338.

[17] Tornell, A., and P. Lane, 1999, The voracity effect, *American Economic Review* 89, 22-46.

[18] Tornell, A., and A.Velasco, 1992, The tragedy of the commons and economic growth: why does capital flow from poor to rich countries?, *Journal of Political Economy* 100, 1208-1231



α_2
45°
DONOR 2'S REACTION CORRESPONDENCE
DONOR 1 'S REACTION CORRESPONDENCE
$\alpha_2 - J(\alpha_1) = 0$
α_K
α_K
α_L
α_H
$\bar{\alpha}$
α_1

FIGURE 1

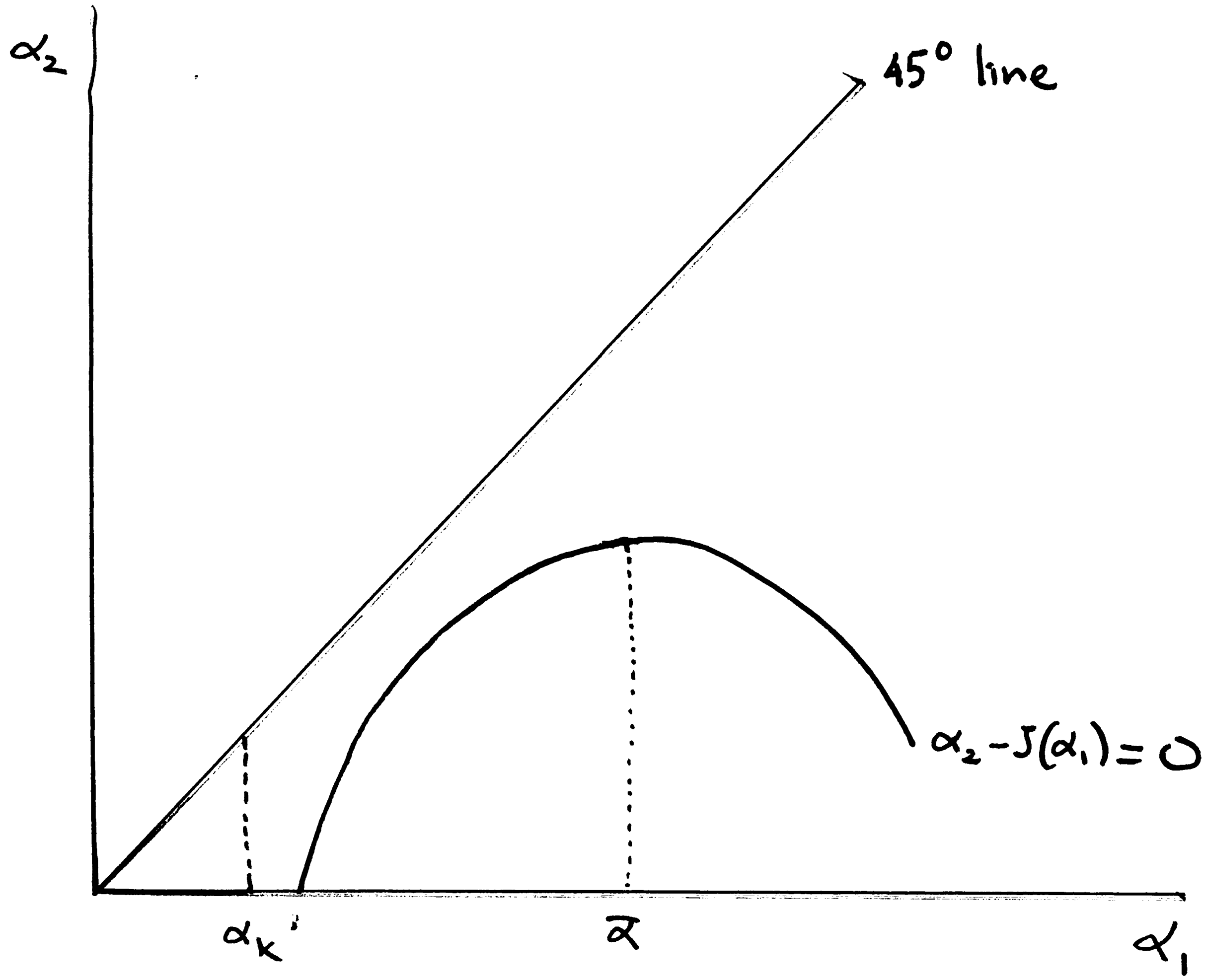
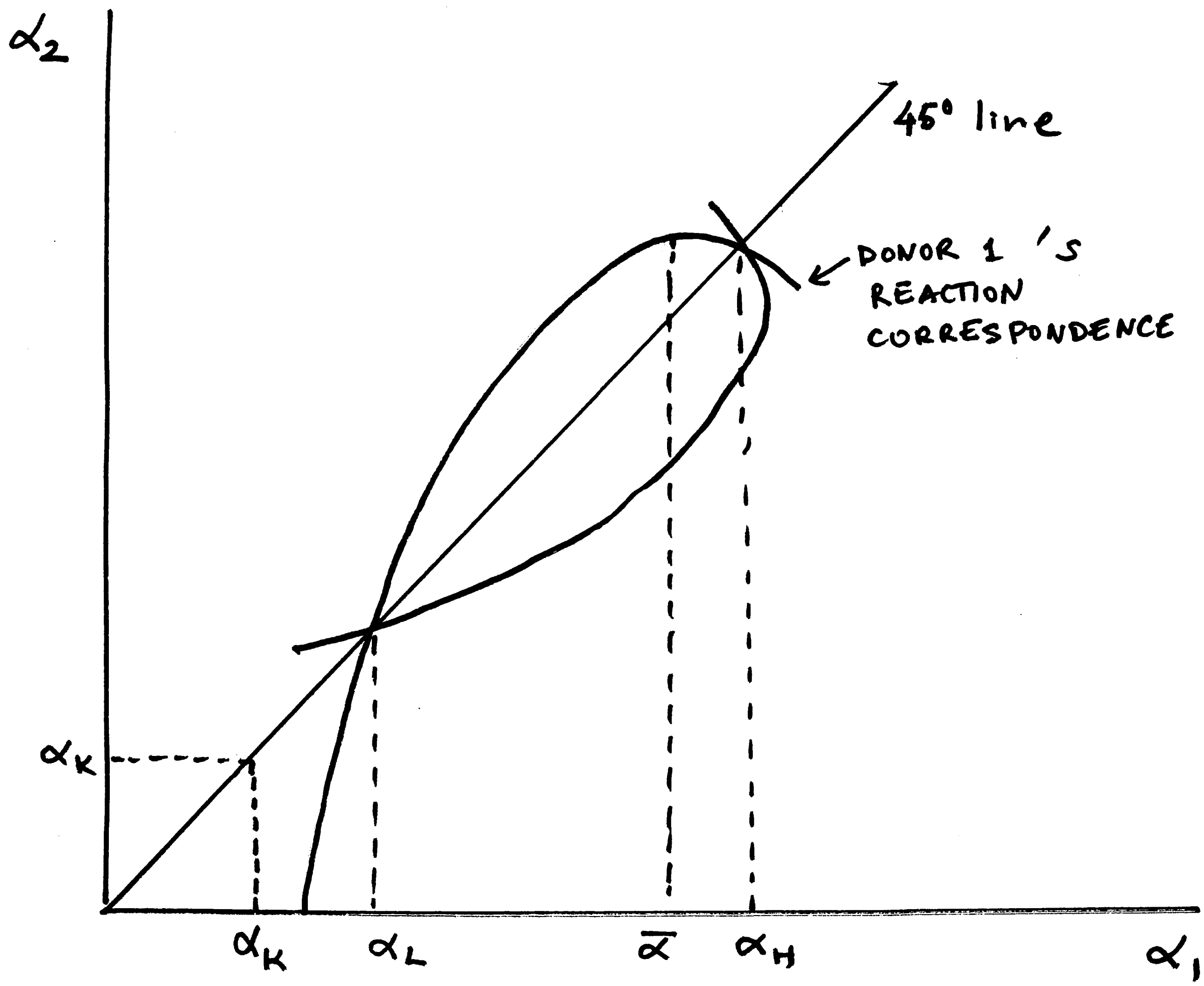


FIGURE 2



α_2
45° line
DONOR 1 'S REACTION CORRESPONDENCE
α_K
α_K
α_L
$\bar{\alpha}$
α_H
α_1

FIGURE 3