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The Public-Private Sector Risk-Sharing in the French Insurance "Cat. Nat. System"

Nathalie de Marcellis-Warin[*] *, Erwann Michel-Kerjan*[†]

Résumé / *Abstract*

Il y a 20 ans, la France créait un système d'assurance des sinistres dus aux catastrophes naturelles qui demeure unique au monde. Le système "Cat.Nat" est constitué d'un partenariat entre l'industrie privée de l'assurance, un réassureur public et le Ministère des Finances.
Nous proposons ici un modèle simple de ce système qui semble refléter concrètement son fonctionnement. Nous montrons que le gouvernement, suivant ses attentes, choisit de moduler sa politique de prise en charge de tels risques afin de conduire les assureurs privés à adopter l'une des deux stratégies diamétralement opposées: (i) transférer tous les risques au réassureur public et ainsi se comporter comme de simples intermédiaires financiers; (ii) décider de conserver le plus possible de hauts risques.

Twenty years ago, the French created a so far unique insurance scheme to cover damages due to natural catastrophes. This so-called "Cat-Nat system" combines private insurance industry, a state-guaranteed public reinsurance and the Treasury. We provide a simple game-theoretic model which seems to capture the situation of the concrete scheme.
We find that, depending on its expectations, the government modulates its policy to induce private insurers adopting one of those two opposite strategies: (i) transfer all the covered risks to the public reinsurer and therefore simply behave as indemnification intermediaries; (ii) conserve the largest share of high risks.

Mots clés : partage de risques catastrophiques, système français d'assurance, parteneriat public/privé, équilibres de signal.

Keywords : *catastrophe risk-sharing, French insurance system, public-private sector partnership, signaling equilibria*.

JEL Classification : D80, G22, G38, H42

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[*] GRID, Ecole Normale Supérieure, France & CIRANO - Center for Interuniversity Research and Analysis on Organization , 2020 University 25th Floor, Montreal (Quebec) H3A 2A5 Tel : (514) 985 4000 # 3120 email: *demarcen@cirano.qc.ca*

[†] IDEP-GREQAM (University of Aix-Marseille II), Laboratoire d'économétrie of the Ecole Polytechnique, 1 rue Descartes, 75005 Paris, France and CIRANO, Montreal, Canada Tel: 33-1-55-55-82-42, Fax: 33-1-55-55-84-28. email: *erwannmk@poly.polytechnique.fr*

Introduction

Storms Lothar and Martin devastated France and western Europe in December 1999 reminiscent of the ice storms which devastated Canada in January 1998 (Lagadec and Michel-Kerjan, 2000). These storms and subsequent large-scale floods led to 92 deaths and heavy financial consequences: more than $7 billion in insured damages and great deal in non-insured damages as well. In a worldwide context, the number of natural disasters becomes evident and catastrophes appear less exceptional: according to the Swiss Reinsurance Company (2000), the year 1999 was the second most costly year of all in the history of world insurance. Every country tried –and is still trying- to create systems for bearing catastrophic losses that ensue from large-scale acts of God, called *cats*[1].

Insurers often consider low-probability/high-consequences events (LP-HC) as being non insurable at an affordable price. On the one hand, potential purchasers tend to underestimate the real level of risk (Camerer and Kunreuther, 1989). The hypothesis that risk-perception influences purchasing of insurance (Kunreuther, 1996) has been shown by empirical studies (Browne and Hoyt, 2000; Ganderton and al., 2000). Moreover, under the public pressure in the aftermath of a catastrophe, the government would have to help both its insured and non-insured citizens (emergency measures, crisis management, and disaster relief to uninsured citizens…). The charity hazard, defined by Browne and Hoyt (2000) as "the tendency of an individual at risk not to purchase insurance or other risk financing as a result of a reliance on expected charity from (…) a government emergency program", induces potential purchasers of insurance to consider the actuarial premiums as being too expensive. That may constitute another factor contributing to market failure. On the other hand, the regulation of premium rates obliges private insurers to sell insurance coverage at a lower price than necessary for business (Priest, 1996; Jaffee and Russell, 1997).

The debate on the comparative advantages of the private property/casualty insurance industry to the government for the provision of insurance coverage for such events is an important one and exists in other areas (Moss, forthcoming). The theory underpinning the role of public and private sectors in the insurance of major natural hazards has received considerable scholarly attention. Although an exhaustive review of the literature is beyond the scope of this article, we will discuss some of the important works.

A first key question may be: *can the private insurance and reinsurance industry handle the problem by itself?* The strengths of the private sector are numerous: insurers can adopt more aggressive investment strategies, damage appraisal can be quicker and more precise owing to their networks of experts, the level of premium rates is adjusted to the risk level, and effective risk reduction measures are achieved by market discipline. However, with the recent major natural disasters and their resulting effect on the solvency of insurance companies, the question of insurers' insolvency has become of great importance. According to King (1993), following the 1992 Hurricane Andrew, no less than nine US property-casualty insurance companies became insolvent.

Traditionally, the insurance industry avoids the insolvency problem by reinsuring the LP-HC risks (Borch, 1990). However, the current reinsurance capacity for coverage of natural catastrophes is limited and prices of catastrophe reinsurance are high (Froot, 1999). Very few reinsurers in place provide protection against industry-wide losses for catastrophic event greater than $5 billion. Froot (1999) gives several arguments to explain the reason why the prices of catastrophe reinsurance are high: insufficient reinsurance capital, reinsurers' market power, inefficiency of the corporate form of reinsurance, high transaction costs, moral hazard and adverse selection at the insurer level. Therefore, insurers seem to have difficulty finding available reinsurance coverage at prices that they consider affordable.

Another means of financing is currently discussed and developed by looking at the $19 trillion capital markets is the development of “securitization” (property-catastrophe-risk financial instruments) (D'Arcy and France, 1992; Niehaus and Mann, 1992). The property-catastrophe options that have been traded on the Chicago Board of Trade (CBOT) since 1992 would enable insurers to hedge against their underwriting risks by attracting capital from other sources of the economy that insurance segment. The property-catastrophe swaps traded on the Catastrophe Risk Exchange (CATEX) in NYC would allow a real diversification of risks and quasi-unlimited reserves also. A recent approach to securitization has been developed in several articles (Froot (ed.), 1999; Harrington and Niehaus, 1999; Cummins, Lewis, and Phillips, 1999; Schlesinger, 1999; Cox, Fairchild and Pedersen, 2000; Cummins, Lalonde and Phillips, 2001; Froot, 2001; Loubergé and Schlesinger; 2001). However, whereas the development of catastrophe modeling and simulation is effective owing to the advances in information technology, the development of financial instruments remains under potential. Still, we have not seen any definitive argument to show that the private insurance and reinsurance industry alone can handle the problem of insolvency in case of extreme financial consequences of natural disasters.

The second question is: *if the public sector should participate, can the government handle the question by itself?* Some of the main argued strengths of the public sector in financing the catastrophic losses due to these events are essentially based on its powerful source of wealth redistribution toward losses already occurred. Its ability to spread the costs of catastrophes over time and to obtain them from the largest social entity in existence, the whole population, based on national solidarity. Indeed, the government is even able to redistribute the cost of disasters among present and future generation of citizens and firms. Public intervention may solve the problem of insolvency of private insurers and reinsurers. The second ability of the public sector is to achieve high diversification by pooling several sources of risk. A third ability is to constrain adverse selection phenomena by the enforcement of insurance purchase. Adverse selection is well known by economists and insurers (Akerlof, 1970; Borch, 1990)[2]. It appears when the insurance company is not able to distinguish between high risk and low-risk purchasers of insurance in its pool of purchasers. As a result the average premium offered by the insurance is higher than the actuarial premium rate associated with low-risk: the low-risk purchasers of insurance leave the pool. Even if this governmental control of adverse selection does not reduce the risk itself, it also constitutes a strong vector of redistribution.

The *National Flood Insurance Program (NFIP)* developed in the United-States by the Federal Emergency Management Agency (FEMA) for insuring people against floods constitutes a good example of public insurance. The objectives of that program were to provide flood insurance at a subsidized rate to homeowners and business and to improve the mitigation measures. Large-scale flood studies were conducted in 1996 by the *NFIP* which identified all the communities that contained areas at risk of serious flood disaster, regardless of size. According to Pasterick (1998, p129), through the 1997 fiscal year, the cost of such a massive study was about $1.154 billion for 18,760 studied communities. Specifically, the *Community Rating System (CRS)* was created in 1990 to recognize and encourage community flood-plain management activities. A city will receive a CRS classification based upon the score obtained for its activities of prevention and mitigation. There are ten classes: class 1 represents the highest score that can be obtained for the best preventive measures, class 10 the worst. Such a system facilitates the public insurance rating by allowing premium reduction (the greatest reduction is obtained for cities of Class 1). The part of subsidies increases with the best scores. According to Pasterick (1998, p151) “currently, about 35% of NFIP policies are subsided to some degree, which costs the program approximately a half billion dollars in annual premiums”. NFIP policies in force stand at 4 million in 1998, with total premium of $1.5 billion and total coverage of $455 billion. Besides such a federal effort to provide public insurance at an affordable price, market penetration remains low and a significant portion of the flood losses remains uninsured. One estimates that the current number of policies represents less than half of potential purchasers of flood insurance

Whereas the public sector presents real strengths, some negative effects likely to appear when the aid program functions alone should be considered also. Priest (1996) advanced that massive subjective payments of disaster aid in questionable contexts and the disaster payments that may be unrelated to the actual economic losses suffered have been denounced by two US government reports studying the operation of government-provided disaster assistance following Hurricane Andrew and the 1993 Mississippi floods. Henriet and Michel-Kerjan (2001) develop a model describing the impact of disaster aid programs on the strategy of community owners of land in hazard-prone areas. The potential buyer, who has no other information than his/her beliefs on the quality of land, will receive governmental indemnification in the event of a natural disaster. The authors demonstrate that knowing that, communities may be incited to misrepresent the risk level of the land they are trying to sell. The government can offer to subsidy risk analysis only if the results are made public. Such a policy would contribute to limiting the proportion of high risk land sold. The aid program is made more efficient by indemnifying only people who suffer from damage in low risk areas.

This brief description presents some advantages and drawbacks of both the public and the private sectors. Its main conclusion is to call emphatically for a public-private sector partnership instead of concluding that unilateral solutions provide definitive answer to the problem posed by natural disasters.

Insured losses caused by both Hurricane Andrew and Northridge Earthquake were estimated at $31 billion. Such levels of insurance claims have lead insurers to look for high amounts of risk financing. In order to avoid insurers becoming insolvent following major natural disasters, Lewis and Murdock (1996) and Cummins, Lewis and Phillips (1999) proposed a risk-specific federal reinsurance program able to expand the supply of reinsurance. Under the program, the federal government sells contingent claims (excess-of-loss contracts) to qualified insurers and reinsurers against the upper layers of $25-50 billion insurance industry losses on a peer occurrence basis. The government pays only when losses are between the trigger and the cap (it pays the difference between losses and the $25 billion trigger), or exceeds the cap (it pays the difference between the cap and the trigger -i.e. $25 billion). Between 1988 and 1998, the insured coastal property values in the United States have increased by 69% (up to more than $3 trillion). If that increase is combined with the assumption of research on natural disasters that foresees a real potential for increased natural catastrophic events over the next twenty years (Gray, 1990), the $25-50 billion range could be attained in the short run. To minimize the information problems, coverage is offered for a single event and the contract is written on losses aggregated over eighteen months after the date of the event.

The proposed federal excess-of-loss (XOL) reinsurance contracts constitute an important contribution to possible partnerships not only in financing the losses due to natural disasters[3] but also in risk transferring as the public intervention is determined *ex ante* by reinsurance treaties.

But the partnership is mainly concerned by the question of insolvency and financial aspect of the disasters with high levels of trigger.

The continuing increase in the costs of aid for governments has caused policy-makers to look closely at indemnification, possible insurance solutions and mitigation programs (Kunreuther, 1997). Such a partnership may concern different aspects of the problem such as improving mitigation measures, *ex ante* risk-sharing or financing the cost of recovery from large-scale natural disasters, and victim indemnification. We are aware that the question of mitigation is extremely relevant, yet beyond the scope of this article[4].

Therefore, another fundamental issue is whether government intervention, when effective, should be only on a basis of *ex post* or also on *ex ante* involvement. In other words, is the role of government only to intervene *ex post* in the aftermath of a disaster to lend when required, or to share *ex ante* the risk with the private market, or both?

In sum, a simple partnership classification may be suggested. On the insurers' side: do they insure the natural hazard or not? On the governmental side: is federal involvement only concerned with *ex post* intervention (loans, grants,etc.) or with risk transfer also? In the later case, does the government operate as a reinsurer or does it act as a first insurer also?

For instance, as insurance against floods in the US is mainly offered by the public sector through the NFIP, the government operates as a first insurer (it sells the contracts) and offers governmental loans in the aftermath of a natural disaster.

In the federal XOL contracts, the insurers sell the policies, not the government, which offers public reinsurance through the stop-loss treaties. In that case, the public-private partnership is effective through governmental reinsurance.

As we will explain in the remainder of the paper and according to the classification described above, the current French system of insurance against natural disasters operates one more step in the public-private partnership for the indemnification component. It makes the public-private sector partnership come into play as soon as the first dollar of losses has to be reimbursed, as well as for very high levels of damage. Only a few related articles have been written in English (Magnan, 1995; Michel-Kerjan, 2001) or even in French (de Marcellis, 1997; Munier, 1997; Jullien, 1998; de Marcellis and Michel-Kerjan, 2000).

The article is organized as follows. We present the French insurance scheme against natural disasters in section 1: the dual coverage, the national solidarity principal, the main actors of the scheme through the existing risk-sharing between the government and the insurance industry. After showing that the French public sector has in fact more information than insurers on the risks of natural disasters in section 2, we start out in section 3 by presenting our basic model of the French public-private sector risk-sharing in the form of a game with Perfect Bayesian Equilibrium (PBE). The framework is presented in this section: the model's hypothesis, the players' action, the objectives of the government and the insurance industry. This section describes what we call the governmental high risk sharing (HRS) payment: to induce the private insurers to bear more high risks, the government can decide to raise the level of premiums paid by insured citizens and firms. The government can vote different policies. We discuss two of them: the first one, in which it uses the insurers as indemnification intermediaries and the second one in which the government is looking for the autonomy of the scheme for repeated events in a short period of time. Section 4 gives a characterization of the corresponding pooling or separating equilibria. In section 5, we discuss pour results and find they seem to capture the situation of the concrete scheme. Section 6 concludes the paper. An appendix provides detailed proofs for the propositions presented in section 4.

1. The French insurance "Cat.Nat. system"

Before 1982, French insurance companies refused to cover damages caused by most natural catastrophes. Following the serious flooding which occurred at the end of 1981, the French government instituted a so far unique compensation system for victims of specific major natural hazards officially considered "natural catastrophes": the "Cat. Nat. system" (*catastrophes naturelles* in French).

1.1 Storm guaranty and Cat.Nat. guaranty: "insurable" versus "not insurable" event

The French government created this specific insurance operated by a public-private sector partnership for natural hazards considered "uninsurable" such as earthquake, flood, drought, subsidence, avalanche, tidal wave or landslide.

In France, the co-existence of two indemnification systems for major natural events has to be considered: the "Cat.Nat guaranty" for uninsurable natural events and the "Storm guaranty" for insurable ones such as windstorm, ice and snow[5]. In France, the expression "natural catastrophes" is more precise than it is usually the case. Paradoxically, a big storm, which is covered by the storm guaranty, is not a "natural catastrophe" from a legal point of view for the French insurance scheme: such an event is covered exclusively by the private insurance without any governmental intervention. There is of course the possibility that both sectors may operate at the same time for the allocation of claims. For instance, the indemnification process in the aftermath of the major storms in December 1999 required the operating of both systems. Indeed, whereas direct insured damage due to the wind was exclusively indemnified by the private scheme (the storm guaranty), the storms involved major floods too. Damage due to floods was reimbursed by the Cat.Nat. scheme.

That scheme is based on a combination of the *national solidarity principle* (every insured person pays the same percentage whatever his location) and a *public-private sector partnership* with the main objective of efficient indemnification: every person should be rapidly reimbursed after the appraisal of damage caused by a natural catastrophe.

1.2 The national solidarity principle

The law of 13th July 1982 imposed the obligation to extend the scope of every Property & Casualty (P&C) and damage to motor vehicle insurance contracts to include the risk of natural catastrophe. A level of extra-charge for the new line "natural catastrophes" is applied on all contracts. This additional premium is calculated on the basis of a single rate for each line of business, whatever the level of the premium paid by the purchasers. Therefore, everybody pays the same surcharge rate whatever the risk. Thus, the system is based on the national solidarity principle: the losses due to natural catastrophes are distributed between those who suffer from the event and those who do not. Thus, the principle of equality in the face of natural disasters proclaimed in the preamble to the French Constitution is enforced. This principle prevents also adverse selection phenomena (Moss, 1999).

1.3 Stakeholders of the scheme: a public-private sector partnership

In addition to the insured citizens and firms, there are three chief actors in this system: the *French government* (the Treasury to be more precise), the *private insurers* and the *public reinsurer*, the *Caisse Centrale de Réassurance (CCR)*, a French state-owned reinsurance company (figure 1). Therefore, both public and private sectors operate. The government plays a key role in the scheme.

First, the indemnification process requires that a "state of natural catastrophe" be declared. Such a state is declared by interministerial decree: the government decides the indemnification process.

Second, the government decides the level of the extra-charge rate for the natural catastrophe line that will be applied on every P&C's insurance contract. The total amount of extra-charge is transferred to the private insurers (like a premium) who will pay the compensation within three months in the event of natural catastrophes. The partnership with the private insurers presents real advantages: the large network of insurers covering all of France, in cities and in the countryside, the technical ease with which they may add and manage a new line of business, and the quick official appraisement of damage by their network of experts.

Third, to solve the problem of insurers' insolvency in participating in the Cat.Nat. system, the government gives private insurers the option of being reinsured against these risks by a public reinsurer, the CCR (not obligatory). One of the characteristics of the scheme is that the French government offers the CCR a non-limited guarantee. That means the government is the lender of last resort (Vandamme, 1998): it will pay if the CCR is "virtually" bankrupt. In dealing with catastrophic events, it should be clearly understood why the governmental guarantee accorded to the CCR is so important to balance the system. For the insurers, it means being reinsured by a reinsurer that will never be bankrupt.

Two main stakeholders will be considered in the remainder of the paper: both the French government (with its unlimited guarantee) and the CCR as the public sector on the one side, the insurers as the private sector on the other side.

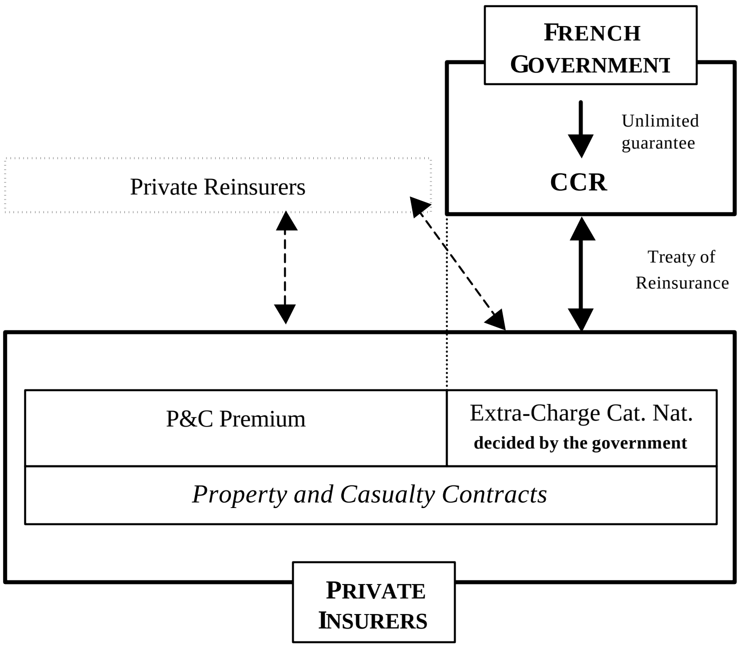

Figure 1: The French Cat.Nat. System

2. Risk knowledge

The French government, which plays a very important role in the "Cat.Nat" system, is assumed to be more informed on this special line of risks than the private insurers. Several arguments back up this assumption.

On the one hand, and even before 1981, a municipality has some historical data on catastrophes it has suffered. Added to the governmental knowledge due to aid programs to victims, the aggregation of that information at the national level increases the risk knowledge of the public sector. On the other hand, the private insurers had refused to insure LP-HC events before 1981, therefore they had no reason to have more information that the public sector on such risks.

Moreover, even if both the government and the private insurers have by now got the twenty years of historical data, the public sector has even more information. Indeed, the 13th July 1982 law provided the implementation of prevention measures[6]. Municipalities should make risk analysis and implement *"Risk Prevention Plans"*, called *PPR* in French. However, the low percentage of PPR elaboration has not induced any modification to the indemnification scheme (Munier, 1997).

An interesting fact is that the French private insurers never had a deep interest in elaborating their own risk analysis for the natural catastrophes. A clear example of that argument is the creation by the mutual and insurance companies in 2000 only of the French "Natural Hazards Mission". One of its main roles is to transfer the results of PPR into their database. That shows clearly that the information on risks described by the PPR have not been considered by the private sector for twenty years of the scheme's operation.

Two main reasons can explain such a deliberate asymmetry of information on natural hazards covered by the Cat.Nat. guaranty. First, the line "natural catastrophes" represents only a small percentage of the total business of French private insurers. Second, because of the unique level of extra-charge over all the French insurance purchasers, the insurer could not make the premium reflect the real risk. In such a context, why should the insurers carry out very expensive risk analysis for so few benefits?

3. A basic model of the French system

3-1. The game

Our theoretical study confronts the public sector with private insurers. We consider a simple game of incomplete information. In order to simplify the model, assumptions are made on both the set of catastrophe risks and the possible actions. We are only interested in the events covered by the Cat.Nat. guaranty during a given period of time. The total insured damage due to some "natural catastrophes" is denoted by D. The game has three players : the *Nature*, the *public sector* and the *private insurers*.

Players' action.

The player 1, the *Nature*, can choose between two expected damages levels p_L.D and p_H.D. We consider that there exist only two possible probabilities of occurrence p_L and p_H with $p_L < p_H$. The player 2, the *public sector,* fixes the level of the extra-charge rate, denoted by ß, taking into account its knowledge on the occurrence of the natural disaster damage. Referring to the game theory literature, we call the strategy (or action) played by the government the "extra-charge policy". ? is the total premiums of the Property and Casualty contracts in France. Therefore, the total extra-charges collected for the line natural catastrophes are ß.?. The government chooses between a low extra-charge policy, denoted by ß, or a high one, denoted by $\overline{b}$. The player 3, the *private insurers*, receives the total extra-charges collected for the line natural catastrophes, ß.?. The insurance industry's behavior is assumed to be summarized by a single action: the reinsurance cession rate to the CCR[7], denoted by α. The private insurers are assumed to be only reinsured by the public reinsurer, the CCR. They choose a cession rate to the CCR α in $[\underline{a};\overline{a}]\subset[0,1]$. They can keep the largest part of the premiums and risks by playing a low cession rate $\underline{\alpha}$ or choose to transfer the major part of them to the CCR and then take the action "a high cession rate". The high limit of reinsurance quota-share allows the system to share a minimum percentage of the risks with the insurers (they keep a minimum percentage (1–α)) whatever their decision of cession. The low rate limits the amount of losses the insurers would bear in case of catastrophic damage if they had decided to keep the largest possible part of risks. Moreover, the government offers the insurers a commission *m* in order to induce them to participate in the system (*m* is the private insurers' *participation commission*).

Players' preferences.

In the standard theoretic framework, the two main stakeholders are (i) the public sector and (ii) the private insurers who decide according their own preferences. The criteria for action are the following.

The French government wishes all natural catastrophe victims to be compensated. However, the establishment of such an insurance scheme based on a national redistribution induces a social cost: the total amount of extra-charges paid by the insured, $\beta\Pi$.

The payment of the participation commission *m* to the private insurers by the government has also to be taken into account. Moreover, as the CCR is a public-owned company, the Treasury searches to balance the public reinsurer.

Finally, as the government accepts a possible *ex post* public intervention by offering an unlimited guarantee to the CCR, we have to consider the effect of such an intervention by introducing a parameter λ which represents the weight attributed by the government to an *ex post* public intervention in order to balance the system[8]. The greater λ, the more reluctant to risk financing the government is. Adding to its own objective, we can consider that the French government (here the Treasury) internalizes all the objectives of the public sector as a whole: the insured citizens and firms, the CCR, and itself.

Taking into account such an argument in the definition of utility U(.) of the Treasury, it can be written as follows:

$$U_p(b;a) = -m - b.\Pi + a.(b.\Pi - p.D) \quad if \quad b.\Pi \geq D \qquad (1)$$

$$U_p(b;a) = -m - b.\Pi + (1-p).a.b.\Pi + l.a.p(b.\Pi - D) \quad otherwise$$

The first equation corresponds to the case in which the total amount of extra-charges is high enough to compensate all the victims of natural catastrophes (with total damage D) that occur with a probability p (L or H) during the studied period of time. The *ex post* governmental intervention is not required because $b.\Pi > D$. The benefit of the public reinsurer, which positively affects the expected public utility, is the difference between the total premiums received from the insurers and the total amount of reimbursement in case of catastrophes. The second equation corresponds to the case in which the extra-charge policy is not high enough. Indeed, the portion of premium received is not enough to compensate the part of damage the public reinsurer has to pay. The CCR is virtually bankrupt. The state guarantee is triggered and the government pays the excess of insured losses that the CCR is not able to pay, that is $\alpha.(D - \beta\Pi)$ timed by the factor λ.

For the private insurers, we use a mean-variance utility function assuming they are risk averse for catastrophic risks[9]. When the insurers believe the probability of damage is p and with an extra-charge policy β, their utility V(.) is:

$$V(a;b;pD) = (1-a).(b\Pi - p.D) - \frac{k}{2}(1-a)^2 p.(1-p).D^2 + m \tag{2}$$

As the reinsurance treaty to the CCR is assumed to be only on quota-share, whereas the portion of risk premium kept by the insurers and the participation commission offered by the government positively affect their utility, the expected payments and the variance in expected reimbursable damage affect it negatively. k/2 is assumed to be positive (catastrophe risk aversion). We finally assume in this model that the commission *m*, which the government offers the insurers to cover their management costs, is always high enough to incite them to participate (i.e. V>0).

Players' information.

The public sector is assumed to be more informed on these risks than the private insurers. As explained in the preceding section, in the model the government is then assumed to be the informed party whereas the private insurers have only prior beliefs on the distribution of types: there is a proportion μ of low types of expected damages and a proportion $(1 - \mu)$ of high types. The game is played within imperfect information of player 3 which is represented by two information sets in the extensive form of the game.

Sequence of decisions.

The game takes place over three "periods" (figure 2). In period one, player 1 (Nature) chooses between two expected damage levels, $p_L.D$ or $p_H.D$. In the second period, player 2 (the government) gets to choose between two policies (nodes G1 and G2 in the extensive form of the game), a low extra-charge policy or a high level: $b \in \{\underline{b};\overline{b}\}$. Finally, in the third period, player 3 (private insurers) who receives either the signal of a high extra-charge policy (nodes I1 and I2) or the signal of a low extra-charge policy (nodes I3 and I4) must choose a reinsurance cession rate $a \in [\underline{a};\overline{a}]$.

The insurers only receive indirect information on the nature of the risks they are going to insure when the government chooses the extra-charge policy. Observing that policy, they revise their prior beliefs using the Bayes's rule. The government knows the prior beliefs of insurers. And both the government and the insurers obtain a certain level of utility according to their action and criteria.

We can now draw the complete extensive form of the game as in figure 2.

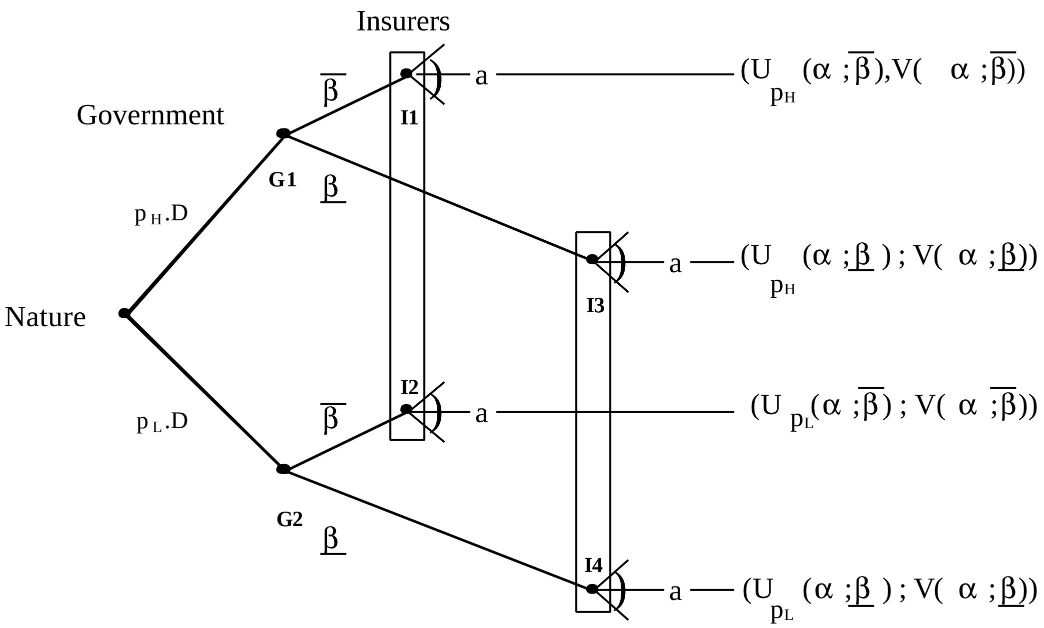

Figure 2. The extensive form of the game.

3-2. Main assumption

As explained above, the government fixes the extra-charge. We make the following assumption concerning the level of this extra-charge. First, the low extra-charge equal the actuarial rate for the low risk ($\underline{b}.\Pi = p_L.D$)[10]. Such a low governmental extra-charge policy, whose advantage is to limit the payment of the citizens and make the system popular, is not sufficient to enforce the balance of the indemnification process without any *ex post* governmental intervention in the event of a natural catastrophe. However, whereas the low governmental policy is not sufficient in case of damage, one may consider that they levy a minimum level of extra-charges which permits to reduce the total *ex post* governmental payment (if compared with a situation in which the system did not exist).

Moreover, the main difficulty for the government in creating this scheme was to find a balance between the expected damages and the extra-charge policy in order to assure the autonomy of the scheme, that is to limit the *ex post* public intervention. This autonomy ends as soon as the total extra-charges levied during the period of time are not high enough to balance the CCR's results. In this case, the state guarantee must be triggered in order to assure victim indemnification (see (1)). To enforce the autonomy of the scheme, the government can define a high extra-charge policy which is sufficiently high so that the state guarantee is not required. So, the second part of this assumption is that $\bar{b}.\Pi \geq D$[11]. In so doing, the French government defines a higher extra-charge policy than the corresponding actuarial premium for high expected losses ($\bar{b}.\Pi > p_H D$). The profit may be higher for the insurers who keep the risks: they may be also incited to be less reinsured in order to keep a higher part of the premiums.

We call such a policy the governmental "high risk sharing payment" (HRS payment for brief) of the scheme. The HRS payment is a key part for a clear understanding of the scheme. Traditionally, the insurers accept only to insure "good risks" and refuse to insure "bad risks". However, with the HRS payment, the opposite situation may occur. Since the government is likely to offer a higher payment than the actuarial premium, the insurers may make more profit than only the participation commission m. Thus, the insurers' retention rate, which represents their reaction function to the governmental extra-charge policy, could be non-decreasing with their prior beliefs on the proportion of high risks $(1-m)$.

4. Public risk management

In this section we make a focus on two policies that may be chosen by the government and we determine the corresponding equilibria. When the government announces its β, this does not necessarily correspond to its knowledge of expected damage. Insurers who observe the government's decision should update their beliefs and base their choice on the posterior distribution: $\Phi(p.D\backslash\beta)$, which depends on the signal β received and compatible with Bayes' rule. Observing $b \in \{\underline{b};\bar{b}\}$, the insurers can use Bayes' rule to update $\mu(.)$ to $\Phi(. \backslash \beta)$.

Definition. *A perfect Bayesian equilibrium with pure strategies is an action profile* $(\beta^* ;\alpha^*)$ *and posterior beliefs* $\Phi(p.D \backslash b)$ *such that*[12]:

(i) $\forall pD \in \{p_L D; p_H D\}$, $b^*(pD) = \arg\max_b U_p(b; a^*(b))$;

(ii) $\forall b \in \{\underline{b};\bar{b}\}$, $a^*(b) = \arg\max_a \sum_{pD} \Phi(pD \backslash b).V(a; b; pD)$

(iii) $\forall b \in Supp b^*$, $\Phi(pD \backslash b)$ can be obtained from the *a priori* distribution $\mu(.)$ (discrete in the model) by using the Bayes' s rule, *whenever feasible*.

(i) and (ii) are the perfection conditions: (i) says that the government takes into account the effect of β on insurers' decisions and determines its best response for each level $pD \in \{p_L D; p_H D\}$, (ii) states that the insurers react optimally to governmental decisions given their posterior beliefs about pD. They choose the cession rate to the CCR maximizing their utility. (iii) corresponds to the application of Bayes' rule by the insurers. It should be noted that if β is not part of government optimal action for some type, observing β is a probability-0 event, and Bayes' rule does not pin down posterior beliefs. Every posterior belief $\Phi(. \backslash b)$ is then admissible and every decision α, which is the best response for certain beliefs, can thus be put into play.

Let us explain a taxonomy of potential perfect Bayesian equilibria.

A *pooling equilibrium* is an equilibrium in which the government chooses the same action whatever the type p.D. The insurers do not update their beliefs when they observe the equilibrium action: $\Phi(p_L.D \setminus \underline{b}) = m = \Phi(p_L.D \setminus \overline{b})$ and $\Phi(p_H.D \setminus \overline{b}) = 1 - m = \Phi(p_H.D \setminus \underline{b})$.

A *separating equilibrium* is an equilibrium in which the government chooses two different actions depending on the type p.D. Observing the nature of the extra-charge played by the government, the insurers know the governmental type.

Moreover, when the observed level of extra-charge is inconsistent with the given equilibrium strategy, it is not possible to use Bayes's rule. We deal with this well-known problem by assuming that the private insurers will then set their beliefs via *forward induction* (see Fudenberg and Tirole, 1991 for a rigorous and complete discussion of perfect equilibrium and its refinements). Under the latter the private insurers view any "surprising" (i.e out-of-equilibrium) action by the government as truly intentional (as opposed to being the result of some mistake). They first rule out the level of expected damage at which a rational government would not depart from the proposed equilibrium. Once their beliefs are updates accordingly, the private insurers' utility maximizing reaction must deter the level extra charge at any other expected damage level. An equilibrium obtain in this manner turns out to be unique modulo the private insurers' current state of mind.

4.1. The indemnification intermediary behavior

First, the government can decide to levy a low amount of extra-charges whatever the level of risk. That case appears precisely at the beginning of a scheme's operation when the government needs to obtain a consensus with citizens and firms in order that the system be accepted. To encourage that acceptance, the government may choose a low extra-charge policy. The question is whether, according to the model, an equilibrium can be achieved with the government always choosing such a low policy. The insurers, receiving no information from the governmental signal $\underline{b}$, thus maximize their utility considering their prior beliefs.

Lemma. *The best response of the private insurance industry to a low extra-charge policy* $\underline{b}$ *decided by the government whatever the type p.D is to transfer the largest portion of the risks to the public reinsurer.*

Proof [13]*:* If the government chooses the low level of extra-charge rate $\underline{\beta}$, insurers will choose the level of cession which maximizes their utility according to this policy. Because the government plays the same action whatever the type p.D, the private insurance industry will choose its optimal level of cession $\alpha(\mu)$ by using their prior beliefs m on p.D.

Considering :

$$V\left(a(m);\underline{b};p.D\right)=m\left[(1-a(m)).\left(\underline{b}.\Pi-p_L.D\right)-\frac{k}{2}.(1-a(m))^2.p_L.(1-p_L).D^2\right]$$

$$+(1-m)\left[(1-a(m)).\left(\underline{b}.\Pi-p_H.D\right)-\frac{k}{2}.(1-a(m))^2.p_{H.}(1-p_H).D^2\right]+m$$

The first derivative can be written as follows:

$$V_a^{'}\left(a(m);\underline{b};p.D\right)=\left(1-a(m)\right).\left[m.k.p_L.(1-p_L).D^2+\left(1-a(m)\right).k.p_H.(1-p_H).D^2\right]$$

$$-m.\left(\underline{b}.\Pi-p_L.D\right)-(1-m).\left(\underline{b}.\Pi-p_H.D\right)$$

Therefore, the sign of the first derivative is positive if and only if the inequality holds:

$$a(m)<\frac{m.\left(\underline{b}.\Pi-p_L.D\right)+(1-m).\left(\underline{b}.\Pi-p_H.D\right)}{m.k.p_L.(1-p_L).D^2+(1-m).k.p_H.(1-p_H).D^2}$$

As $\underline{\beta}.\Pi = p_L.D$, the inequality becomes:

$$a(m)<1-\frac{(1-m).\left(\underline{b}.\Pi-p_H.D\right)}{m.k.p_L.(1-p_L).D^2+(1-m).k.p_H.(1-p_H).D^2}$$

As $\underline{b}.\Pi < p_H D$, the fraction is strictly negative. The inequality is equivalent to $\alpha(\mu) < 1 + a$, with $a > 0$. As $\alpha \in [0,1]$, the inequality always holds true whatever the beliefs out of the equilibrium and the first differential is always positive. Therefore, private insurers choose to transfer the largest portion of the risks to the public reinsurer when they receive no information from the governmental action.

■

The lemma is very intuitive. As the insurers know the government has more information on the risks, they know that the government could be induced to levy less extra-charge than the situation requires.

However, to determine whether an equilibrium can be reached, we have to show that the government has an interest in playing that action. Using the lemma, we have to show under which conditions, if any, the government has no interest in deviating from that action. Under the main assumption of the model, we find that whereas the government decides the same extra-charge policy for Cat.Nat guaranty whatever the level of risk, the private insurers will basically behave as indemnification intermediaries of the scheme. This is equivalent to the following formal proposition. The demonstration is offered in appendix.

Proposition 1. Insurers as intermediaries.
There exists a pooling equilibrium defined by the following actions and beliefs in which the government chooses a low extra-charge policy $\underline{b}$ *, the private insurers choose the largest possible cession rate* $\overline{a}$ *; the insurers' beliefs are* $\Phi\left(p_L.D \setminus \underline{b}\right)=m$ *and* $\Phi\left(p_H.D \setminus \underline{b}\right)=1-m$.

Proof: See Appendix.

We find that conditions for no deviation depend on the specification of the insurers' beliefs out of the equilibrium. Taking $n(p_H.D \setminus b)=1$ as the insurers' beliefs out of the equilibrium, the condition for no governmental deviation from the equilibrium is $\underline{a} \leq \min(r_L; r_H)$, with the rate bounds [14] $r_L = 1-(l-1)(1-p_L).\bar{a}.p_L$ and $r_H = \bar{a}.p_L + \frac{(1-p_L).(1-l\,\bar{a}.p_H)}{(1-p_H)}$.

It should be noted that when λ, the weight attributed by the government to an *ex post* financing through the trigger of the state-guarantee, is lower than one, both the bound rates r_L and r_H are higher than one. The proposition is true whatever the cession rate range of α. That case appears when the government prefers financing the system *ex post* instead of paying *ex ante* to improve the prevention and mitigation measures.

This pooling equilibrium is not sustainable if $\underline{a}$ is higher than either r_L or r_H. Indeed, in that case, taking into account the insurers' beliefs out of the equilibrium, the government would have an interest in deviating. In the situation presented by proposition 1, the government levies *ex ante* small extra-charges and will have to pay *ex post* the amount corresponding to the state guarantee that is triggered. When the exogenous parameter λ increases, the value of the bound rates r_L and r_H decreases more and more. From a particular level of l, according to the insurers' action out of this equilibrium, the government is incited to deviate from this equilibrium, which falls (see the proof of the proposition in the appendix).

On the other side, the insurers who have no other information on the nature of the risk than their prior beliefs, will prefer to transfer the largest part of the risk to the CCR. In that case, the French scheme uses the network and the know-how of private insurers as what we could call a "financial indemnification intermediary" per contra the payment of a participation commission *m* for managing that Cat.Nat. line in their portfolio. The insurers levy the extra-charge premiums on all purchasers and transfer the largest part of them to the public reinsurer, here the CCR. In case of damage, the indemnification comes mainly from the CCR and the state guarantee is necessary to indemnify the victim firms and citizens.

4.2 The autonomy of the system

Second, the government may want to assure the autonomy of the Cat.Nat. system, especially when the events are numerous – i.e. when the probability is high in the model. Defining a high extra-charge policy to induce that autonomy, the government may induce the insurers to keep the largest part of the risks, that is to play the action $\underline{\alpha}$ in the model.

As developed in the preceding paragraph, the pooling equilibrium can be obtained with the government playing the action $\underline{\beta}$. But, in so doing, the government knows that the insurers will take an intermediary course, which is not the government's objective. In other words, the equilibrium candidates for the scheme's autonomy can only be the separating equilibria. In a separating equilibrium, the government's choice reveals its type and gives a signal to the insurers.

The members of the government ought never to decide a low extra-charge policy when they know that the probability of damage is high. Indeed, private insurers would decide to transfer the highest part of risk to the CCR. Moreover, because the total premiums transferred to the public reinsurer would not be high enough to counterbalance the cost of indemnities, the state guarantee would always be triggered in that case. Reciprocally, when a high extra-charge policy is decided although the probability of damage is low, the insurer will keep the highest part of premiums. Whereas the government levies more charges than necessary, the insurers will choose to transfer a minimal portion of premiums. This policy could never be beneficial from a governmental point of view.

The government would like the insurers to conserve the greatest part of the risks in order to allow the autonomy of the scheme and to limit the amount paid when the state guarantee is triggered. The government is then ready to offer the private insurance industry a HRS payment (the difference between $\bar{b}.\Pi$ and $p_H.D$). We find that the condition is $(\bar{b}.\Pi - p_H D) \geq (1-\underline{a})k.p_H.(1-p_H)D^2$. This general expression of the trade-off can be written in order to consider the catastrophe risk aversion of private insurers k/2 (with the simple assumption $\bar{b}.\Pi = D$): $k \leq \dfrac{1}{(1-\underline{a})\,p_H.D}$.

The existence and the nature of the separating equilibria mainly depend on the level of HRS payment the government may offer. The following proposition gives the necessary condition to obtain an equilibrium in which the private insurance industry seeks to bear the largest share of the high risks. Under the main assumption of the model, the government, looking for the scheme's autonomy for high risks, will choose a specific extra-charge policy for each level of risks. Moreover, it offers a sufficient HRS payment which induces the insurers to conserve the largest share of high risks. This is equivalent to the following formal proposition.

Proposition 2. Looking for the scheme's autonomy

When the governmental HRS payment is sufficiently large, so that $(\bar{b}.\Pi - p_H.D)$ *is higher than* $(1-\underline{a}).k.p_H.(1-p_H)D^2$, *there exists a separating PBE such that the action profile* $\left\{\left(\underline{b}(p_L.D), \bar{a}(\underline{b})\right); \left(\bar{b}(p_H.D), \underline{a}(\bar{b})\right)\right\}$ *and posterior beliefs* $\Phi(p_L.D \setminus \underline{b}) = \Phi(p_H.D \setminus \bar{b}) = 1$ *define it.*

Proof. See Appendix.

As for the proposition 1, we find that conditions for no governmental deviation depend on the specification of the insurers' beliefs out of the equilibrium (see appendix). Taking $n(p_L.D \setminus b) = 1$ as the insurers' beliefs out of the equilibrium and $\bar{b}.\Pi = D$, the conditions necessary for the government not to deviate from that equilibrium are $\underline{a} \leq r_L$ and $\bar{a} \geq r_{sep}$, with

$$r_{sep} = \frac{(1-p_L) - \underline{a}.(1-p_H)}{l.p_H.(1-p_L) - (1-p_H)p_L}.$$

This separating equilibrium is not sustainable as soon as $\bar{a}$ is lower than the bound r_{sep} because taking into account the insurers' beliefs out of the equilibrium, the government would have an interest in deviating. That proposition may be interpreted as follows.

On the one hand, when the probability of damage is low, the extra-charge policy decided by the government equals the actuarial premium. The insurers have agreed to participate in that case because they will receive a participation commission such that their utility V will be positive. Nevertheless, the insurers have no real interest in bearing the major portion of the risks without making any profit with the premiums. They prefer to act as an indemnification intermediary. That explains the action profile $\left(\underline{b}(p_L.D),\overline{a}(\underline{b})\right)$ in the proposition 2.

On the other hand, when the probability of damage is high, because the government chooses a high extra-charge policy, the insurers have to consider the trade-off between keeping the largest portion of premiums (with a profit $(1-\underline{a}).(\overline{b}\Pi - p_H.D))$ or transferring a larger portion of the risk to the CCR. The trade-off will depend on both the governmental HRS payment and the insurers' risk aversion k/2. The proposition 2 deals with that trade-off by taking into account the necessary and sufficient conditions for the government and the insurance industry to play a specific equilibrium in which the insurers conserve the largest portion of the risks.

5. Discussion

Our model simplifies the present mechanism of the French insurance scheme against natural catastrophes. Nevertheless, it seems to capture the situation of the concrete scheme, both in its infancy and in its more recent application in France.

First, as has been written, when the government decides a low extra-charge policy, the insurers are induced to transfer most of the risk to the CCR, making the CCR vulnerable to virtual bankruptcy. At the system's infancy, as the risk knowledge was poor, it can be considered that the governmental action was taken whatever the potentiality of a catastrophe. That situation corresponds to the pooling equilibrium: insurers can not learn the type of risks by observing governmental policy. According to the first proposition, the pooling equilibrium requires that the government play the low extra-charge policy and that the insurers play the high cession rate to the CCR. The interesting fact is that, at the birth of the system, the CCR offered to private insurers the possibility of cession rates between 40 % and 90 %. The upper-limit 90 % rate corresponded to a concession made to insurers who agreed to insure those risks only because they would be quasi-entirely reinsured. The 40% rate allowed a minimum risk sharing between insurer and the public reinsurer. For this situation, the rate of extra-charge set by the government was low at 5.5% of the basic policy premium. The insurers' cession rate was between 85% in 1982/83 and 75% in 1986 which was high (CCR's Annual Report, 2000). That may correspond to the results obtained in the pooling case of the model (proposition 1).

Second, repeated major flooding and drought phenomena occurred between 1990 and 1996. In the present model, those repeated major natural hazards correspond to the high probability of damage. Therefore, the government should choose a high extra-charge policy knowing the probability of damage is high. According to our theoretical approach, to induce the private insurers to choose a low rate of cession, the government has to decide a level of extra-charge sufficiently high considering the insurers' catastrophe risk aversion so as to offer them the possibility of making higher profits than the participation commission. The facts are the following: during the last decade, the rate of extra-charge set by the government was equal to 9%. The percentage of extra-charge can be considered high enough when it is compared with the other situation (5.5 %). The average cession rate to the public reinsurance company remained stable, with an average of 43 % (CCR Annual Report, 2000), which can be considered a low rate of cession to the CCR (the lower bound being 40%). Such an evolution in the scheme seems to confirm the validity of proposition 2.

Further extensions can be considered. From a public risk management point of view we may wish to apply the scheme to other countries or to imagine related systems. As explained above, in France, the purchase of Property and Casualty insurance is mandatory for all persons and firms: all property owners and tenants must purchase liability insurance[15]. As Michel-Kerjan pointed out (2001), this is a key element for understanding the strength of the system. By basing the national program of insurance against natural catastrophes on this kind of insurance contracts, the government includes the highest number of people and firms in the system. Such a specification means that there is an extra-charge applicable on every contract. This would not be possible if the insurance were not required for everybody. Because of that, the parameter Π may be considered an important characteristic of the studied country (Moss, 1999)[16].

As explained in the paper, whereas the government accepts a possible *ex post* public intervention by offering an unlimited guarantee to the CCR, the government may prefer to use public expenditure to improve mitigation programs.

Therefore, because λ can be interpreted as a relative mark of efficiency of mitigation policy, it constitutes another central parameter of a discussion on public risk management of catastrophic risks. All things being equal, if we consider that the couple (Π,λ) characterizes a given country (or state), it might be interesting to test the results of our model by applying to it different characteristics λ and Π. We may suggest a possible interpretation of that extension. Intuitively, when the level of total premiums on which an extra-charge policy is applied is large, the model may more often lead to a pooling equilibrium: for a given governmental weight attributed to finance *ex post* the cost of a catastrophe, the government could be more susceptible to choose a low extra-charge rate whatever the type given by the nature. By the way, they could use insurers as indemnification intermediaries. Inversely, they will choose to inform the private insurers of the true type if there is only a low financial basis Π for the application of the catastrophe extra-charge policy[17].

Reciprocally, let us consider a given level of premium basis Π (as made in this paper). When the prevention and mitigation program (*ex ante* approach) are efficient (capable of significantly decreasing loss and probability of loss), it is cheaper for the government to intervene through mitigation or regulation of safety than *ex post* for financing the system. The higher the value of l , the more often they will prefer the separating equilibrium to the pooling one. Moreover, it can be shown that the social welfare, defined as the sum of U and V, in the separating equilibrium will always be higher (or at least equal) than the social welfare associated with the pooling equilibrium as soon as the parameter λ is higher than 1. In other words, the government should improve the public-private sector partnership by sharing both information and risks with the insurers. In so doing, insurers may be incited to contribute to influencing their purchasers to promote mitigation …

6. Conclusion

In this paper, we have attempted to whet readers' appetite as concerns this so far unique system called the French "Cat.Nat. system". To our knowledge, this paper constitutes one of its rare modelizations in the literature. The model of course simplifies the present mechanism of the French insurance scheme against natural catastrophes. Nevertheless, we have shown how our results may explain the situation of the scheme, both in its infancy and in its more recent application in France. Moreover, by introducing the governmental HRS payment we put forth the strategic behaviors of the chief actors and the risk sharing, thus giving a model of insurers' counter-intuitive behavior (agreeing to bear only the high risks and transferring the small ones to the CCR).

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7. Appendix

Proof of proposition 1.

To prove that $(\underline{b};\overline{a})$ constitutes the action profile at the pooling equilibrium, we have to show that considering some private insurers' beliefs out of equilibrium, say $(n;1-n)$, the government has no interest in deviation.

- When Nature chooses $pD = p_L D$.

According to the lemma, the members of the government know that if they choose the low extra-charge policy $\underline{b}$ the insurers will play the largest cession rate action, and that they will have to support the CCR because $\underline{b}.\Pi - D < 0$. It is never in the government interest to deviate considering the insurers' beliefs n and action out of equilibrium $a(n)$ if $U_L(\underline{b};\overline{a}) \geq U_L(\overline{b};a(n))$.

As, by playing the low extra-charge, the state guarantee will be triggered, the inequality can be written as follows: $-\underline{b}.\Pi + (1-p_L)\overline{a}\,\underline{b}.\Pi + l.\overline{a}.p_L.(\underline{b}.\Pi - D) \geq -\overline{b}.\Pi + a(n).(\overline{b}.\Pi - p_L.D)$.

With $\underline{b}.\Pi = p_L D$ and $\overline{b}.\Pi = D$, we have : $a(n) \leq 1 + \dfrac{(1-p_L)\overline{a}p_L D + .l\,\overline{a}.p_L.D.(p_L - 1)}{D - p_L.D}$.

That is $a(n) \leq 1 - \dfrac{(l-1)(1-p_L).\overline{a}.p_L}{(1-p_L)} = 1-(l-1)(1-p_L).\overline{a}.p_L \equiv r_L$.

So, $U_L(\underline{b};\overline{a}) \geq U_L(\overline{b};a(n))$ if $a(n) = r_{L.}$

Under H5, α ? $[\underline{a};\overline{a}] \subset [0,1]$. As soon as $\underline{a} \le r_L$, by taking $n(p_L D \setminus b) = 0 = 1 - n(p_H D \setminus b)$ out of the equilibrium, insurers will be led to play the action low cession rate. In that case, the government would not have any interest in deviation from the equilibrium and vice versa. Note that as soon as $r_L > \overline{a}$, we can take any belief n out of the equilibrium. The insurers would always play an action out of equilibrium that would lead the government not to deviate from the equilibrium. Reciprocally, to say that the government has no interest in deviation implies that the insurers' cession rate out of equilibrium is lower than a given rate, here r_L. Conversely, if $\underline{a} > r_L$, the government will always be incited to deviate from the candidate.

- When Nature chooses $pD = p_H D$.

Without any deviation from such an action, the government will obtain $U_H(\underline{b};\overline{a}) = -\underline{b}.\Pi + (1-p_H)\overline{a}\,\underline{b}.\Pi + l.\overline{a}.p_H.(\underline{b}.\Pi - D)$. If they decide to deviate and then choose the high extra-charge policy $\overline{b}$, they must consider the action $a(n)$ of insurers out of the equilibrium and they will obtain $U_H(\overline{b};a(n)) = -m - \overline{b}.\Pi + a(n).(\overline{b}.\Pi - p_H.D)$.

Therefore, the government has no interest in deviation from $(\underline{b};\overline{a})$ if and only if $U_H(\underline{b};\overline{a}) \ge U_H(\overline{b};a(n))$, that is:

$$a(n) \le \frac{D.(1-p_L) + (1-p_H)\overline{a}.p_L D - l\,\overline{a}.D.p_H.(1-p_L)}{D.(1-p_H)}$$

That is, $a(n) \le \overline{a}.p_L + \dfrac{(1-p_L).(1-l\,\overline{a}.p_H)}{(1-p_H)} = r_H$

So, $U_H(\underline{b};\overline{a}) \ge U_H(\overline{b};a(n))$ if $a(n) = r_H$.

As shown above, as soon as $\underline{a} \le r_H$, we can take $n(p_L D)\setminus b) = 0 = 1 - n(p_H D)\setminus b)$ out of the equilibrium in order to lead insurers to play the strategy low cession rate: in that case the government would not have any interest in deviation from the equilibrium and vice versa. Note that as soon as $r_H > \overline{a}$, we can that any belief n out of the equilibrium. The insurers would always play an action out of equilibrium that would lead the government not to deviate from the equilibrium. Reciprocally, to say that the government has not interest in deviation implies that the insurers' cession rate out of equilibrium is lower than a given rate, here r_H.Conversely, if $\underline{a} > r_H$, the government will always be incited to deviate from the candidate.

To conclude, if $\underline{a} \le \min\{r_L; r_H\}$ and $n(p_H D \setminus b) = 1$ for insurers' beliefs out of the equilibrium, the proposition 1 is proven. _

Proof of proposition 2.

- When the government chooses the low level of extra-charge rate $\underline{b}$ for $p_L D$.

As $\Phi(p_L D \setminus \underline{b}) = 1$, the insurers choose the level of cession $a \in [\underline{a};\overline{a}]$ which maximizes $V(a;\underline{b};p_L D) = (1-a).(\underline{b}\Pi - p_L D) - \frac{k}{2}(1-a)^2 p_L.(1-p_L).D^2 + m$.

As $\underline{b}.\Pi = p_L D$, the utility of insurers is strictly increasing with $a \in [0;1]$. Therefore, the private insurance industry will choose to transfer the largest part of the risks to the CCR, that is the action $\overline{a}$.

The member of the government know that if they deviate in playing $\overline{b}$, they will have to take into account the insurers' beliefs out of the equilibrium $(n;1-n)$ and action $a(n)$. It is never in the government's interest to deviate considering the insurers' beliefs and action out of equilibrium $a(n)$ if $U_L(\underline{b};\overline{a}) \geq U_L(\overline{b};a(n))$.

The demonstration is the same as for the corresponding case in proposition 1 above. It needs $a(n) \leq r_L$. As soon as $\underline{a} \leq r_L$, by taking $n(p_L D \setminus b) = 0 = 1 - n(p_H D \setminus b)$ out of the equilibrium insurers will be led to play the action low cession rate. In that case the government would not have any interest in deviation from the equilibrium.

- When the government chooses the high level of extra-charge rate $\overline{b}$ for $p_H D$.

As $\Phi(p_H D \setminus \overline{b}) = 1$, the insurers choose the level of cession $a \in [\underline{a};\overline{a}]$ which maximizes $V(a;\overline{b};p_H D) = (1-a).(\overline{b}.\Pi - p_H D) - \frac{k}{2}(1-a)^2 p_H.(1-p_H).D^2 + m$.

Here, as the utility function of the insurers is concave with a $(V''_a = -k.p_H.(1-p_H).D^2 < 0)$, the first order condition is sufficient for the maximization and is given by $\left(V'_a(a;\overline{b};p_H D)\right)_{a=a^*} = 0$, which can be written $a^* = 1 - \frac{(\overline{b}.\Pi - p_H D)}{k.p_H.(1-p_H)D^2}$.

As $a \in [\underline{a};\overline{a}]$, there are three cases to discuss: (i) $a^* \geq \overline{a}$; (ii) $a^* \in]\underline{a};\overline{a}[$, and (iii) $a^* \leq \underline{a}$. In proposition 2, we are interested only in the case in which the insurers choose the lowest possible rate offered by the CCR, that is $\underline{a}$. As the utility function is concave, this only happens in the third case (iii).

Writing the inequality (iii) $a^* = 1 - \frac{(\overline{b}.\Pi - p_H D)}{k.p_H.(1-p_H)D^2} \leq \underline{a}$ in order to show the HRS payment, it becomes : $(\overline{b}.\Pi - p_H D) \geq (1-\underline{a})k.p_H.(1-p_H)D^2$.

This general expression of the trade-off can be written (with the simple assumption $\bar{b}.\Pi = D$) : $k \le \frac{1}{(1-\underline{a})\, p_H .D}$.

So, the necessary and sufficient condition for the insurers to play the action $\underline{a}$ in that case is that this inequality holds (i.e the insurers have to be not too risk averse).

Therefore, it remains to be shown that the government has no interest in deviating from the action $\bar{b}$, that is $U_H(\bar{b};a) \ge U_H(\underline{b};a(n))$ where $a(n)$ would be the action played by the insurers taking into account of their beliefs n out of the equilibrium.

$U_H(\bar{b};\underline{a}) \ge U_H(\underline{b};a(n))$ can be written as follows:
$-\bar{b}.\Pi + \underline{a}.(\bar{b}.\Pi - p_H D) \ge -\underline{b}.\Pi + (1-p_H)a(n).\underline{b}\Pi + .\,l\,..a(n).p_H(\underline{b}.\Pi - D)$.

With $\underline{b}.\Pi = p_L D$ and $\bar{b}.\Pi = D$, the inequality becomes $\frac{(1-p_L)-\underline{a}.(1-p_H)}{l.p_H.(1-p_L)-(1-p_H)p_L} \le a(n)$.

We note r_{sep} such a lower-bound. As soon as $\bar{a} \ge r_{sep}$, we can take $n(p_L D \setminus b) = 1$ out of the equilibrium in order to lead insurers to play the action high cession rate out of the equilibrium: in that case the government would not have any interest in deviating from the equilibrium. Note that as soon as $\underline{a} \ge r_{sep}$, we can choose any belief n out of the equilibrium. The insurers would always play an action out of equilibrium that would lead the government not to deviate from the equilibrium. Reciprocally, say that the government has not interest in deviation induces that the insurers' cession rate out of equilibrium is higher than a given rate, here r_{sep}. Conversely, if $\bar{a} \le r_{sep}$, that equilibrium falls.

References

Akerlof, George. (1970). "The market for lemons: qualitative uncertainty and the market mechanism", *Quarterly Journal of Economics*, 74, 488-500.

Borch, Karl. (1968). Economics of Uncertainty, Princeton: Princeton University Press.

Borch, Karl. (1990). "Economics of Insurance". In: K.K. Aase and A. Sandmo, eds. Amsterdam: North-Holland).

Browne, Mark J., and Robert E. Hoyt. (2000). "The Demand for Flood Insurance: Empirical Evidence", *Journal of Risk and Uncertainty*, 20:3, 291-306.

CCR. (2000). Annual report. Paris.

Cox, Samuel, Joseph Fairchild, and Hal W. Pedersen. (2000). "Economic Aspects of Securitization of Risk", *Austin Bulletin*, 30(1): 157-193.

Camerer, Colin. and Howard Kunreuther. (1989). « Decision Process For Low Probability Events : Policy Implications », *Journal of Policy Analysis and Management*, 8, 565-592,

Cummins, David., David Lalonde, and Richard, Phillips. (2001). "The Basis Risk of Catastrophic-Loss Index Securities", paper presented at the 3rd NBER Insurance Project Workshop, February 16, 2001, Cambridge (MA).

Cummins, David. J., Christopher M. Lewis, and Richard D. Phillips. (1999). "Pricing Excess-of-Loss Reinsurance Contracts against Catastrophic Loss". In: Kenneth Froot (ed.), The Financing of Catastrophe Risk. Chicago: Chicago University Press, 93-141.

D'Arcy, Stephen. P., and V. Grace France. 1992. "Catastrophe Futures: A Better Hedge for Insurers", *Journal of Risk and Insurance*, 59:4, 575-601.

De Marcellis, Nathalie. (1997). “Catastrophes naturelles et systèmes d’indemnisation : évaluations et perspectives”. In *La prévention des risques naturels*, Rapport du Comité interministériel de l’évaluation des politiques publiques. Paris: La Documentation Française.

De Marcellis, Nathalie, and Erwann Michel-Kerjan. (2000). "Tempête ou catastrophe naturelle? Implications sur les systèmes d'indemnisation", *Revue Préventique*, 49.

Dionne, Georges. and Scott Harrington. (1990). Foundations of Insurance Economics: Readings in Economics and Finance. London: Kluwer Academic Publishers.

Dionne, Georges., Neil. Doherty and Nathalie Fombaron. (2000). "Adverse Selection in Insurance Markets", *Working paper* 00-05, Chaire de gestion des risques, Montreal: HEC.

Froot, Kenneth, (2001). “The market for catastrophe risk: a clinical examination”, *Journal of Financial Economics*, 60: 2-3, 529-571.

Froot, Kenneth, ed. (1999). The Financing of Catastrophe Risk. Chicago: Chicago University Press.

Fudenberg, Drew., and Jean Tirole. (1991). Game Theory. Cambridge, MA: MIT Press.

Ganderton, Philip. T., David S. Brookshire, Michael McKee, Steve Stewart, and Hale Thurston. (2000). "Buying Insurance Disaster-Type Risks: Empirical Evidence", *Journal of Risk and Uncertainty*, 20:3, 271-289.

Gray, William J. (1990). "Strong association between West African rainfall and U.S. land-fall of intense hurricanes", *Science*, 249: 1251-56.

Harrington, Scott, and Greg Niehaus. (1999). “Basis Risk with PCS Catastrophe Insurance Derivative Contracts”, *Journal of Risk and Insurance*, 66(1): 49-82.

Henriet, Dominique, and Erwann Michel-Kerjan. (2001). "Natural Catastrophes and Governmental Aid. Impact on the Strategy of Municipalities at Risk", Unpublished Working Paper, Marseilles: Institut d’Economie Publique.

Jaffee, Dwight., and Thomas Russell. (1997). "Catastrophe Insurance, Capital Markets, and Uninsurable Risks", *Journal of Risk and Insurance*, 64(2): 205-230.

Jullien, Bruno. (1998). “Analyse Economique du Système Cat. Nat.”, *Risques*, 34, 63-68.

King, Rawle. (1993). Insurance Markets After Hurricane Andrew. Washington, DC: Congressional Research Service.

Kleindorfer, Paul. R., and Howard C. Kunreuther. (1999). "Challenges Facing the Insurance Industry in Managing Catastrophic Risks". In: Kenneth Froot (ed.), The Financing of Catastrophe Risk. Chicago: Chicago University Press, 149-189.

Kunreuther, Howard. C. (2000) "Insurance as a Cornerstone for Public-Private Sector Partnerships", *Natural Hazards Review*, 1: 126-136.

Kunreuther, Howard. C. (1997). “Rethinking Society's Management of Catastrophic Risks”, *Geneva Papers on Risk and Insurance-Issues and Practices*, 22(83): 151-176.

Kunreuther, Howard. C. (1996). “Mitigating Disaster Losses through Insurance”, *Journal of Risk and Uncertainty*, 12: 171-187.

Lagadec, Patrick., and Erwann Michel-Kerjan. (2000). “D'un continent à l'autre”, in the French newspaper *Le Monde*, 11th January, 17.

Lewis, Christopher., and P. Davis. (1998). "Capital market instruments for financing catastrophe risk : New directions ?", Working paper. Philadelphia: Wharton School, University of Pennsylvania.

Lewis, Christopher. M., and Kevin. C. Murdock. (1996). "The Role of Government Contracts in Discretionary Reinsurance Markets for Natural Disasters", *Journal of Risk and Insurance*, 63: 567-597.

Loubergé, Henri., and Harris Schlesinger. (2001). “Optimal Catastrophe Insurance”, mimeo, University of Geneva.

Magnan, Serge (1995). “Catastrophe Insurance System in France”, *Geneva Papers on Risk and Insurance-Issues and Practices*, 20:77, 474-480.

Michel-Kerjan, Erwann. (2001). "Insurance against Natural Disasters: Do the French Have the Answer? Strengths and Limitations", *Cahier du Laboratoire d'économétrie*, 2001-007, Paris: Ecole Polytechnique.

Moss, David. A. When All Else Fails: Government as the Ultimate Risk Manager, forthcoming. Cambridge, MA: Harvard University Press.

Moss, David. A. (1999). "Courting Disaster? The Transformation of Federal Disaster Policy since 1803". In: Kenneth Froot (ed.), The Financing of Catastrophe Risk. Chicago: Chicago University Press, 307-355.

Munier, Bertrand. (1997), “Réflexion sur la prévention des risques naturels et l’indemnisation des dommages”. In *La prévention des risques naturels*, Rapport du Comité interministériel de l’évaluation des politiques publiques. Paris: La Documentation Française.

Niehaus, George., and Steven. V. Mann. (1992). "The Trading of Underwriting Risk: An Analysis of Insurance Futures Contracts and Reinsurance", *Journal of Risk and Insurance*, 59:4, 601-627.

Pasterick, Edward T. (1998). "The National Flood Insurance Program" In: Howard Kunreuther and Richard J. Roth (eds.), Paying the Price, The Status and Role of Insurance Against Natural Disasters in the United States. Washington: Joseph Henry Press, 125-154.

Petak, William. (1998). "Mitigation and Insurance". In: Howard Kunreuther and Richard J. Roth (eds.), Paying the Price, The Status and Role of Insurance Against Natural Disasters in the United States. Washington: Joseph Henry Press, 155-170.

Priest, George. L. (1996). "The Government, the Market, and the Problem of Catastrophic Loss", *Journal of Risk and Uncertainty*, 12: 219-237.

Schlesinger, Harris. (1999). “Decomposing Catastrophic Risk”, *Insurance: Mathematics and Economics*, 24: 95-101.

Swiss Reinsurance Company. (2000). "Natural catastrophes and man-made disasters in 1999: Storms and earthquakes lead to the second-highest losses insurance history." In *Sigma* 2. Zurich Swiss Re.

Vandamme, René. (1998). “Enjeux et défis de la réassurance en France”, *Risques*, vol. 34.

Notes.

[1] The tragic events of September 11, 2001 and their consequences lead such a question to be deepest relevant than ever for both the public sector and the (re)insurance industry. That relevance is truth not only for acts of God such as natural disasters but also for all large-scale risks, whatever their origins.

[2]For a recent survey of adverse selection phenomena in insurance, see Dionne, Doherty and Fombaron, 2000.

[3] One of the main drawbacks of the proposed program may be this delayed settlement: Are insurance and reinsurance companies able to wait for such a long period of time to receive the payment from the government?

[4] For recent articles on the subject, see for instance Kleindorfer and Kunreuther (1999), Petak (1998).

[5] In 1981, companies began to cover natural events which they considered « insurable ». An insured party could ask the insurance company for an extra-policy to be insured against those events. Nevertheless, only a small part of people were specifically insured against these risks. Such a situation forced the government to vote a law (law of the 25 June 1990) making storm insurance compulsory.

[6] See the discussion and criticism of the present French risk analysis and mitigation programs in Michel-Kerjan (2001).

[7] The CCR’s equalization reserves are assumed to be zero in the model. The introduction of such a parameter makes the model more realistic but leads to results difficult to interpret economically.

[8] For this, the literature on risk-managers’ choices between mitigation and reparation can help us. Mitigation measures, which permit the reduction of loss probability *ex ante*, are often less costly than a reparation *ex post*. The government may prefer spend public expenditures to improve mitigation programs.

[9] In the literature, insurers are often considered to be risk neutral. For catastrophic risks, the reinsurance demand explicitly shows explicitly their risk aversion. We use a mean-variance utility function because it is easily manipulated and it provides good intuition. But we know that this type of function has an undesirable property: it does not respect the stochastic dominance first-order condition (Borch, 1968). However, with a more correct version of the utility function, calculus are much more complicated and results are similar.

[10] This choice, in the event of a unique level of catastrophic damage, is not efficient from an insurance theory point of view (Borch, 1990). Moreover, here we consider the situation for which the disaster losses are partially financed through extra-charge and the government provides financial assistance to the total extra-charge.

[11] By taking into account of the CCR's equalization reserve R, the inequality becomes $\bar{b}.\Pi + R - a D \geq 0$. So the government has to define a high extra-charge policy higher than $D - R/a$, with alpha the action decided by the insurers.

[12] For a rigorous definition and complete discussion of perfect Bayesian equilibrium, see Fudenberg and Tirole (1991).

[13] All proofs are put in appendix. This proof is in the main text to help the reader understanding the notation.

[14] If we consider the CCR's equalization reserves noted R, the bound are quite complicated. For instance, $r_L = 1 - \frac{[l\, \bar{a}.D(1-p_L) + (1-l).R)]\, p_L.\bar{a}}{\bar{a}.D(1-p_L) - R} + \frac{\bar{a}^2.D(1-p_L).p_L}{\bar{a}.D(1-p_L) - R}$ and $r_H = \frac{(\bar{a}.D(1-p_L) - R) - [l\, \bar{a}.D(1-p_L) + (1-l).R)]\, p_H.\bar{a} + \bar{a}^2.D(1-p_H).p_L}{\bar{a}.D(1-p_H) - R}$.

[15] Proprietors who rent an apartment or house have to verify that their tenants have purchased liability insurance: this is a pre-requisite for renting.

[16] Moss suggests that a system similar to the French one could be established in the United-States. The introduction of an extra-charge on every US property insurance policy, with a rate varying between 5 and 20% (with an average of about 13%) and a two-thirds of the basic extra-charge on base premiums for motor vehicles is studied. According to the author (1999, pp.346-347), "if these surcharges had been applied to all property and casualty lines [as in France], the surplus at the end of 1993 would have totaled $148 billion. "

[17] The mathematical proofs of these arguments are not presented here in order to facilitate reading.

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