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Relative Wealth, Status Seeking, and Catching Up

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Résumé / Abstract

Nous démontrons que si l'utilité est une fonction de la richesse relative, peut-être à cause de la recherche du standing, alors, sous certaines hypothèses concernant la courbature de la fonction d'utilité et de la fonction de production, les gens pauvres peuvent rattraper les gens riches. Nous donnons des conditions suffisantes pour que la distribution de richesse finale soit indépendante de la distribution initiale, ainsi que les conditions suffisantes pour la stabilité au sens du point de selle.

We show that, if relative wealth appears in the utility function, for example due to status seeking, then under certain conditions on the curvature of the utility function and the production function, the poor will eventually catch up with the rich. We give sufficient conditions for the final distribution of wealth to be independent of the initial distribution, and conditions for saddlepoint stability in a two-class model.

Mots Clés: La recherche du standing, la richesse relative, le rattrapage

Key Words: *Status seeking, relative wealth, catching-up*

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1. Introduction

Many economists have pointed out that it is not wealth *per se* that is wanted; rather wealth (in the sense of relative wealth) is valued because it gives access to non-market goods such as status and influence. The relationship between relative wealth and non-market goods was aptly expressed in Adam Smith's 'The Theory of Moral Sentiments':

"To what purpose is all the toil and bustle of the world?...It is our vanity that urges us on...It is not wealth that men desire, but the consideration and good opinion that wait upon riches" .[1]

Since the non-market goods are arguments in the direct utility function of economic agents, wealth necessarily appears in their *reduced-form utility function*[2]. The purpose of our paper is to demonstrate how the recognition of relative wealth in the reduced-form utility function can explain a number of phenomena, such as poor families'catching-up with richer families.

Our paper is related to the coruscating articles by Akerlof (1976), Cole et al. (1992), and Konrad (1992). Akerlof argues that status seeking results in the "Rat Race" which reduces the welfare of all participants. Cole et al. show how wealth may appear in the reduced form utility function. Konrad considers a model with two classes of agents: those who care about relative wealth, and those who care only about their consumption. He shows that the latter may benefit from the existence of the former. He also shows that society as a whole may over-accumulate capital. Our paper contains a new contribution: we provide an analysis of conditions under which poor individuals (or countries) will be able to catch up with those who are initially richer. The question of catching up was studied by Stiglitz (1969) under the asssumption that individuals do not save optimally. Stiglitz showed

[1] Quoted by Cole et al. (1992, p. 1092). They also quote: "The boy with the cold hard cash is always Mister Right because we are living in the material world and I am a material girl." [Madonna, 'Material Girl'].

[2] For an interesting model leading to reduced form preferences, see Cole et al. (1992)

that if all individuals save a constant fraction of their income, then eventually the poor will catch up with the rich. Kemp and Shimomura (1992) demonstrated that catching up will not occur if individuals save optimally (and care only about their consumption). In this paper, we show that if individuals care enough about their relative wealth, then catching up will take place under optimal saving.

2. The Model

2.1. Assumptions and Notation

Individuals differ only in their initial wealth. Each supplies one unit of labor.The measure of the set of all individuals is normalized to unity. There are two groups of individuals: those who are initially poor, and those who are initially rich. Their measures are α_1 and α_2; $\alpha_1 + \alpha_2 = 1$. The initial capital stock of a poor individual is $k_1(0)$ and that of a rich one is $k_2(0) > k_1(0)$. The question that interests us is whether the poor will catch up with the rich in the long run.

Let c_i denote the consumption level of type i individual, and k_i his wealth. Per capita wealth is $k = \alpha_1 k_1 + \alpha_2 k_2$. Define relative wealth of i as $z_i = k_i/k$. The utility function of i is $U(c_i, z_i) = u(c_i) + v(z_i)$, where $u(.)$ and $v(.)$ are strictly concave and increasing. The strict concavity of $v(.)$ means that a poor person gets more pleasure from a marginal increase in his relative wealth than a rich person. This provides a strong incentive for the poor to accumulate.

The agregate production function is $y = f(k)$. It has the usual neoclassical properties. Let c denote per capita consumption. Then $\dot{k} = f(k) - c$. In a competitive equilibrium, the rental rate is given by $R = f'(k)$ and the wage rate is $W = f(k) - kf'(k)$.The rate of interest r is equal to the rental rate, because there is no depreciation. Individuals take the paths of $k(t)$, $W(t)$, $R(t)$ as given. Individual i solves

$$\max_{c_i(t)} \int_0^\infty \left[u(c_i(t)) + v\left(\frac{k_i(t)}{k(t)}\right)\right] e^{-\rho t} dt \qquad (1)$$

subject to $k_i(0) = k_{i0}$, $\dot{k}_i(t) = R(t)k_i(t) + W(t) - c_i(t)$, and

$$\lim_{t\to\infty} k_i(t) \exp\left[-\int_0^t r(s)ds\right] = 0 \tag{2}$$

The Hamiltonian for the optimization problem of individual i is

$$H = u(c_i) + v\left[k_i/k\right] + \psi\left[rk_i + W - c_i\right]$$

Hence we get the Euler equation

$$\frac{1}{\beta(c_i)}\frac{\dot{c}_i}{c_i} = [\rho - r(t)] - \frac{1}{u'(c_i)k}\frac{dv}{dz_i} \tag{3}$$

where $\beta(c_i)$ is the inverse of the elasticity of marginal utility of consumption:

$$\beta(c_i) \equiv \frac{1}{\sigma(c_i)} \equiv -\frac{u'(c_i)}{c_i u''(c_i)} > 0 \tag{4}$$

Since $r = f'(k)$, and $W = f(k) - kf'(k)$, we have the following system of four differential equations:

$$\dot{k}_i = rk_i - c_i + W = f'(k)k_i - c_i + f(k) - kf'(k) \text{ for } i = 1, 2 \tag{5}$$

$$\dot{c}_i = c_i\beta(c_i)\left[f'(k) - \rho + (1/k)\frac{v'(z_i)}{u'(c_i)}\right] \text{ for } i = 1, 2. \tag{6}$$

2.2. Steady state

It is useful to define the capital share and the elasticity of marginal utility of relative wealth

$$\gamma(k) = kf'(k)/f(k) \text{ and } \eta(z) = -zv''(z)/v'(z) \tag{7}$$

If there is a symmetric steady state with $k_1^* = k_2^* = k^*$ and $c_1^* = c_2^* = f(k^*)$, then k^* satisfies

$$f'(k^*) - \rho + \frac{v'(1)}{k^* u'(f(k^*))} = 0 \tag{8}$$

Let us define the function $\phi(k) \equiv ku'(f(k))\left[f'(k) - \rho\right]$.Then

Proposition 1:There exists a unique symmetric steady state $k^* > 0$ if (i) $\phi(0) + v'(1) < 0$, $\phi(\infty) + v'(1) > 0$ and (ii) for all $k \geq 0$,

$$\beta(f(k)) \geq \gamma(k) \tag{9}$$

Proof: Since $\phi(k)$ is continuous, condition (i) ensures the existence of a $k^* > 0$ that satisfies (8). Condition (ii) ensures that $\phi'(k) > 0$ and thus k^* is unique. To show that $\phi'(k) > 0$, we compute

$$k\frac{d\ln\phi(k)}{dk} = 1 - \frac{\gamma(k)}{\beta(f(k))} + \frac{kf''(k)}{f'(k) - \rho}$$

Since $f'(k^*) - \rho = -\frac{v'(1)}{k^*u'(f(k^*))} < 0$, a sufficient condition for $d\ln\phi(k)/dk$ to be positive at k^* is $\beta(f(k^*)) \geq \gamma(k^*)$.

Proposition 2: If, for all non-negative c_h,z_h, the following condition holds:

$$\beta(c_h)\eta(z_h) \geq 1 \tag{10}$$

i.e., the elasticity of marginal utility of relative wealth is at least as great as the elasticity of marginal utility of consumption, then all individuals will have identical steady-state wealth and consumption levels. (That is, asymmetric steady states do not exist.)

Proof: Note that any steady state (c_1, c_2, k_1, k_2) is a solution to the following system of four equations

$$c_i = f'(k)k_i + [f(k) - kf'(k)] \text{ for } i = 1, 2 \tag{11}$$

$$0 = f'(k) - \rho + \frac{v'(k_i/k)}{ku'(c_i)} \text{ for } i = 1, 2. \tag{12}$$

where $k = \alpha_1 k_1 + \alpha_2 k_2$. Suppose there is an asymmetric steady state, $(c_1^*, c_2^*, k_1^*, k_2^*)$ where $k_1^* \neq k_2^*$ and $c_1^* \neq c_2^*$. Then, for a given value $k^* = \alpha_1 k_1^* + \alpha_2 k_2^*$, consider, in the (k_h, c_h) space, the straight line

$$c_h = f'(k^*)k_h + [f(k^*) - k^*f'(k^*)] \tag{13}$$

and the curve

$$0 = f'(k^*) - \rho + \frac{v'(k_h/k^*)}{k^* u'(c_h)} \tag{14}$$

If there is an asymmetric steady state, then these two graphs must cut each other twice (at least); one of these points is (k_1^*, c_1^*) and the other is (k_2^*, c_2^*). Now the slope of (13) is $f'(k^*)$, and the slope of (14) is

$$\frac{dc_h}{dk_h} = \frac{u'v''}{k^* v' u''} = \beta(c_h)\eta(z_h)\frac{c_h}{k_h} \tag{15}$$

Now if the curve (14) cuts the line (13) twice, then at one of these points, say point A, the curve (14) cuts the line (13) from above. (See Figure 1). At point A, the slope of the curve (14) is smaller than the slope ($= c_h/k_h$) of the ray OA that goes through the origine O. It follows that at A, $dc_h/dk_h < c_h/k_h$. In view of (15), this condition cannot be met if (10) holds.

PLEASE PLACE FIGURE 1 HERE

2.3. Catching up: stability analysis

We now examine the stability properties of symmetric steady states.We linearize the system (5) (6) and then evaluate all derivatives at the symmetric steady state. We have the following matrix:

$$J \equiv \begin{bmatrix} f'(k^*) & 0 & -1 & 0 \\ 0 & f'(k^*) & 0 & -1 \\ a_{31} & a_{32} & a_{33} & 0 \\ a_{41} & a_{42} & 0 & a_{44} \end{bmatrix}$$

where

$$a_{31} = \beta\alpha_1 c^* A + B$$

$$a_{32} = \beta\alpha_2 c^* A, a_{33} = a_{44} = \rho - f' = -\frac{v'}{k^* u'} < 0$$

$$a_{41} = \beta\alpha_1 c^* A, a_{42} = \beta\alpha_2 c^* A + B$$

with

$$A \equiv f'' - \frac{(1-\eta)v'(z^*)}{k^2 u'(c^*)}, B \equiv \frac{v''(z^*)}{k^2 u'(c^*)} = -\frac{\eta(1)v'(1)}{k^2 u'(c^*)} \text{because } z^* = 1$$

Proposition 3: If

$$\beta(c^*) \geq \max\left[\gamma(k^*), \frac{1}{\eta(1)}\right] \tag{16}$$

then the equation $\det[xI - J] = 0$ has two positive real roots and two negative real roots, implying that the steady-state is stable in the saddlepoint sense. This implies that the poor will be able to catch up with the rich.

Proof: Subtracting the third row of $xI - J$ by the first row times $[x - a_{33}]$, and subtracting the fourth row by the second row times $[x - a_{44}]$, we obtain

$$\det[xI - J] = \det\begin{bmatrix} x - f' & 0 & 1 & 0 \\ 0 & x - f' & 0 & 1 \\ -a_{31} - Y & -a_{32} & 0 & 0 \\ -a_{41} & -a_{42} - Y & 0 & 0 \end{bmatrix}$$

where

$$Y \equiv (x - f')(x + f' - \rho) = (x - f')\left(x - \frac{v'}{ku'}\right) \tag{17}$$

Thus, $\det[xI - J] = (a_{31} + Y)(a_{42} + Y) - a_{41}a_{32} = Y^2 + bY + d$, where $d \equiv (\beta c^*)^2 B(B + A > 0, b \equiv \beta c^*(A + 2B)$. The characteristic equation is a quadratic in Y. We get

$$Y_{1,2} = \frac{-b \pm \sqrt{(\beta c^*)^2 A^2}}{2}$$

$$Y_1 = -B\beta c^* > 0$$

$$Y_2 = -[B + A]\beta c^* = -\beta c^* \left[f'' - \frac{(1-\eta)\theta v'(z^*)}{k^2 u'(c^*)} + \frac{\theta v''(z^*)}{k^2 u'(c^*)} \right]$$

Substituting Y_1 into (17), we get

$$0 = (x - f')\left(x - \frac{\theta v'}{k^* u'}\right) - Y_1 = x^2 - x\left(f' + \frac{v'}{k^* u'}\right) + \frac{\theta v'}{(k^*)^2 u'}\left[k^* f' - c^* \beta(c^*)\eta(1)\right] \quad (18)$$

Assume (16) holds. Then $k^* f' - \beta\eta c^* \leq k^* f' - c^* = -[f(k^*) - k^* f'(k^*)] < 0$, and equation (18) has two real roots of opposite sign. Similarly, substituting Y_2 into (17), we get

$$x^2 - \left(f' + \frac{v'}{k^* u'}\right)x + Q = 0 \quad (19)$$

$$Q \equiv \frac{v'}{(k^*)^2 u'}\left[k^* f' - c^* \beta(c^*)\eta(1)\right] + \beta c^* \left[f'' - \frac{(1-\eta)v'}{(k^*)^2 u'}\right]$$

Hence

$$Q = c^* \beta(c^*) f'' + \frac{v'}{k^2 u'}\left[k^* f' - c^* \beta(c^*)\right]$$

where $c^* = f(k^*)$. Thus $Q < 0$ if (9) holds.

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